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Geometry (50, 52, 53)

78T-D7 Alan H. Schoen, Southern Illinois University, Carbondale, Illinois 62901. Penrose SUN and STAR patterns Preliminary report.

The Penrose SUN and STAR patterns are described by Martin Gardner (Sci. Amer. 236, No. 1, Jan., 1977, pp. 110-121) as the two infinite patterns, composed of kites and darts, which are generated "if you add pieces [to the SUN or STAR] so that pentagonal symmetry is always preserved". The following algorithm defines a recursive scheme for constructing either SUN or STAR pattern, given a central core for the pattern. Algorithm: Define C_n (n is a positive integer), a simply connected region tiled with kites and darts, which satisfies: (i) the tiling in C_n has D_5 symmetry; (ii) C_n is enclosed by a cyclic chain of five worm-segments $p_n(i, i+1)$ (i=1,2,..,5 in modulo 5 arithmetic, both here and below) whose long axes coincide respectively with the edges $u_n(i, i+1)$ of a regular pentagon P_n whose vertices are numbered consecutively from 1 to n; (iii) along the edges $v_n(i-1, i+1)$ of a regular pentagram Q_n, inscribed in P_n, lie the long axes of five worm segments $q_n(i-1, i+1)$; (iv) the tiling in each "triangular" domain $T_n(i)$, which is enclosed by $p_n(i, i+1)$, $q_n(i-1, i+1)$, and $q_n(i-2, i)$, is related by reflection in $v_n(i-1, i+1)$ to the tiling in a congruent domain. To expand the pattern, reflect C_n and the four $p_n(j, j+1)$ for which $j \neq i$, in $u_n(i, i+1)$; define vertex i of P_{n+1} as the image of vertex i+2 of P_n obtained by reflection in $u_n(i-1, i)$; define $u^*_n(i, i+1,i)$ as the image of $u_n(i-1, i)$ obtained by reflection in $u_n(i, i+1)$; the tiling in the gap $g_n(i, i+1)$ at the center of each $p_{n+1}(i, i+1)$ and also in the contiguous gap $G_n(i, i+1)$ is related by reflection in $u^*_n(i, i+1;i-1,i)$ to the tiling in a congruent domain. Corollary: The sequential arrangement of long (L) and short (S) bow-ties in the skeletal worm segments described above is given by:

$p_n = p_{n-1}\ L\ q_{n-2}\ L\ p_{n-1}$, and $q_n = q_{n-1}\ L\ p_{n-1}\ L\ q_{n-1}$ (STAR pattern);

$p_n = p_{n-1}\quad q_{n-2}\quad p_{n-1}$, and $q_n = q_{n-1}\quad p_{n-1}\quad q_{n-1}$ (SUN pattern);

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