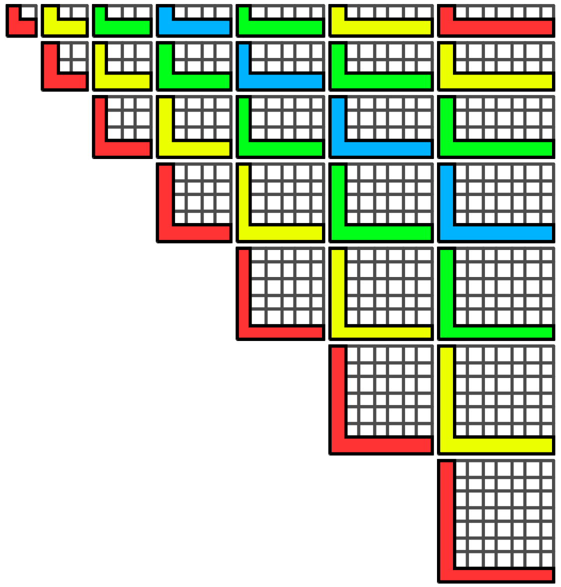
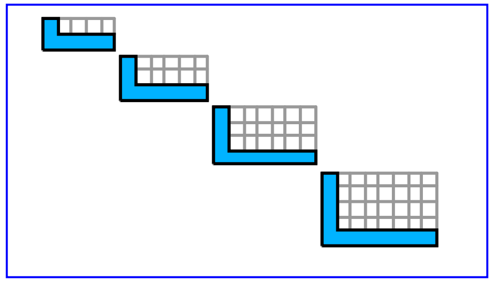


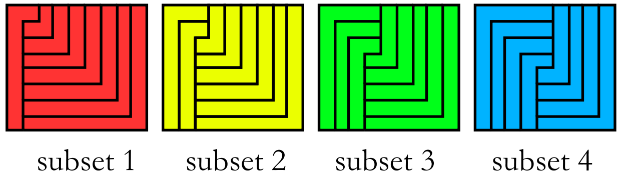
LOMINOES©

The twenty-eight **LOMINOES** of the *standard set* L8 (left) include one specimen of every L-shaped *polycube* with arms of unit square cross-section that can be cut from a 1 x 8 x 8 grid of cubes. The *augmented set* L8† contains four additional pieces (below) that are duplicates of the **LOMINOES** in the central NW/SE diagonal strip of L8.

L8 +  = L8†

By removing one or more columns of pieces from the right side of the triangular array (above left), one can construct any standard set of **LOMINOES** for $2 \leq n < 8$. Every standard set Ln of **odd** order n can be arranged to tile $(n-1)/2$ rectangles with proportions 1 x n x $(n+1)$. These tilings are called *pronic rectangular subsets*. Every standard set Ln of **even** order n can, like L8, be transformed into the corresponding augmented set by adding duplicates of the $n/2$ pieces in the central NW/SE diagonal strip of Ln, and this augmented set can then be partitioned into $n/2$ pronic rectangular subsets.

At right are the four 1 x 8 x 9 *canonically colored* pronic subsets into which L8† can be partitioned.

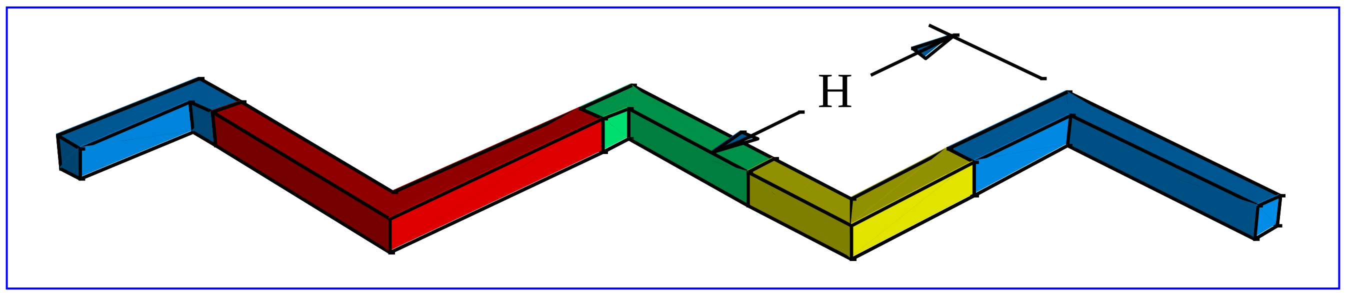


LOMINOES sets offer a variety of 1D, 2D, and 3D puzzle challenges that are described below, but you will undoubtedly discover your own new ways to use them.

A. **1-DIMENSIONAL PUZZLES**

A1. **SAWTOOTHS**

Arrange the thirty-two pieces of L8† to tile a **SAWTOOTH**, a five-piece segment of which is shown below. The SAWTOOTH **slant height *H*** is equal to ten units. (The *unit of length* is **1/2 inch,** the thickness of each LOMINO.)



Construct a SAWTOOTH as a one-dimensional ***four-color map*** tiling. (No two adjoining pieces are of the same color.)

A2. **FENCES**

A FENCE is a circuit, free of self-intersections, composed of LOMINOES laid end-to-end. A FENCE is called *self-avoiding* if every piece is incident only on the two pieces at its ends, and *non-self-avoiding* otherwise. The *self-avoiding* FENCE at the right is composed of all the pieces of L8†. The area enclosed by this fence is equal to 471.

A FENCE is called complete if it is composed of all of the pieces of a single L*n* or L*n*† set, and *incomplete* otherwise.

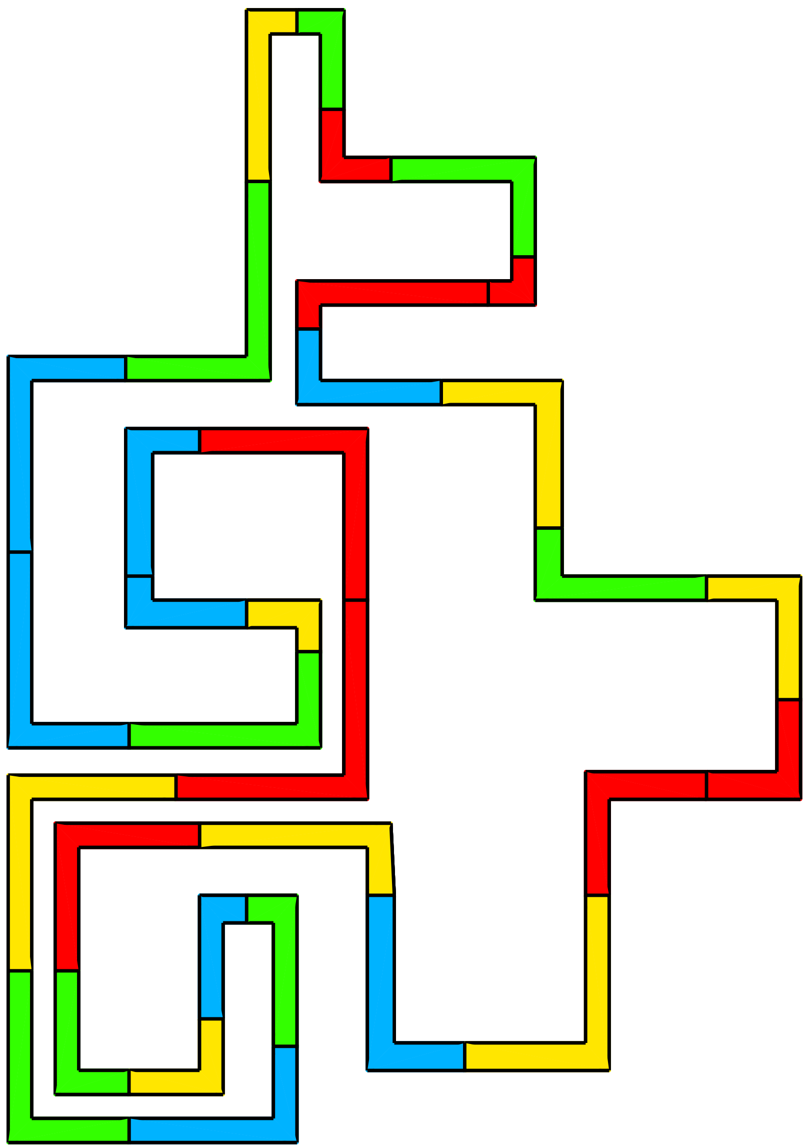
A FENCE is called a **CORRAL** if it is composed of all of the pieces of one pronic set (*cf.* p. 1).

1. What are the maximum and minimum values of the area that can be enclosed by a *complete self-avoiding* FENCE tiled by (a) L8? (b) L8†?

2. Using the eight LOMINOES of either pronic subset 2 or 4, tile a CORRAL.

3. Prove that it is impossible to tile a CORRAL with the eight LOMINOES of subset 1 or 3.

4. Construct a **MATCHED FENCE** from the pieces of L8 – a self-avoiding FENCE in which the two contiguous arms of every pair of adjacent LOMINOES are of the same length.

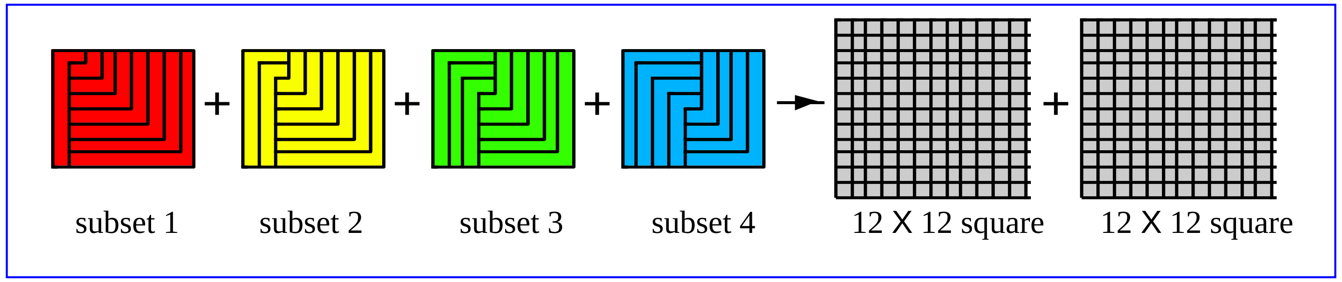


L8† FENCE

B. **2-DIMENSIONAL PUZZLES**

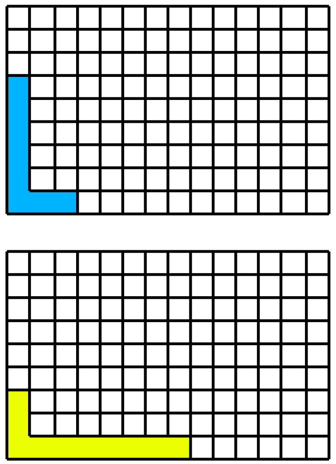
B1. **SQUARES**

Arrange the thirty-two pieces of L8† to tile **two 12 X 12 squares**.

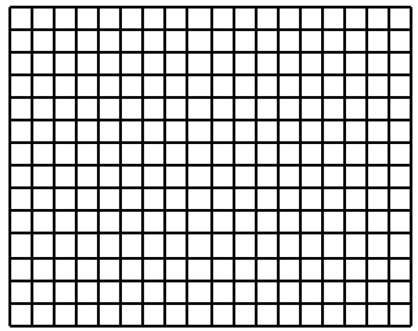


B2. **RECTANGLES (both filled and holey)**

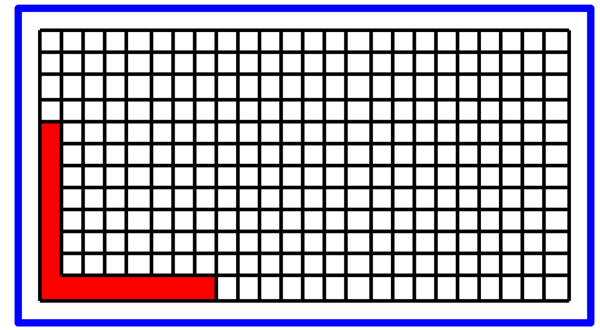
Tile each of the rectangular arenas shown below. The tiling **bordered in blue** requires all thirty-two pieces of the augmented set L8†. Each of the other tilings requires only the twenty-eight pieces of the standard set L8. Try the 14 x 18 rectangle first. It is the least difficult of these three challenges. The holey cases are somewhat more difficult than the filled ones.



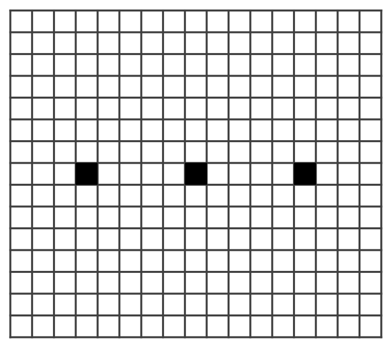
Two 9 x 14 rectangles
One L8 set



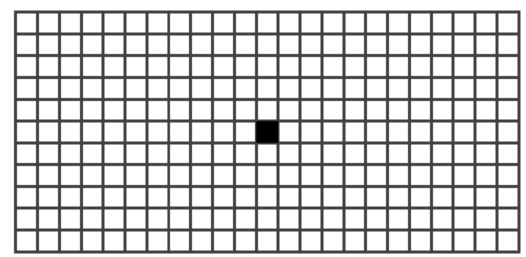
14 x 18 rectangle
One L8 set



12 x 24 rectangle
One L8† set



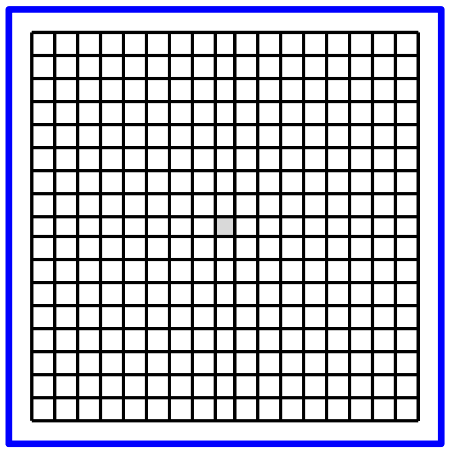
15 x 17 holey rectangle
One L8 set



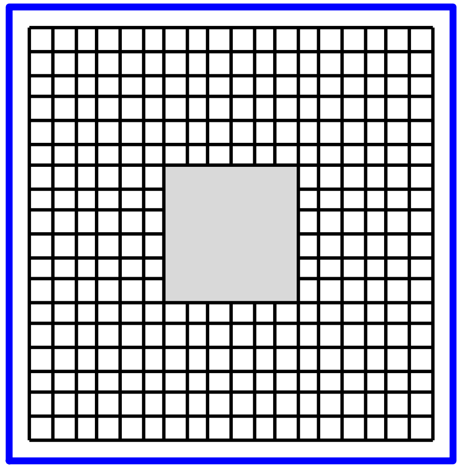
11 x 23 holey rectangle
One L8 set

B3. **SQUARE ANNULI**

Each of these two tilings requires the thirty-two pieces of the augmented set L8†.

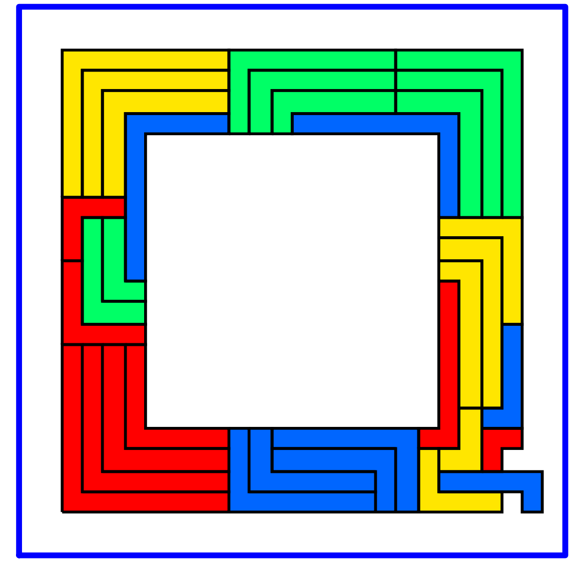


17^2 - 1^2 annulus
One L8† set



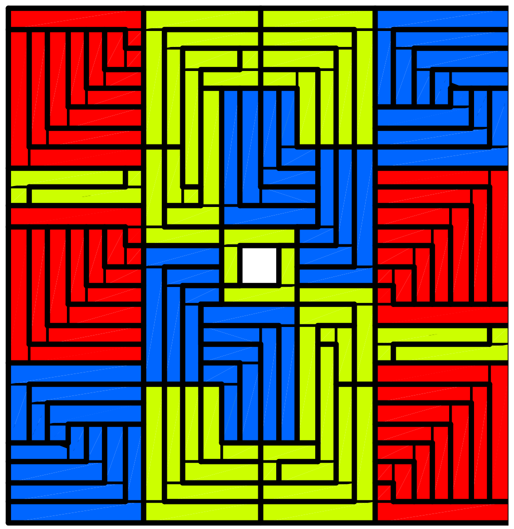
18^2 - 6^2 annulus
One L8† set

In 2009 George Bell succeeded in finding a tiling of the 22^2-14^2 **square annulus**, using one L8† set. An earlier unsuccessful attempt of mine, which is illustrated at right with canonically colored pieces, includes several large monochromatic sub-assemblies. In George's tiling there are no such orderly sub-assemblies!

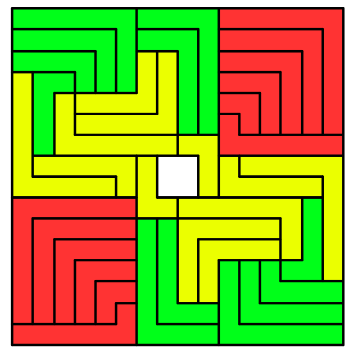


The 22^2 - 14^2 square annulus
not quite tiled by one L8† set

Using two L8† sets, tile the 24 x 24 **square** in a *C2-symmetric* pattern. Two examples of tilings with C2 symmetry are shown below. One is the 26^2 - 2^2 square **annulus** tiled by four L7 sets; the other is the 16^2 - 2^2 square **annulus** tiled by two L6† sets.

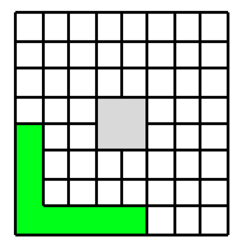


26^2 - 2^2 square annulus
Four canonically colored L7 sets

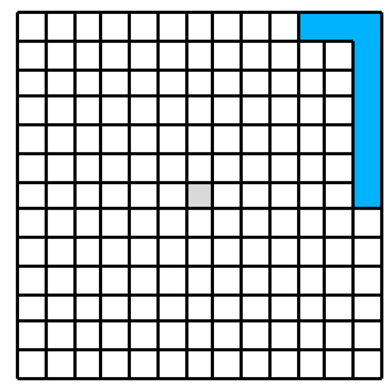


16^2 - 2^2 square annulus
Two L6† sets (30 pieces)
canonically colored

Two easy tilings (right): the 8^2 - 2^2 square annulus tiled by one L5 set and the 13^2 - 1^2 square annulus tiled by one L7 set.



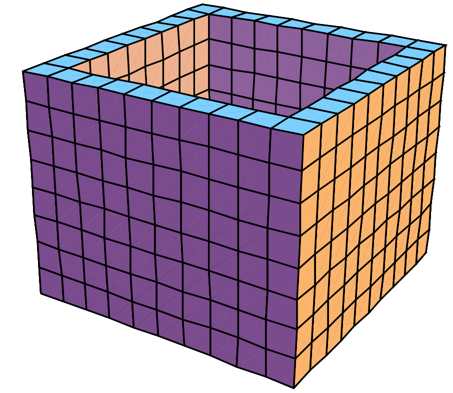
8^2 - 2^2 square annulus
One L5 set (10 pieces)



13^2 - 1^2 square annulus
One L7 set (21 pieces)

C. 3-DIMENSIONAL PUZZLES

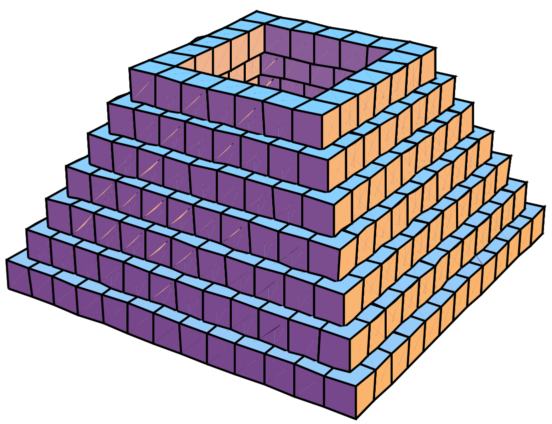
C1. A **TOWER** is a collimated stack of 4-rings (square annuli composed of four LOMINOES) of overall width (‘ringwidth’) ten. Find packing solutions for (a) the eight-story TOWER $T_1[8^\dagger|10,10]_8$ (right) using the thirty-two pieces of L8†, and also for (b) the seven-story TOWER $T_1[8|10,10]_7$ using the twenty-eight pieces of L8. There are thousands of different packing solutions for each of these TOWERS. There are 960 packings of $T_1[8^\dagger|10,10]_8$ in which every 4-ring is *pied*, *i.e.,* composed of four differently colored LOMINOES. Try to find **one**! (*It’s not easy.*)



The eight-story TOWER

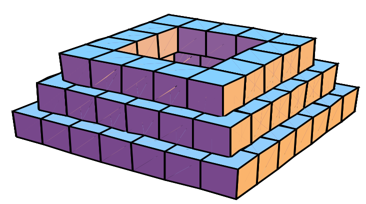
$T_1[8^\dagger|10,10]_8$, which is a stack of eight 4-rings

C2. TOWERS are much easier to pack than **ZIGGURATS**. The seven-story ZIGGURAT ${}_1[8|7,13]_1$ (below left) is a collimated stack of seven 4-rings, each composed of four of the twenty-eight pieces of L8. The top 4-ring has ringwidth seven and the bottom 4-ring has ringwidth thirteen.

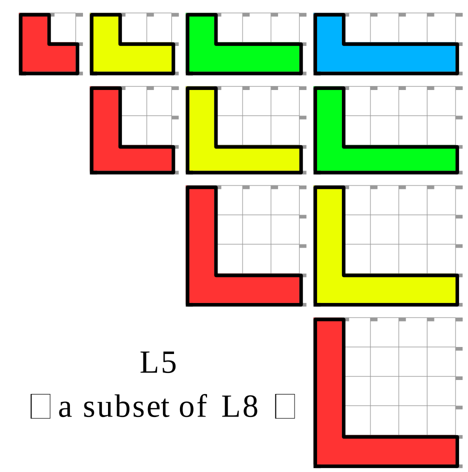


The L8 ZIGGURAT ${}_1[8|7,13]_1$, which is a stack of seven 4-rings

C3. For an easy warm-up exercise, try to pack the three-story ZIGGURAT ${}_1[5|5,7]_1$ (below) with the ten pieces of L5 (right). *There’s only one solution!*



The L5 ZIGGURAT ${}_1[5|5,7]_1$

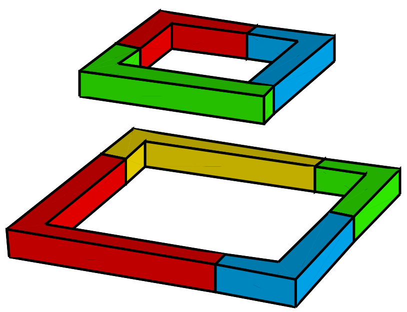


L5

□a subset of L8 □

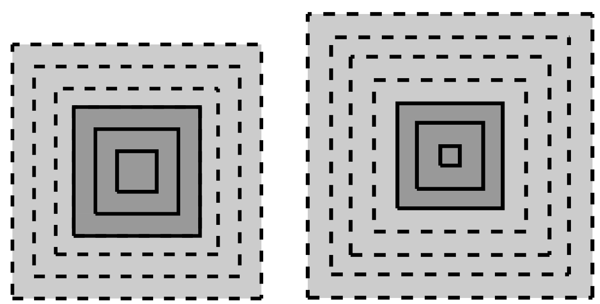
C4. Square rings of sufficiently small ringwidth require only two or three LOMINOES instead of four (*cf.* the two examples shown below left). Unlike the seven-story ZIGGURAT $_1[8|7,13]_1$ (*cf.* p. 5), the nine-story ZIGGURAT $_1[8|4,12]_1$ (right) includes 2-rings, 3-rings, and 4-rings. It is not easy to find a packing for $_1[8|4,12]_1$. It has only 59 solutions, while $_1[8|7,13]$ has 384 solutions. Figuring out where to place the 2-rings and 3-rings is just one of the difficult challenges.

You can top off either $_1[8|4,12]_1$ or $_1[8|7,13]_1$ with the gray **cap rings** that are included with the LOMINOES set) and thereby convert it into a genuine **pyramid** (bottom right). Cap rings also serve as **templates** for tiling the square rings used to construct ZIGGURATS, as explained below:



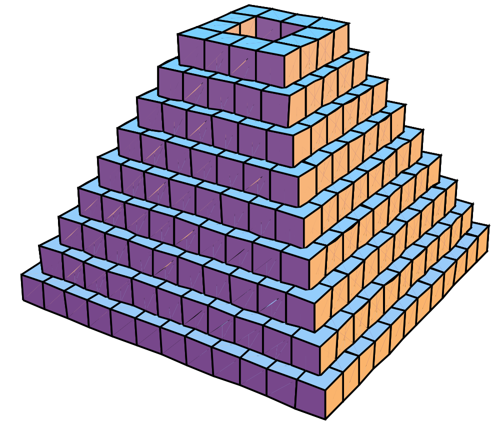
A 3-ring of ringwidth 7
and a 4-ring of ringwidth 11

To construct a ZIGGURAT, start in two dimensions, using the cap rings as **tiling templates** (**pattern cores**) for two sets of concentric square rings (*cf.* illustration below). Tile the square rings of even and odd ringwidths in separate arrays. After the square rings are completed, stack them in an alternating sequence to assemble the ZIGGURAT.

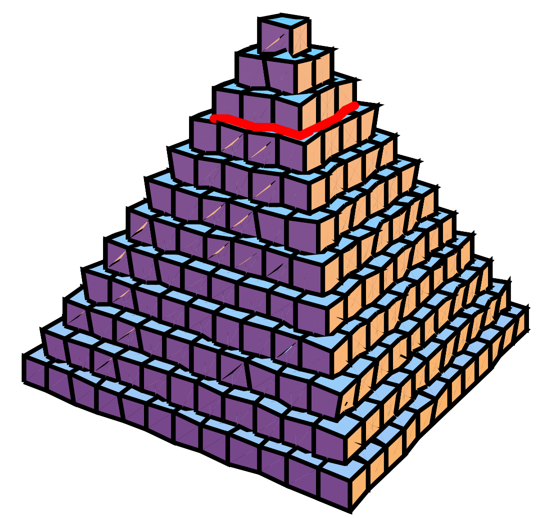


even square rings *odd* square rings

CAP RINGS (dark gray)
surrounded by the 4-rings (light gray) of $_1[8|7,13]_1$



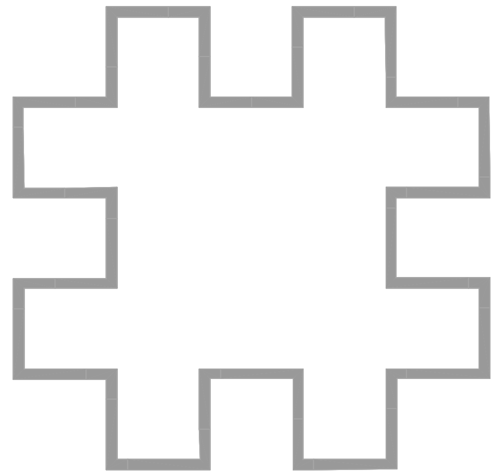
The L8 ZIGGURAT $_1[8|4,12]_1$
(uncapped)



The L8 ZIGGURAT $_1[8|4,12]_1$
(capped)

APPENDIX

The SAWTOOTH on p. 2 takes up lots of table space! It is convenient to transform it into a more compact shape by reversing the direction of the 90° turn at twelve of its twenty-eight corners. We call the resulting closed-circuit shape (right) a FILIGREE. (Any sequence of LOMINOES that works for one of these tilings also works for the other.)



For much more information about LOMINOES, including solutions for some of the puzzles described here, refer to the 138-page CD book.

Alan H. Schoen
schoenah@gmail.com

LOMINOES is manufactured in the U.S.A. from a closed-pore blend of polyethylene and ethylene vinyl acetate.

<u>WARNING</u>: It is strongly recommended that these LOMINOES (*<u>especially the smallest pieces</u>*) be kept away from very small children, because of the possibility of a choking hazard.