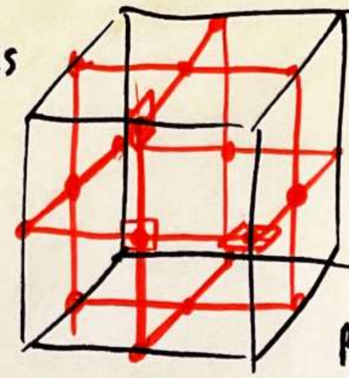
9 PM

5-30-67

4-(111) net appears to have 12 atoms in the basis, i.e., 12 nodes per unit cell.



My nodal polyhedron theorem for homogeneous nets implies that the nodal polyhedron (but not *necessarily* the Voronoi polyhedron, although I think that should work here also) will assume 12 different orientations in a space-filling arrangement, and that the symmetry of each nodal polyhedron, ~~[illegible]~~ *with respect to the* [point] *node on which it sits*, shall be the same as that *of the node in the net.*

---

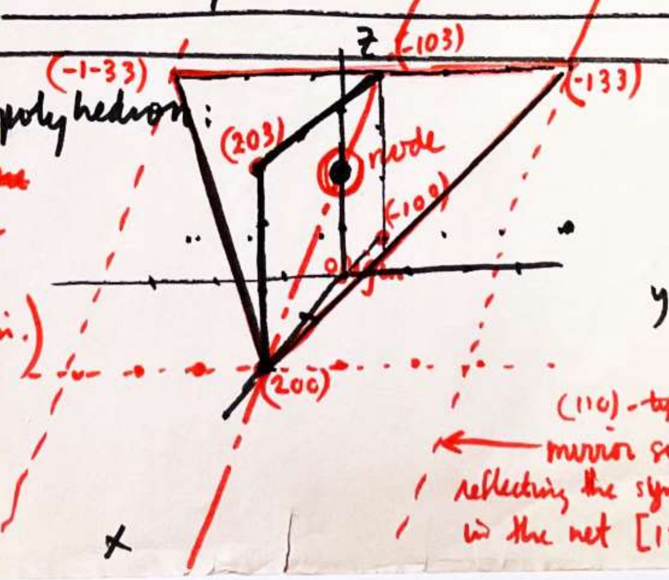
2 hours later

The nodal polyhedron satisfies the theorem!! This makes 17 cases of homogeneous nets, and they all satisfy the theorem. (5 are 2 dim., and 12 are 3-dimensional.) *I suspect the theorem is valid for homog. nets of all dimensions.*

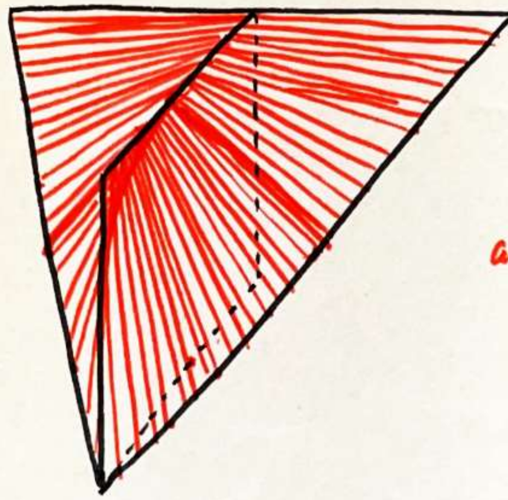
Here's the nodal polyhedron:

~~It's distinctly shown here~~

(It's shown on the next page with the node centered at the origin.)



(110)-type mirror symmetry plane, reflecting the symmetry of the node in the net [12 distinct configurations



This is the general appearance of the nodal polyhedron!

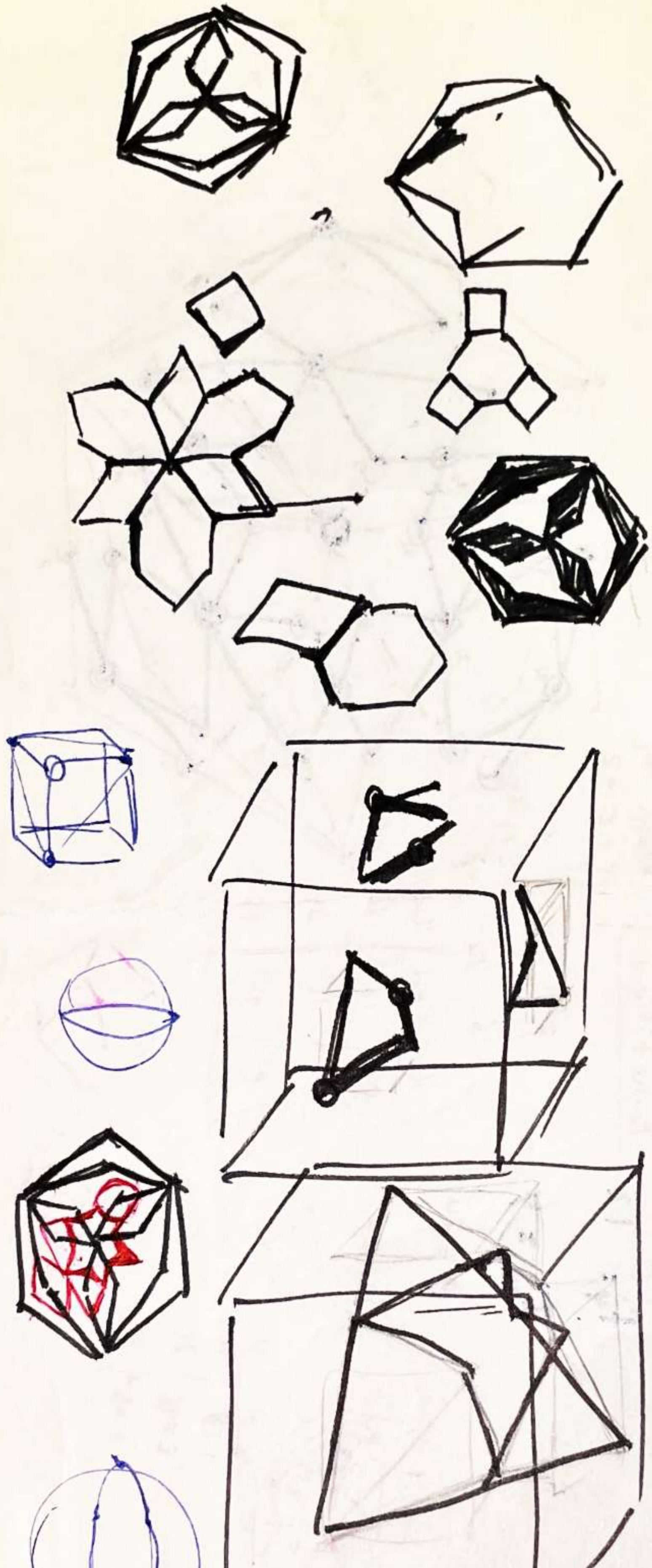
[The 2 front faces are shown. The two rear faces are omitted, for clarity.]

The nodal polyhedron theorem states:

All [17] homogeneous nets* in 2 and 3 dimensions satisfy the following: Span all closed polygons in the net with a minimal surfaces, starting with the polygons having the smallest number of sides, until the space is tesselated into closed polyhedral regions [which are not necessarily all of the same sort]. Then assign an "interstitial point" to the centroid of each of those closed "interstitial" polyhedral regions, or cavities. Join, with a straight line, the interstitial point in each cavity to those interstitial points which lie at the centers of adjoining interstitial polyhedral cavities (i.e., those cavities which share a common polygonal face). Space is then tesselated into identical space-filling nodal polyhedra, each one containing a single node of the original homogeneous net. The node does not necessarily lie at the centroid of the nodal polyhedron, but the nodal polyhedron exhibits the symmetry — with respect to the position of the node it contains — of the node in the net.

---

* A homogeneous net is defined [by me] to be an array of points which are symmetrically equivalent [i.e., each node can be mapped into every other by the same symmetry operation, plus a translation, aside from possible parity differences for different nodes], with symmetrically equivalent edges joining only nearest neighbor nodes



$f_6 = 4$

$f_4 = 6$

$F_{catalan} = C_{original}$

$C_{catalan} = F_{original} + E_{original}$ $= 2F_{original} + C_{orig} - 2$

$E = F + C + 2$

$f_4 = 12$

$f_{12} = 4$

$c = 34$

~~$e = 48$~~

~~24~~

$12 \cdot \left[2 + \frac{1}{3} + 1\right]$

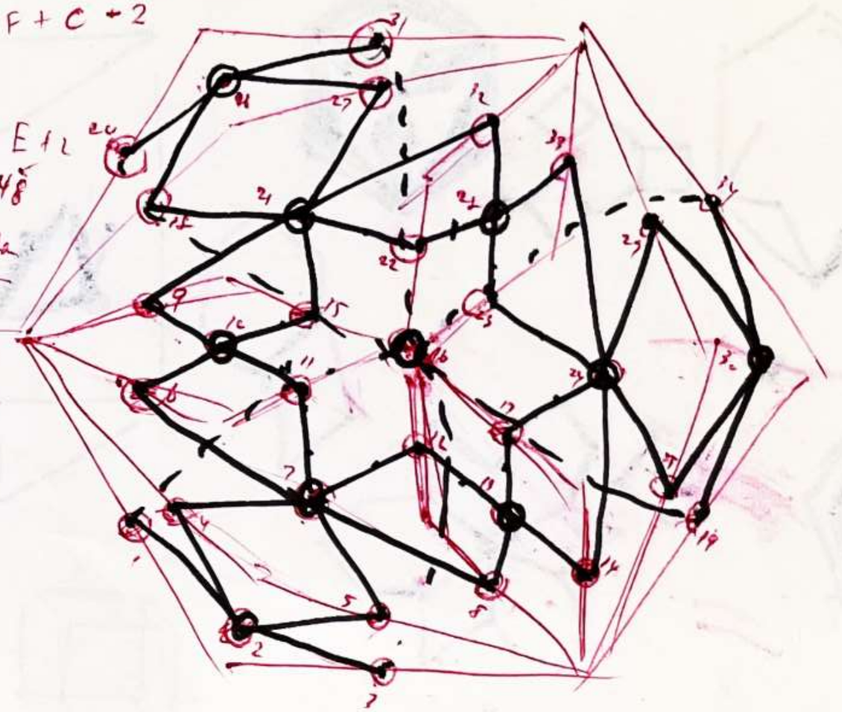
$= c = 40$

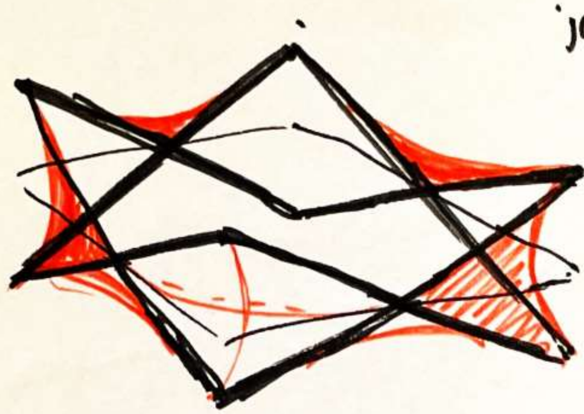
$F + C = E + 2$ ee

16 34 48

	Original dual	Catalan
f	10	16
c	16	34
e	24	48

ext-cut	Catalan
f = 8	18
c = 18	32
e = 24	48



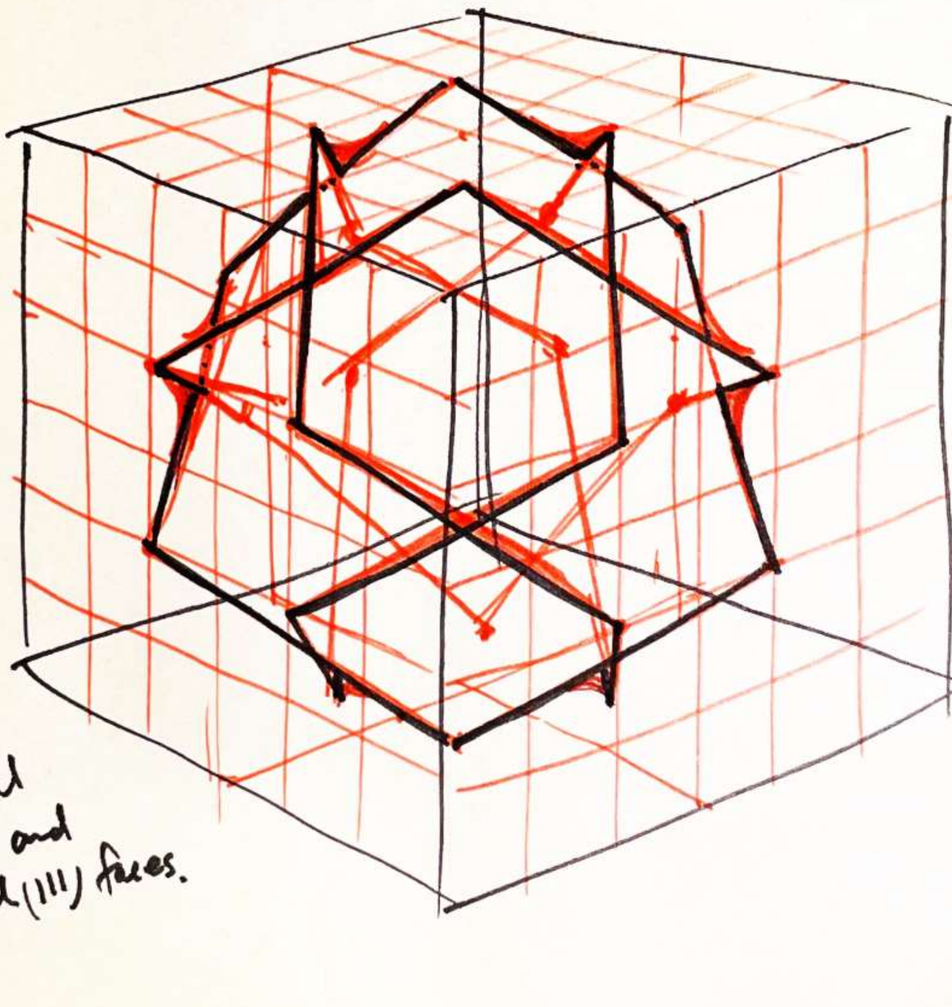


This beautiful hexahedron is the "glue" cavity that joins the cavities which are modifications of the truncated octahedron in the 4 : (111) net.

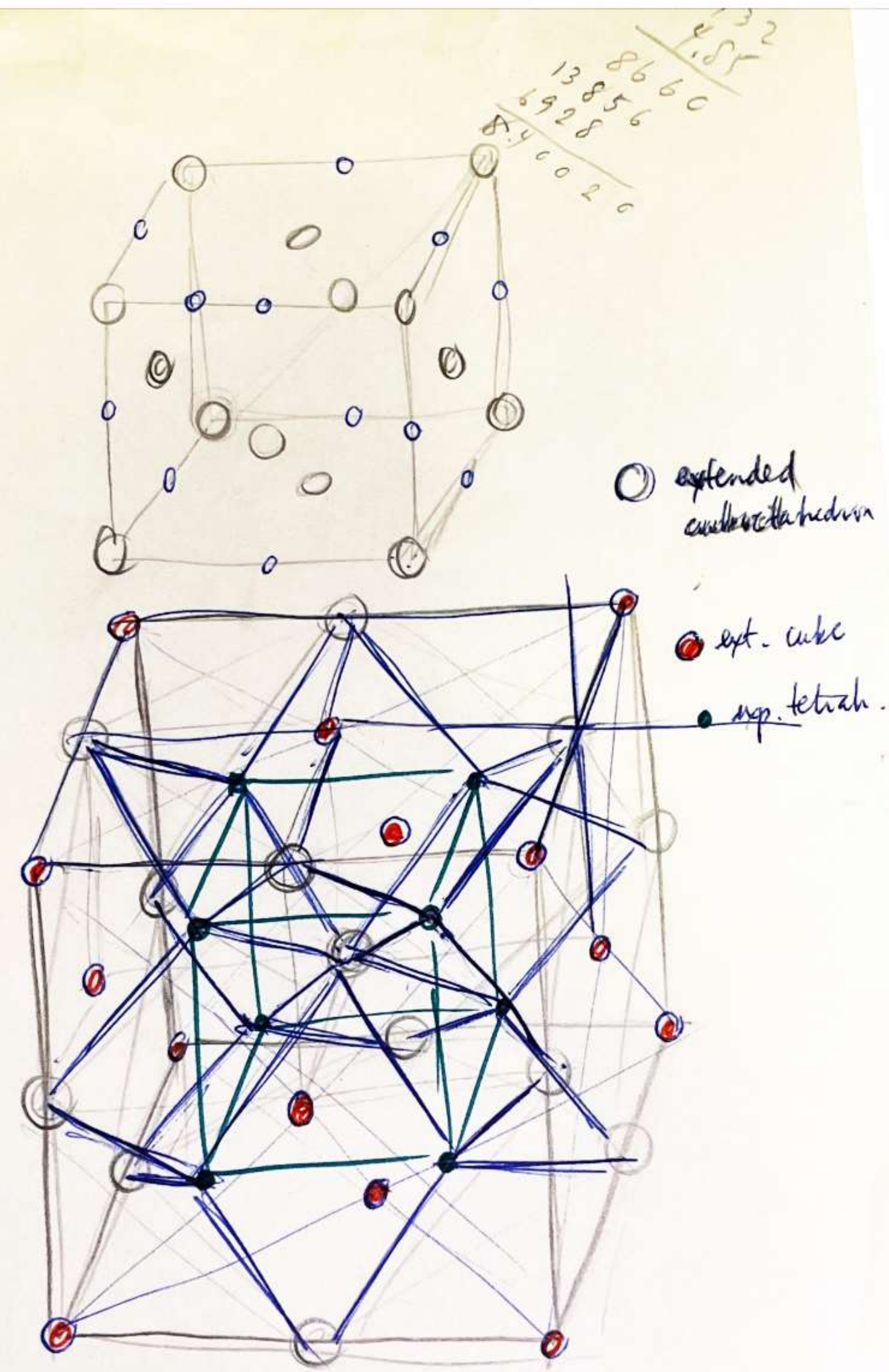
It is made up of the edges of two intersecting skew quadrilaterals, each of which consists of the 4 (111) edges of the tetragonal bisphenoid.

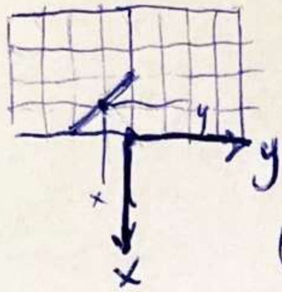
---

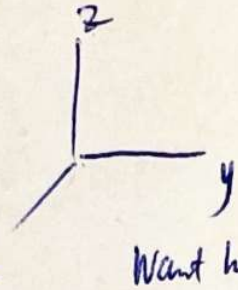
36 vertices
48 edges
14 faces
(F + V = E + 2)



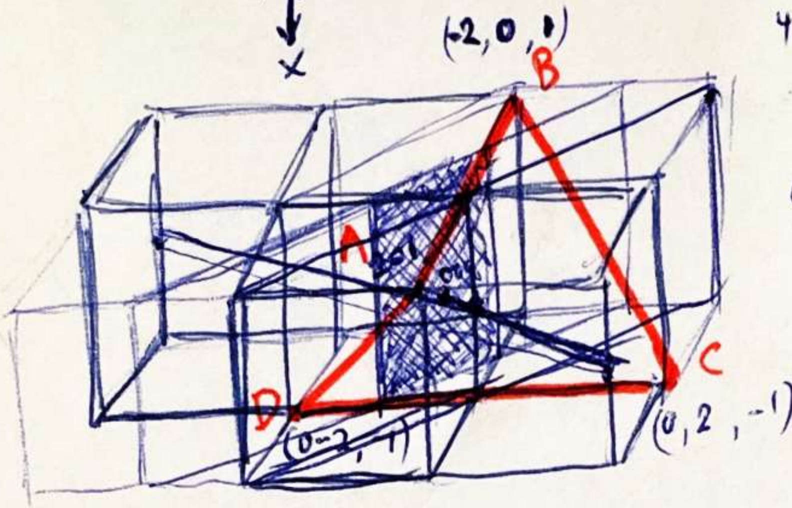
Here's the clathrate-like 14-hedron, with octagonal (100) faces and hexagonal (111) faces.


extended cuboctahedron
ext. cube
exp. tetrah.



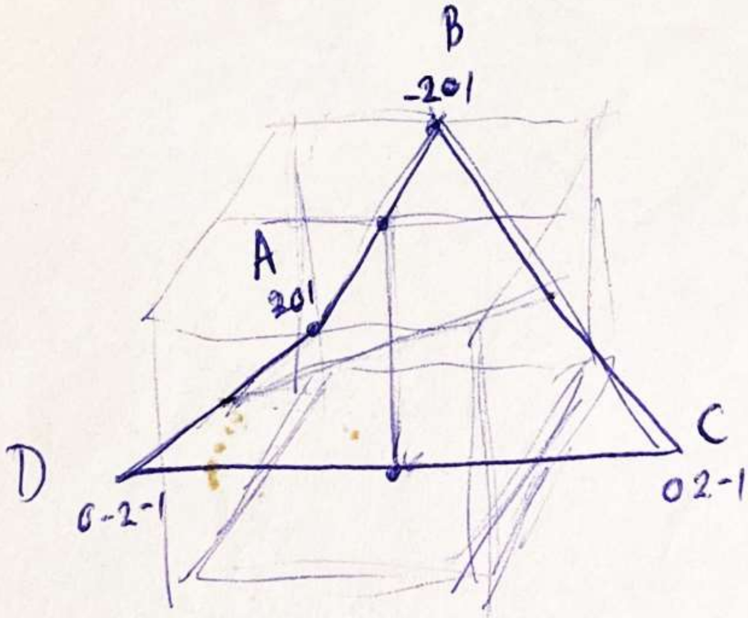


Want heights of 4 points -201
201
0-2-1
02-1

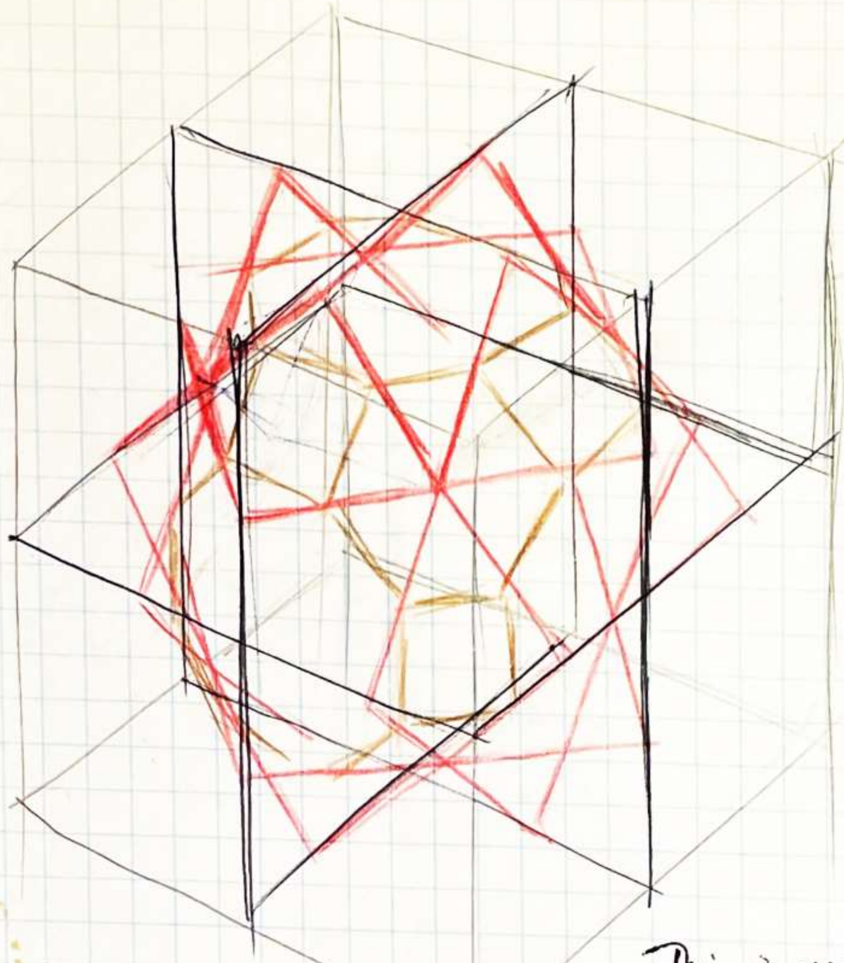


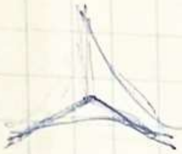
on (110) plane through origin:

$x + y = 0$



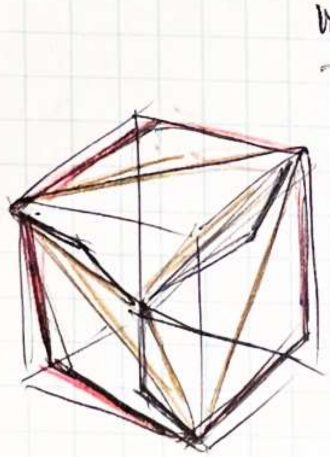
"extended cube"





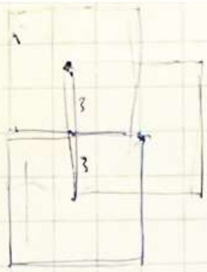
This is ~~xxx~~ the "dual" of the extended octahedron

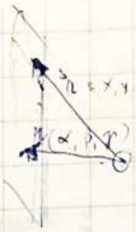
vertex figures define 26-hedron

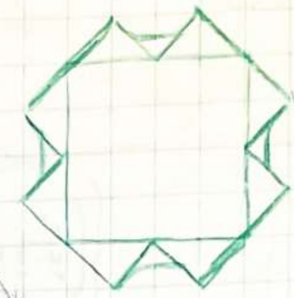


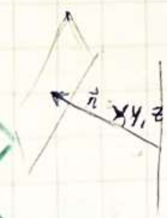
(because the "extensions" lead to intersection of edges in a plane at the "duality"-related planes [e.g. (111) vs (100)]

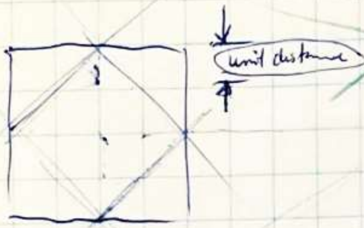
preserve the point group symmetry of the original polyhedron)

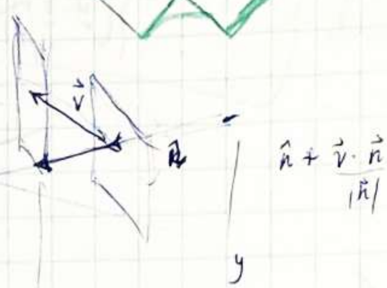


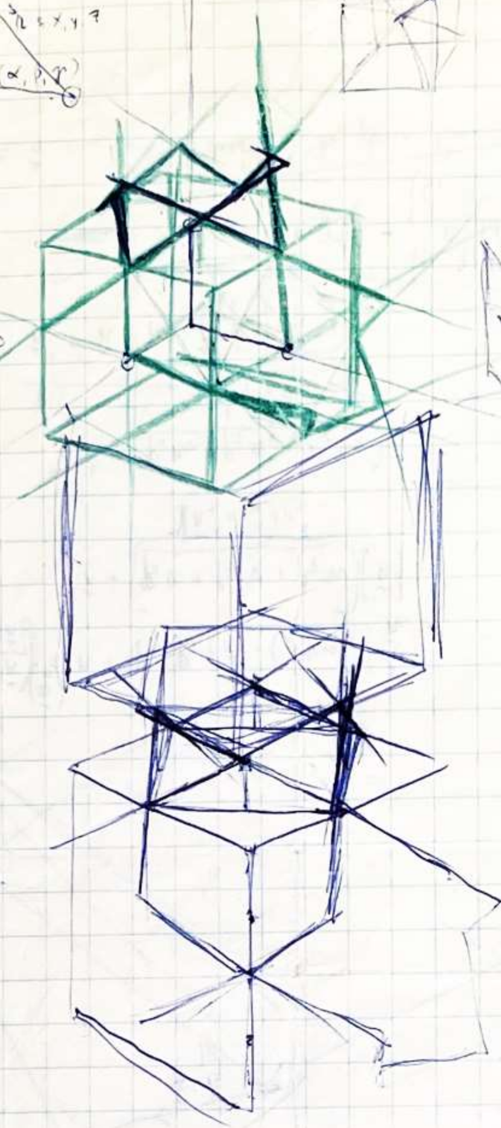












Eqns of planes are derived by specifying all 12 planes in each unit cell and then applying the three primitive translation vectors of the bcc space lattice:

~~±1 ±1~~
$(\pm1, \pm1, 0)$,
$(\pm1, 0, \pm1)$,
& $(0, \pm1, \pm1)$

$\pm x \pm y = 2$
$\pm x \pm z = 2$
$\pm y \pm z = 2$

Prim transl. vectors:

-3 3 3
3 -3 3
3 3 -3

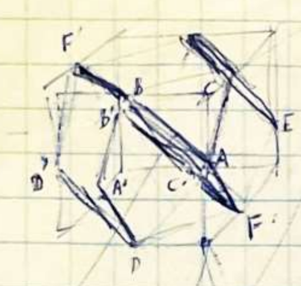
bcc clathrate cages
Each cage is of height 3/2 × reference cube edge length.

Since entire net is made up of ∞ straight lines which intersect, eqn of net can be given by eqns of intersecting planes:

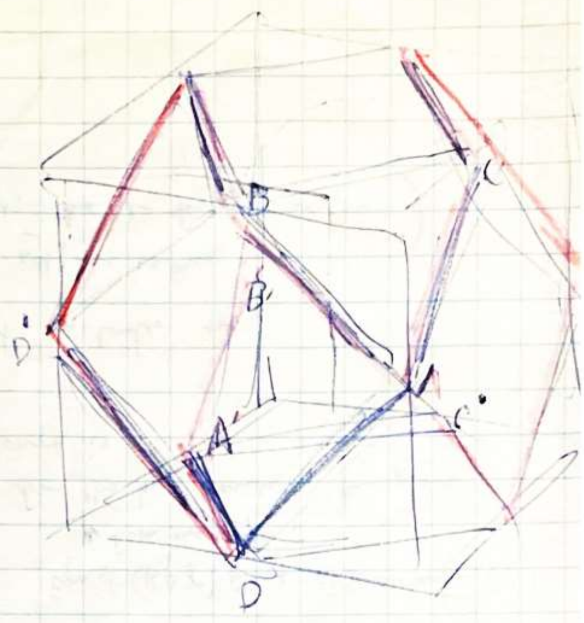
$\alpha x + \beta y + \gamma z - n^2 = 0$ Eqn of $(\alpha\beta\gamma)$ plane at a distance (⊥) n from the origin.

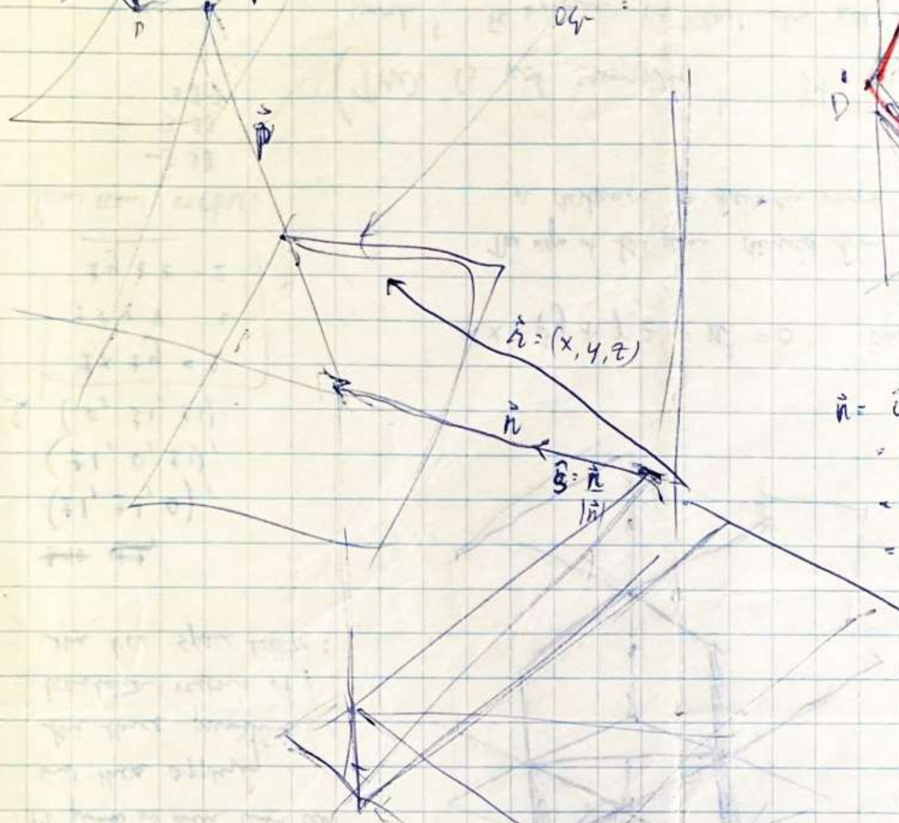
The eqn of the plane derived from this by translating the plane a distance $\vec{p}$ from the origin is

(This is not complete! It includes too much! For example, it includes all the vectors of the rhombic dodec, incl. the 3-connected vectors.)



$(\bar{3})$, 1/2s (CCW 120° then invert)





$$\vec{n} = \hat{\imath} n_x + \hat{\jmath} n_y + \hat{k} n_z = (n_x\ n_y\ n_z)$$
$$= \hat{\imath}(\vec{n}\cdot\hat{\imath}) + \hat{\jmath}(\vec{n}\cdot\hat{\jmath}) + \hat{k}(\vec{n}\cdot\hat{k})$$
$$= \hat{\imath}\, n\cos\alpha + \hat{\jmath}\, n\cos\beta + \hat{k}\, n\cos\gamma$$
$$= |\vec{n}|\,[\hat{\imath}\cos\alpha + \hat{\jmath}\cos\beta + \hat{k}\cos\gamma]$$
$$= |\vec{n}|\,[\hat{\imath} u + \hat{\jmath} v + \hat{k} w]$$
$$|\vec{n}|(u, v, w)$$

$$u = s_x = \frac{n_x}{n} = \frac{n_x}{\sqrt{n_x^2 + n_y^2 + n_z^2}}$$

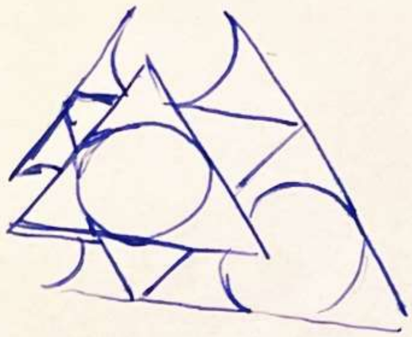
(Use α for α & also for cos of ∡ $= \vec{n}\cdot\hat{\imath}$!)

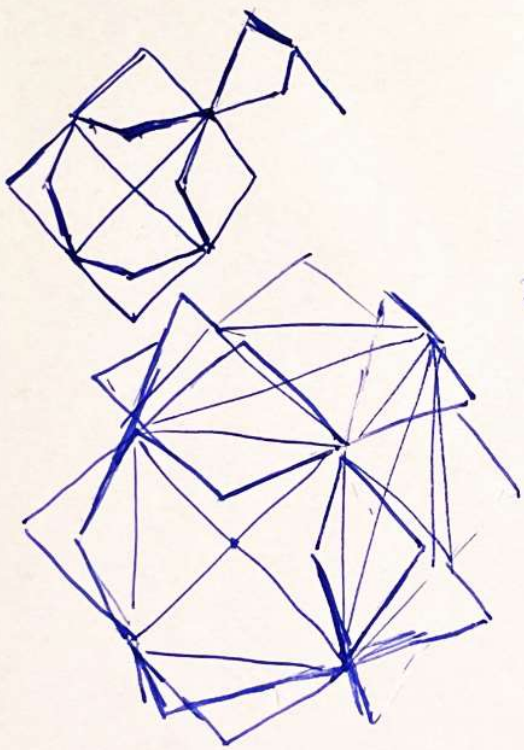
$$\vec{n}' = \vec{n} + \left(\vec{p}\cdot\frac{\vec{n}}{|\vec{n}|}\right)\left(\frac{\vec{n}}{n}\right) = \vec{n} + (p_x, p_y, p_z)\cdot(u\,v\,w)\left(\frac{\vec{n}}{n}\right)$$
$$= \vec{n} + \frac{[p_x u + p_y v + p_z w]}{\sqrt{n_x^2 + n_y^2 + n_z^2}}[\vec{n}]$$
$$= \vec{n} + \left(\frac{p_x n_x + p_y n_y + p_z n_z}{n_x^2 + n_y^2 + n_z^2}\right)\vec{n}$$
$$\vec{n}' = \vec{n}\left[1 + \frac{\vec{p}\cdot\vec{n}}{n^2}\right]$$

$$\alpha x + \beta y + \gamma z - \left\{[\vec{n}]\left[1 + \frac{\vec{p}\cdot\vec{n}}{n^2}\right]\right\}^2 = 0$$

eqn of plane $\vec{n}$ after translation by $\vec{p}$

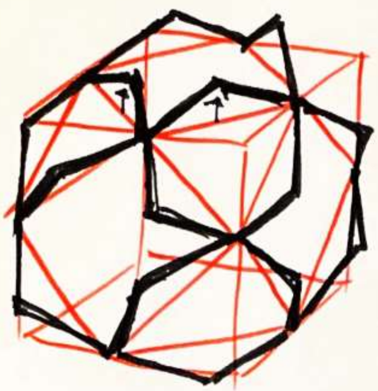
where $\vec{n}$ is normal distance to plane from origin



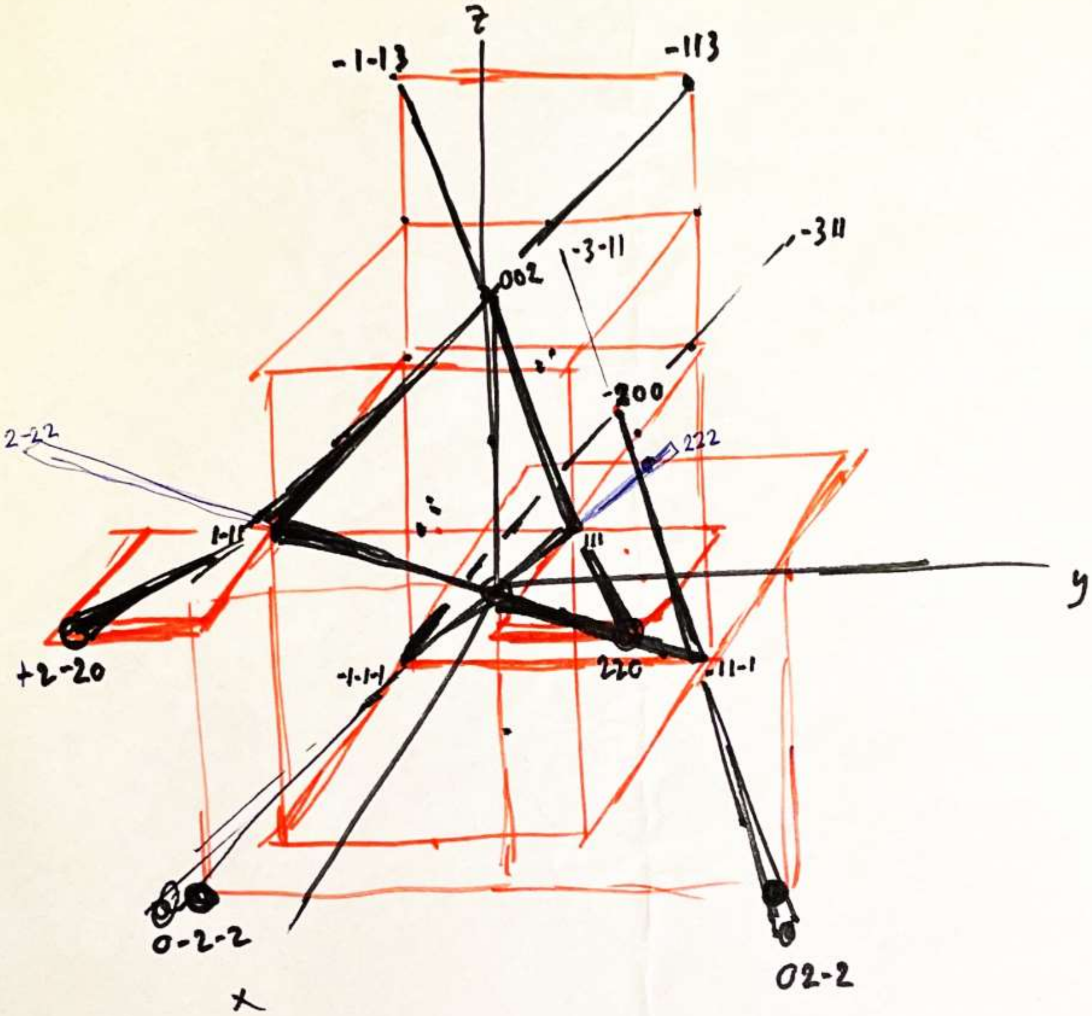


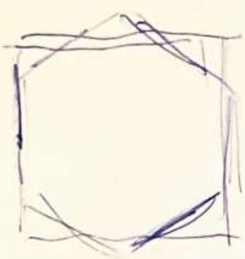
36 faces
(saddle rh. dodec
+ 12 tetrahedr)

This figure is easily derived from the cuboctahedron, as shown.



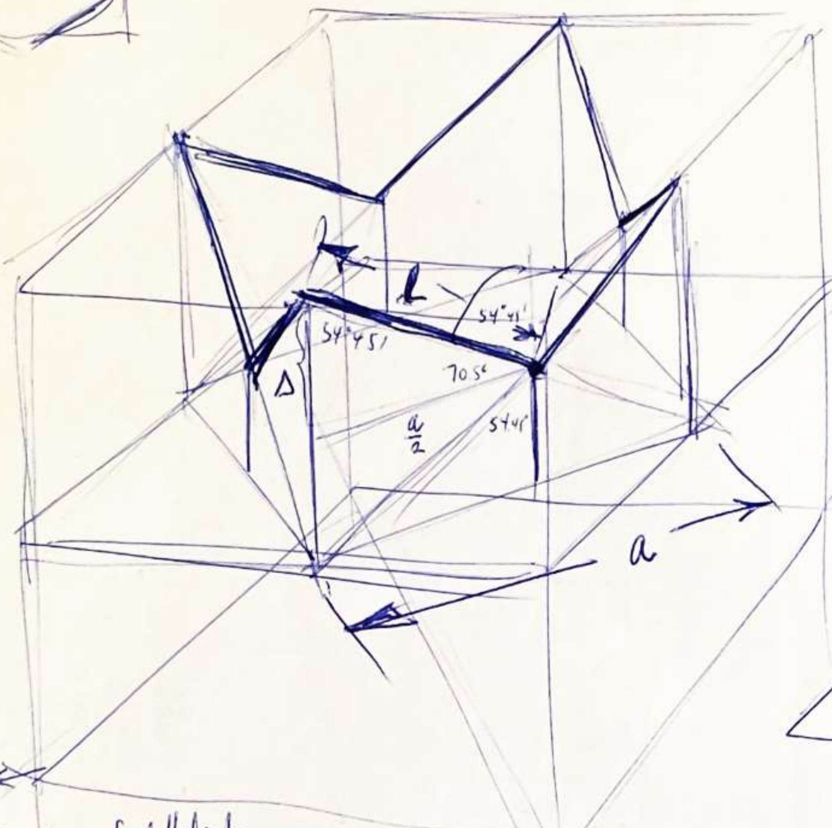
If minimal surfaces span the hexagonal and quadrilateral polygons, but not the octagonal ones, a surface (≠ IPMS) topologically ~~to the~~ identical to the Schwarz-Neovius simple-cubic IPMS will result.


z
-1-13
-113
002
-3-11
-311
-200
222
2-22
1-11
111
y
+2-20
-1-1-1
220
11-1
0-2-2
02-2
x



side l of octagon = side l of $109\frac{1}{2}°$ regular skew hexagon ("◊")

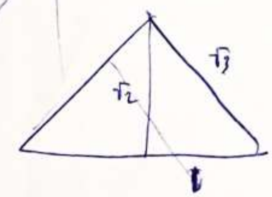
"$1\frac{1}{16}$" = 1.0625"



$$\frac{l}{a/2} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\therefore l = \frac{\sqrt{6}}{2}\,\frac{a}{2} = \frac{\sqrt{6}}{4}a$$

$$= .6122a$$



$$4\,\overline{)2.449}\quad .6122$$

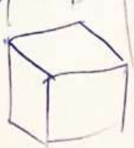
$$a = \frac{4}{\sqrt{6}}l = \frac{4\sqrt{6}}{6}l = \frac{2\sqrt{6}}{3}l$$

$$a = 1.6326\,l$$

$$3\,\overline{)2.4492}\quad .8163$$

1.6326

.864"



base of shillelagh for quadrilateral

l = 1.0625"

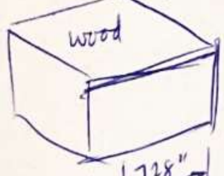
a = 1.728"

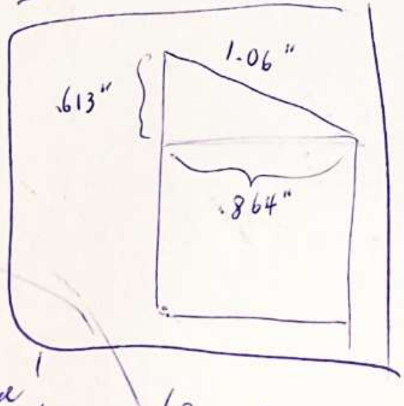
Δ = .613"

$a = (1.6326) \times (1.0625"$

1.63
× 1.06
978
163
1.7278

base of shillelagh for octagon





$$\frac{\Delta}{l} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = .577$$

Check

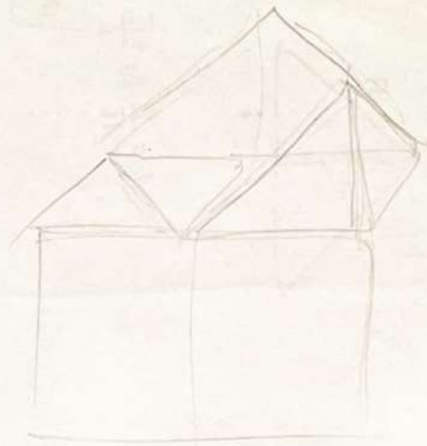
.613
× 1.414
2452
613
2452
613
.866782

1.0625
× .577
74375
74375
53125
.6130625

Make 8 of these!

(for octagon shillelagh)

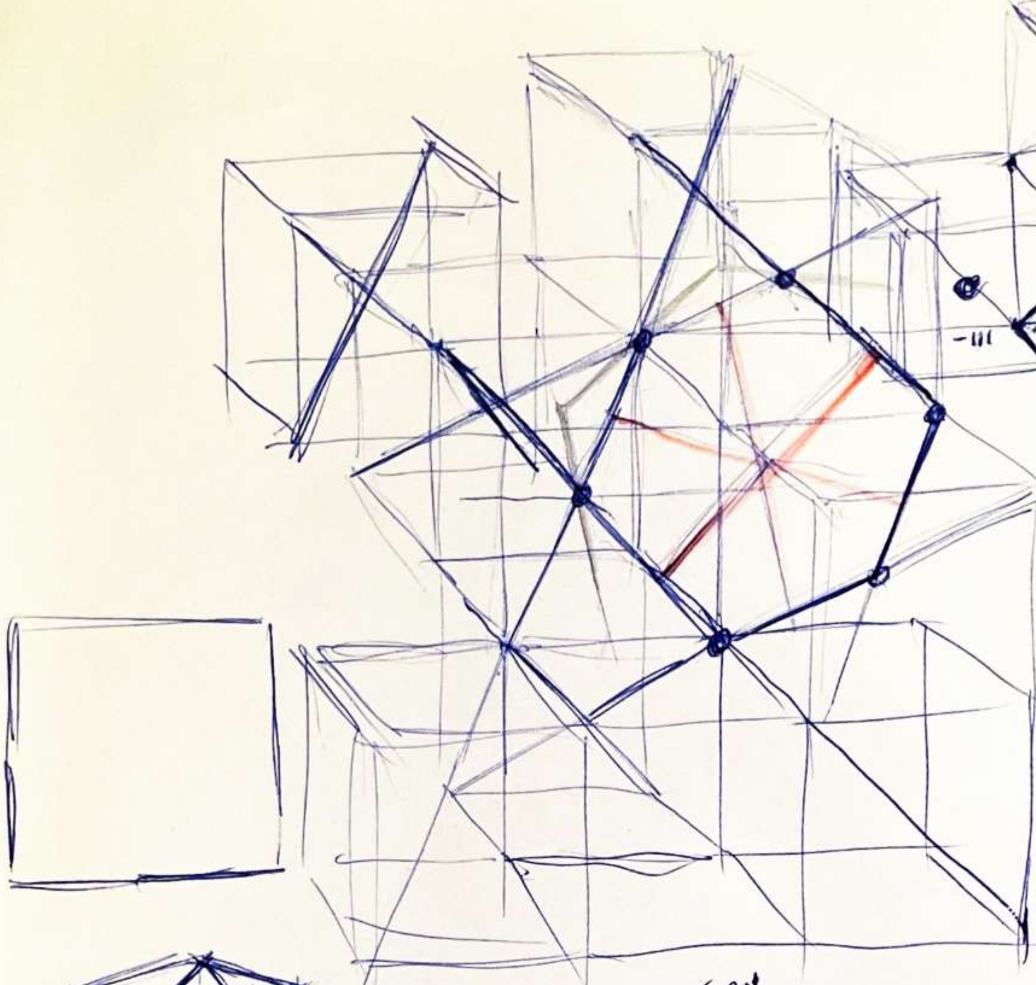
+ 4 more for skew quadrilateral

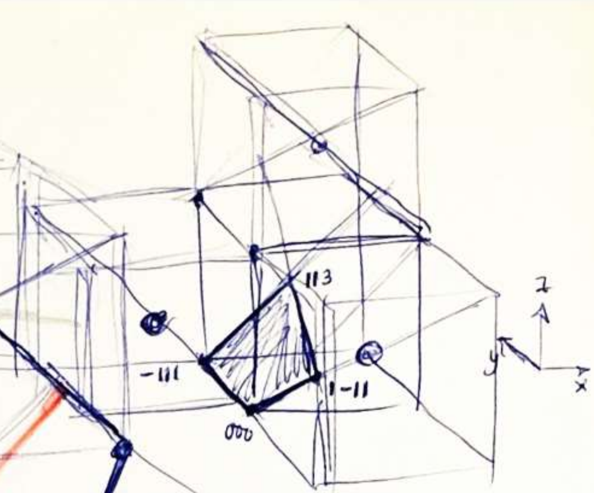


1.35
1.414

540
135
540
135

190890 1.909





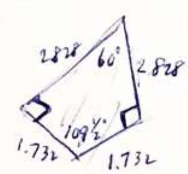
−111
202
113

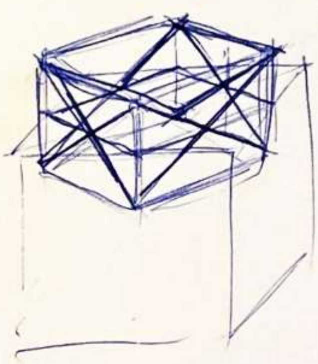
1−11
022
113 ✓ where

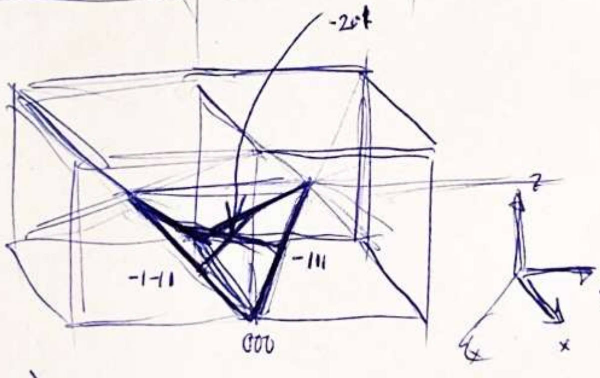
113
1−11
022

length = $\sqrt{8} = 2\sqrt{2}$
= 2.828

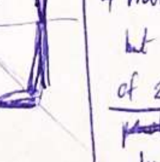
$l\left[\overrightarrow{(000),(1\text{-}11)}\right] = \sqrt{3} = 1.732$

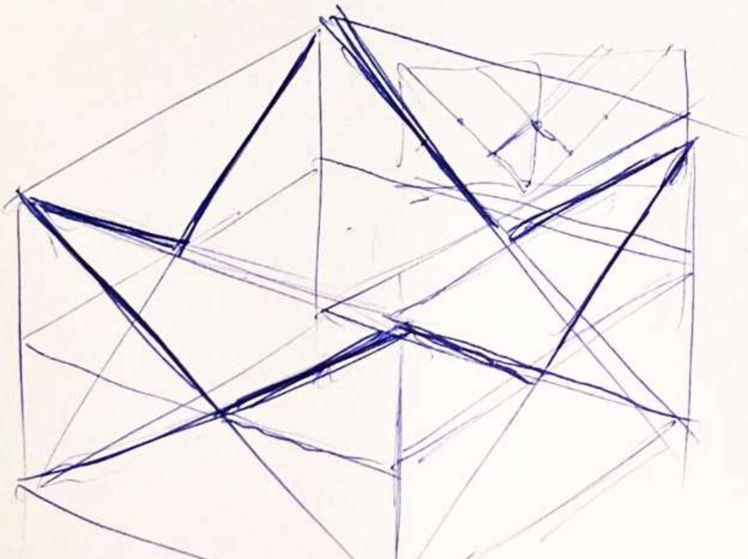


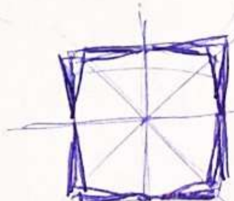




−201
−111
−1−10







4 mirror planes
but no axes
of 2-fold symmetry
~~parallel to the~~
lying in the
median plane of
the skew octagon !

Calculate Dirichlet region zone by zone, i.e.,
first the nearest neighbors, then the 2nd nearest, etc.

-11-1 -1-1-1
200 200
11-1 1-1-1
-11-1 -1-1-1
02-2 0-2-2
-200
111
-111
-200
-311
-200
-11-1
-1-11
-200
-3-11

1) Node at origin.

4 n.n. at 111, -11-1, -1-1-1, 1-11

$\vec{r}\cdot\vec{u} - u^2 = 0$

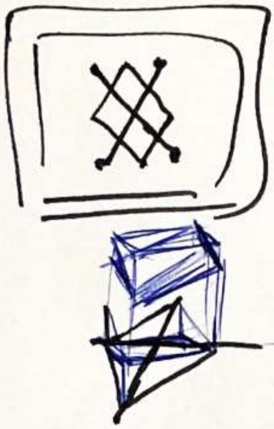
n.n. planes at $\boxed{\vec{r}\cdot\vec{u} - u^2 = 0}$

(111) $x+y+z = 3/2$

(-11-1) $-x+y-z = 3/2$

(-1-1-1) $x+y+z = -3/2$

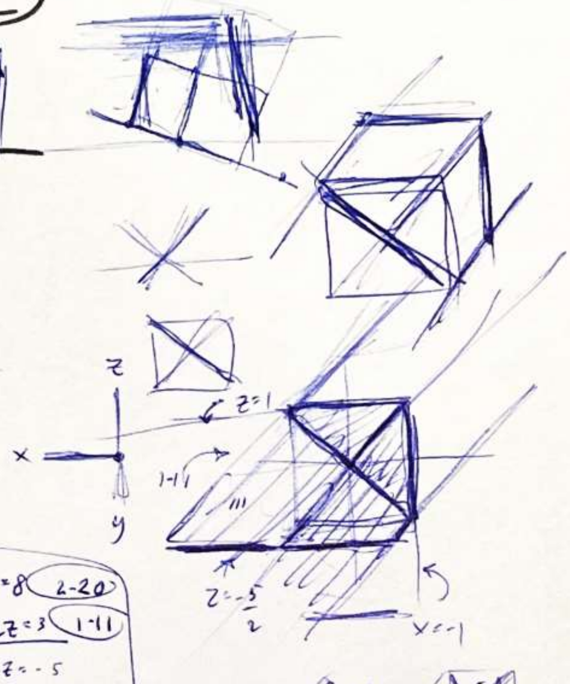
(1-11) $x-y+z = 3/2$



2) 2 2nd nn at 002 and -200

(002) $z = 1$

(-200) $x = -1$



3) 4 3rd nn at -2-20 220 02-2 0-2-2

220 -2-20

$2x+2y = 8$

$2x+2y+2z = 3$

$2z = -5$

$z = -5/2$

$2x+2y = -8$

$2x - 2$

$2x+2y = 8$ (220)

$2x+2y+2z = 3$ (111)

$2z = -5$

$z = -5/2$

$2x-2y = 8$ (2-20)

$2x-2y+2z = 3$ (1-11)

$2z = -5$

$z = -5/2$

$2x+2y = 8$ (220)

$-2x+2y-2z = 3$ (-11-1)

$4y - 2z = 11$

$x+y+z = 3/2$

$-x+y-z = 3/2$

$z = 1$

$2y = 3$

$y = 3/2$

$z = 1$

$x = y - z - 3/2 = 3/2 - 1 - 3/2 = -1$

$(-1, 3/2, 1)$

$x+y+z = 3/2$

$x-y+z = 3/2$

$z = 1$

$2x+2z = 3$

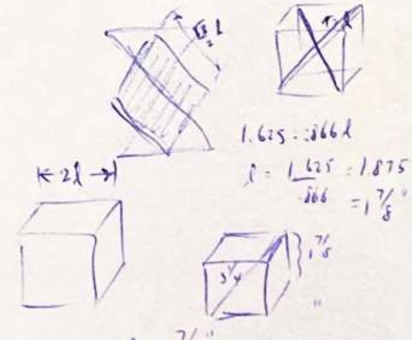
$2x = 1$

$x = 1/2$

$y = 0$

1875

866) 1.625



1.625 = .866 l

l = 1.625 / .866 = 1.875 = 1 7/8"

2l = 1 7/8"

l = 15/16"

$\frac{3/2}{\frac{1}{2}\sqrt{2}} = \frac{x}{2.12}$

$x = \frac{2.12}{\sqrt{2}} = \frac{(2.12)\sqrt{2}}{2} = (1.06)$

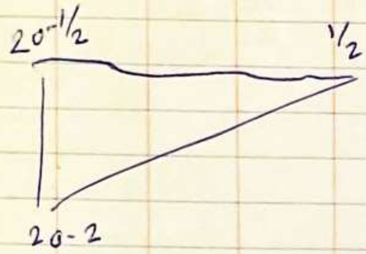
2.12

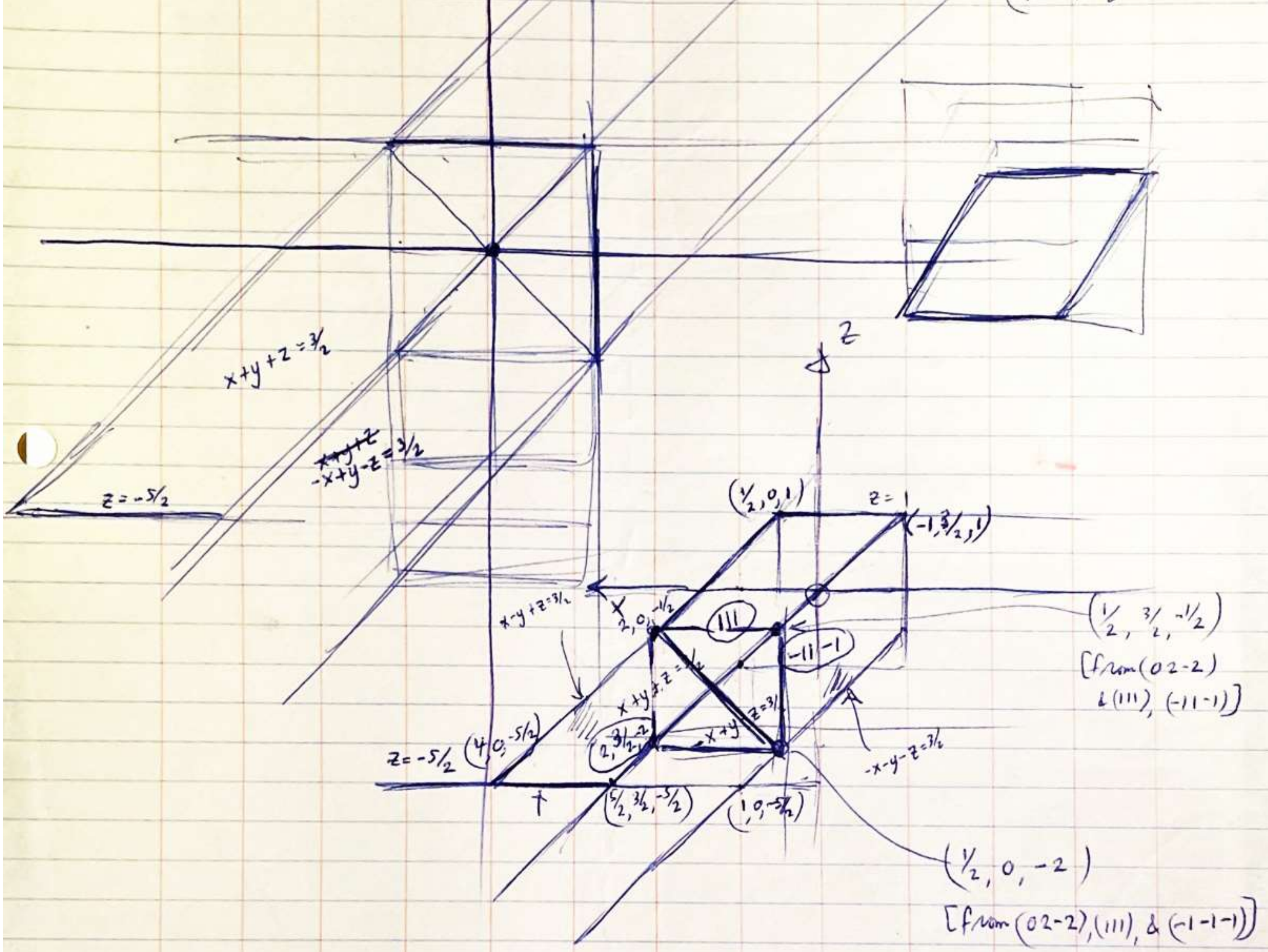
$1.414 \times 1.06 = 1.49884$

$2)\overline{2.4493}$ = 1.2247

3.6735

$(3\sqrt{2})\frac{\sqrt{3}}{2} = \frac{3\sqrt{6}}{2}$





1.414

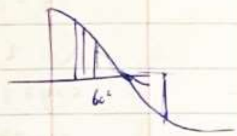
$\sqrt{72} = \sqrt{8 \cdot 9} = 3 \cdot 2\sqrt{2} = 6\sqrt{2}$

$18 = 37$

$(0\,-1\,1) \cdot (-1\,0\,1)$

$= \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$

$\cos^{-1}(\frac{1}{2}) =$



4 0 -1
1 0 2
3 0 -3

-2 3 2
4 3 -4
-6 0 6

4 0 -1
4 3 -4
0 -3 3

-2 3 2
1 0 2
-3 3 0

~~(1 0 2)~~ $\cdot$ (-3 3 0)

$(1\,0\,-1)(-1\,1\,0)$

$\frac{-1}{2}$

$(1\,0\,-1) \cdot (1\,-1\,0)$

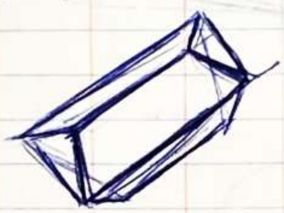
1

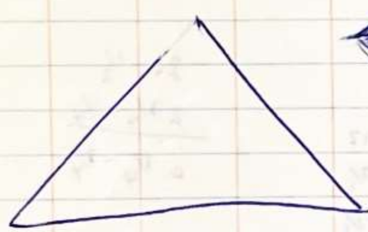
4 0 -1
4 3 -4
0 -3 3

-2 3 2
4 3 -4
-6 0 6

$(0\,2\,-2)\ (2\,2\,0)(2\,-2\,0)$

$2y - 2z = 4$
$2x + 2y = 4$
$2x - 2y = 4$
$4x = 8$
$x = 2$
$2y = 2x - 4 = 0$
$2z = 2y + 4 = -4$
$z = -2$
$(2\,0\,-2)$



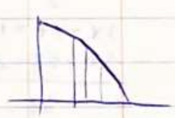


4 0 -1
4 -3 -4
0 3 3

4 3 -4
4 -3 -4
0 6 0

$(0\,1\,1) \cdot (0\,1\,0)$

$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = .707$



$(0\,2\,-2)(-1\,1\,-1)$ ~~(...)~~ $-1\,-1\,-1$

$(0\,-1\,1) \cdot (0\,-1\,0)$

$\frac{1}{\sqrt{2}} =$

$(0\,-1\,-1) \cdot (0\,1\,-1)$

$-1 + 1\ 0$

$2y - 2z = 4$
~~x~~ $-x + y - z = 3/2$
$x + y + z = -3/2$
$2y = 0$
$y = 0$
$2z = -4$
$z = -2$
$x = -3/2 - z = -3/2 + 2 = 1/2$

$1/2\ \ 0\ -2$

$(0\,2\,-2)\ (1\,1\,1)(-1\,1\,-1)$

$2y - 2z = 4$
$x + y + z = 3/2$
$-x + y - z = 3/2$
$2y = 3$
$y = 3/2 \quad z = y - 2 = 3/2 - 2 = -1/2$
$x + z = 0 \quad x = 3/2 - y - z = 3/2 - 3/2 + 1/2 = 1/2$
~~$x - z = 2$~~
$2x = 2$
$x = 1$
$z = -1$

$1\ 3/2\ -1$

$1/2\ 3/2\ -1/2$

$(0\,2\,{-2})\quad(1\,1\,1)\quad(-1\,1\,{-1})$

~~$2y - 2z = 4$~~
~~$x + y + z = 3/2$~~
~~$x + y + z = -3/2$~~
~~$2x + 2y + 2z = 0$~~
~~$2y - 2z = 4$~~
~~$2x + 4y = 4$~~

$2y - 2z = 4$
$x + y + z = 3/2$
$-x + y - z = 3/2$
$2y = 3$
$y = 3/2$
$2z = 2y - 4$
$= 3 - 4 = -1$
$z = -1/2$
$x = y - z - 3/2$
$= 3/2 + 1/2 - 3/2 = 1/2$
$x = 1/2$

$(1/2\ \ 3/2\ \ {-1/2})$ ←

$(0\,2\,{-2})(-1\,1\,{-1})\ {-1}\,{-1}\,{-1}$

$2y - 2z = 4$
$-x + y - z = 3/2$
$x + y + z = -3/2$
$2y = 0$
$y = 0$
$2z = -4$
$z = -2$
$x = y - z - 3/2$
$= 0 + 2 - 3/2 = 1/2$

$(1/2\ \ 0\ \ {-2})$ ←

~~$(3\,3\,{-1})$~~

$(x = 2)\ (1\,1\,1)\ (0\,2\,{-2})$
$x = 2$
$x + y + z = 3/2$
$2y - 2z = 4$
$y + z = 3/2 - 2 = -1/2$
$y - z = 2$
$2y = 3/2$
$y = 3/4$
$z = y - 2 = 3/4 - 2 = -5/4$

$(2\ \ 3/4\ \ {-5/4})$

$2\ 0\ {-1/2}$
$2\ \ 3/4\ \ {-5/4}$
$0\ \ 3/4\ \ {-3/4}$

4 4 16

$(2\,2\,{-4})$
$(1\,1\,1)$
$(1\,{-1}\,1)$

$2x + 2y - 4z = 12$
$x + y + z = 3/2$
$x - y + z = 3/2$
~~$2x + 2z = 3$~~
$2x + 2y + 2z = 3$
$6z = -9$
$z = -9/6 = -3/2$
$2y = 0$
$x = 3/2 + 3/2 = 3$
$3\ 0\ {-3/2}$ no

$2\,2\,{-4}$
$-1\,1\,{-1}$
$-1\,{-1}\,{-1}$

$2x + 2y - 4z = 12$
$-x + y - z = 3/2$
$x + y + z = -3/2$
$2y = 0$
$y = 0$
$2x - 4z = 12$
$2x + 2z = -3$
$6z = -15$
$z = -15/6 = -5/2$
$x = -3/2 - z = -3/2 + 5/2 = 1$

$1\ 0\ {-5/2}$

∴ 22-4 doesn't truncate

$2\ 0\ {-2}$
$2\ \ 3/4\ \ {-5/4}$
$0\ {-3/4}\ {-3/4}$

$(0\,{-1}\,{-1})\cdot(0\,{-1}\,1)$

$(2\,2\,0)(0\,2\,{-2})\ x = 2$
$2x + 2y = 4$
$2y - 2z = 4$
$x = 2$
$2y = 4 - 4 = 0$
$2z = -4$
$z = -2$
$2\ 0\ {-2}$

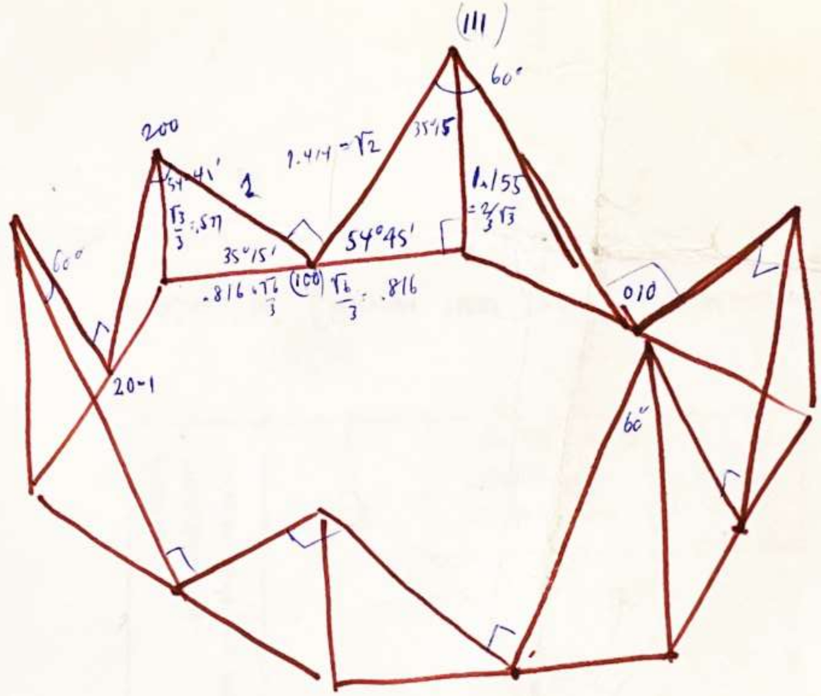
$(0\,2\,{-2})(0\,{-2}\,{-2})(-1\,1\,{-1})$
$2y - 2z = 4$
$-2y - 2z = 4$
$-x + y - z = 3/2$
$-4z = 8$
$z = -2$
~~$-x + y = 3/2 - 2 = -1/2$~~
$2y = 4 + 2z = 4 - 4 = 0$
$x = y - z - 3/2 = 0 + 2 - 3/2 = 1/2$

$(1/2\ \ 0\ \ {-2})$

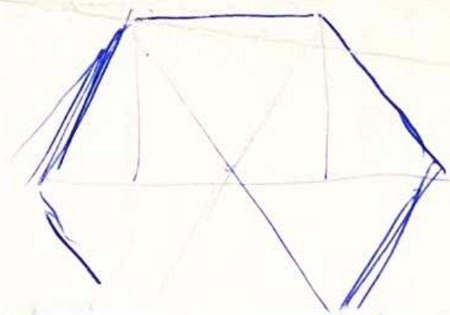
$x = 2\quad 0\,2\,{-2}\quad 0\,{-2}\,{-2}$
$x = 2$
$2y - 2z = 4$
$-2y - 2z = 4$
$-4z = 8$
$z = -2$
$2y = 4 - 4 = 0$

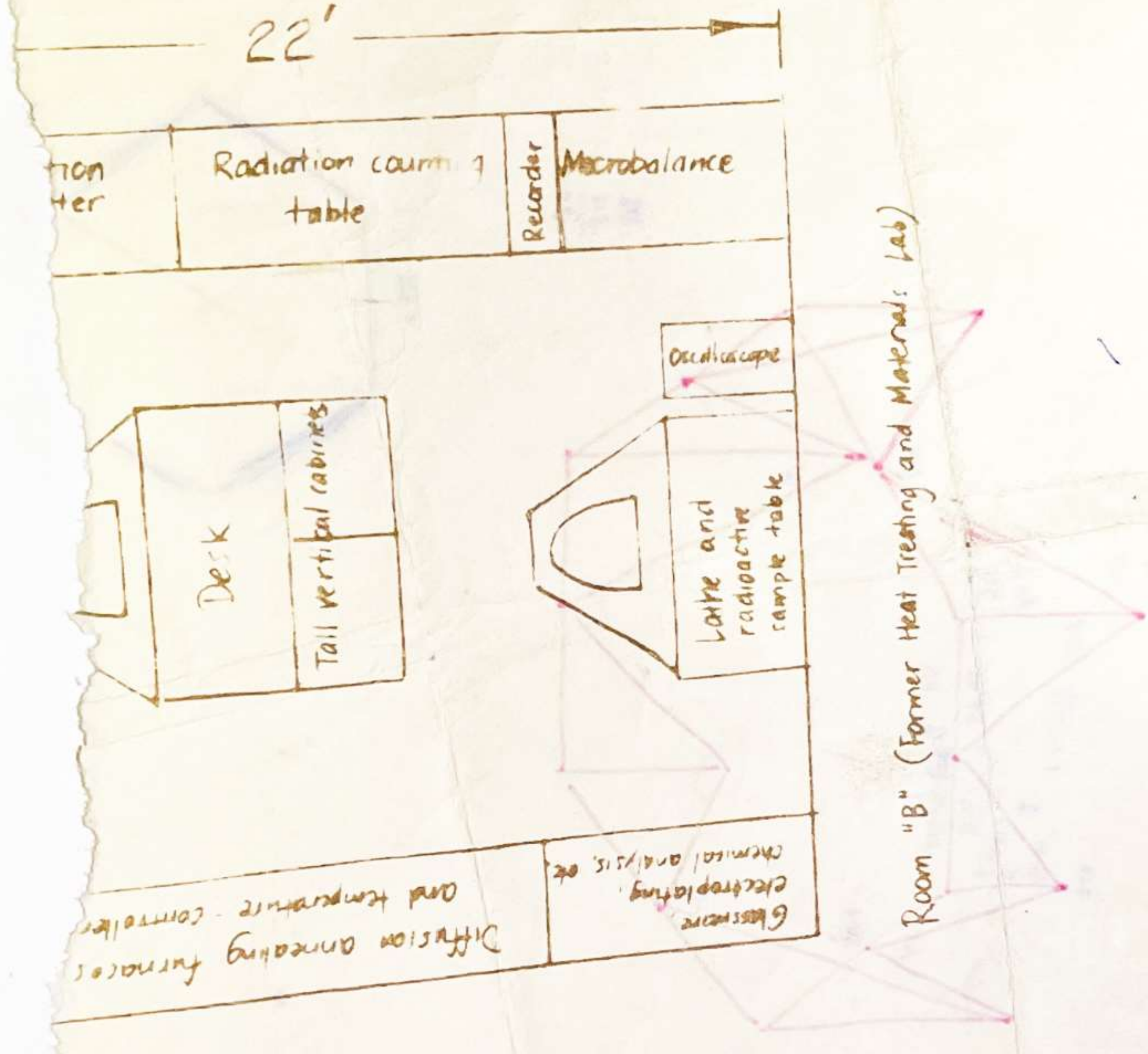
$(2, 0, -2)$ ☆

$(2\,2\,0)(1\,1\,1)(-1\,1\,{-1})$
$2x + 2y = 4$
$x + y + z = 3/2$
$-x + y - z = 3/2$
$2y = 3\quad y = 3/2$
$2x = 4 - 3 = 1$
$x = 1/2$
$(1/2\ \ 3/2\ \ {-1/2})$
$z = 3/2 - x - y = 3/2 - 1/2 - 3/2 = -1/2$

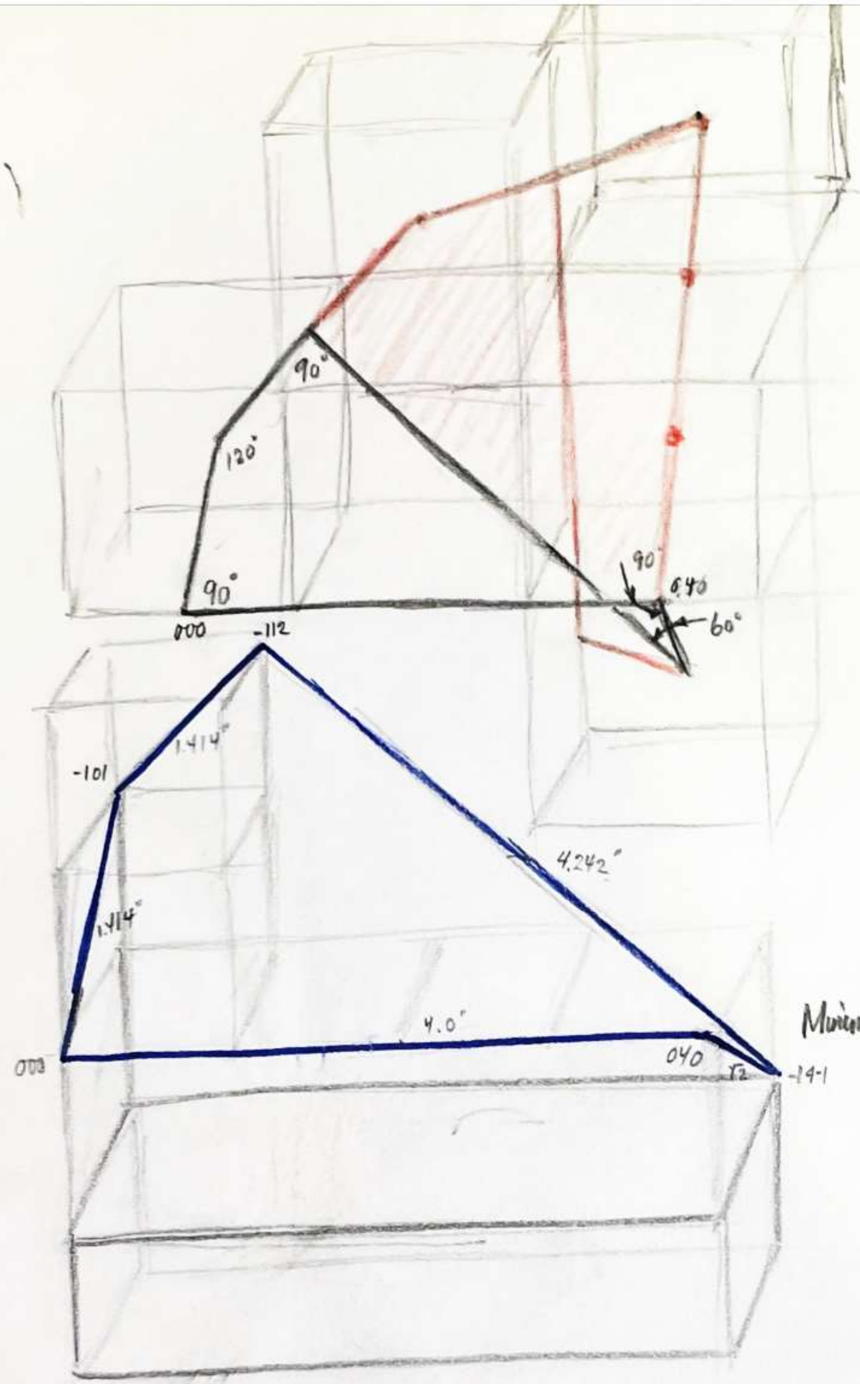

(111)
60°
35°15
200
1.414 = √2
54°41'
2
1.155
= 2/3 √3
√3/3 = .577
35°15'
54°45'
.816 = √6/3
(100)
√6/3 = .816
010
60°
20-1
60°

.816
.816
1.632


1.632


22'
tion
ter
Radiation coun g
table
Recorder
Microbalance
Oscilloscope
Desk
Tall vertical cabinets
Lathe and
radioactive
sample table
Diffusion annealing furnaces
and temperature - controller
Glassware
electroplating
chemical analysis, etc
Room "B" (Former Heat Treating and Materials Lab)

8



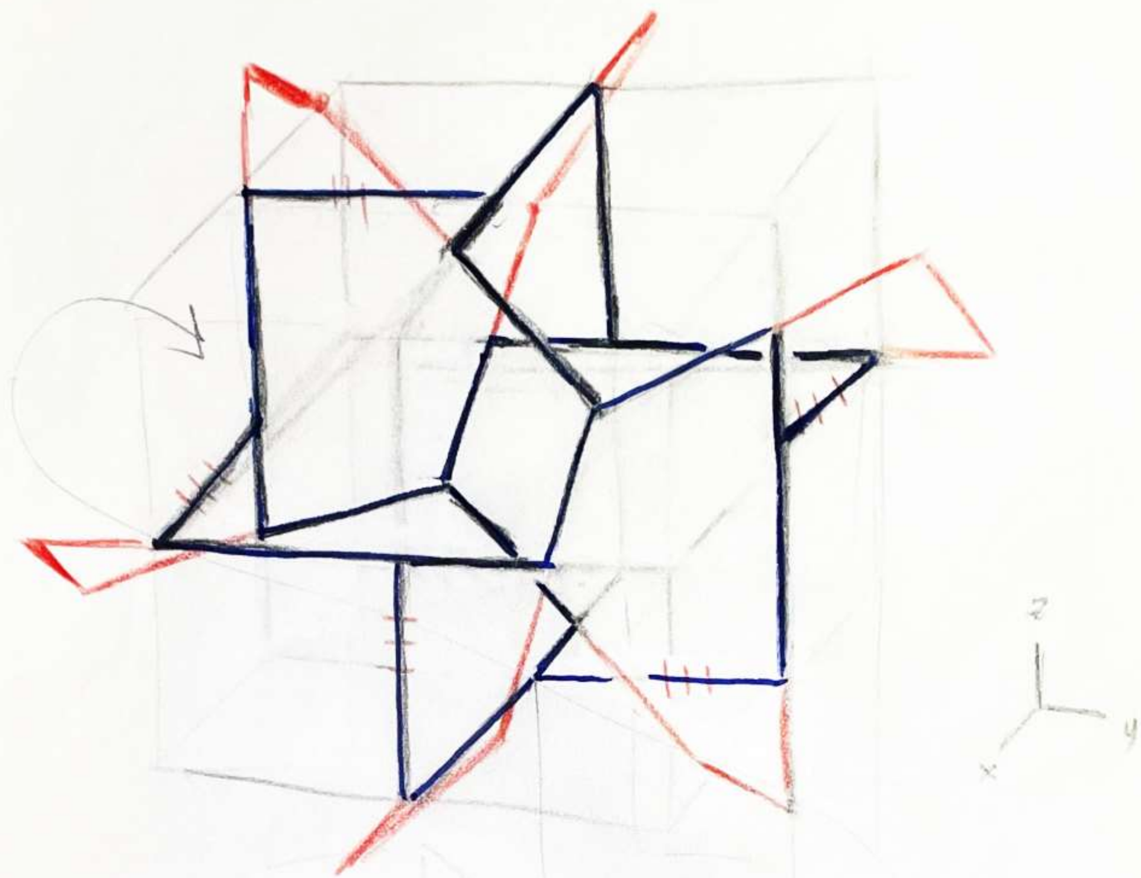
1.414
3
4√2

A new IPMS 10-21-68

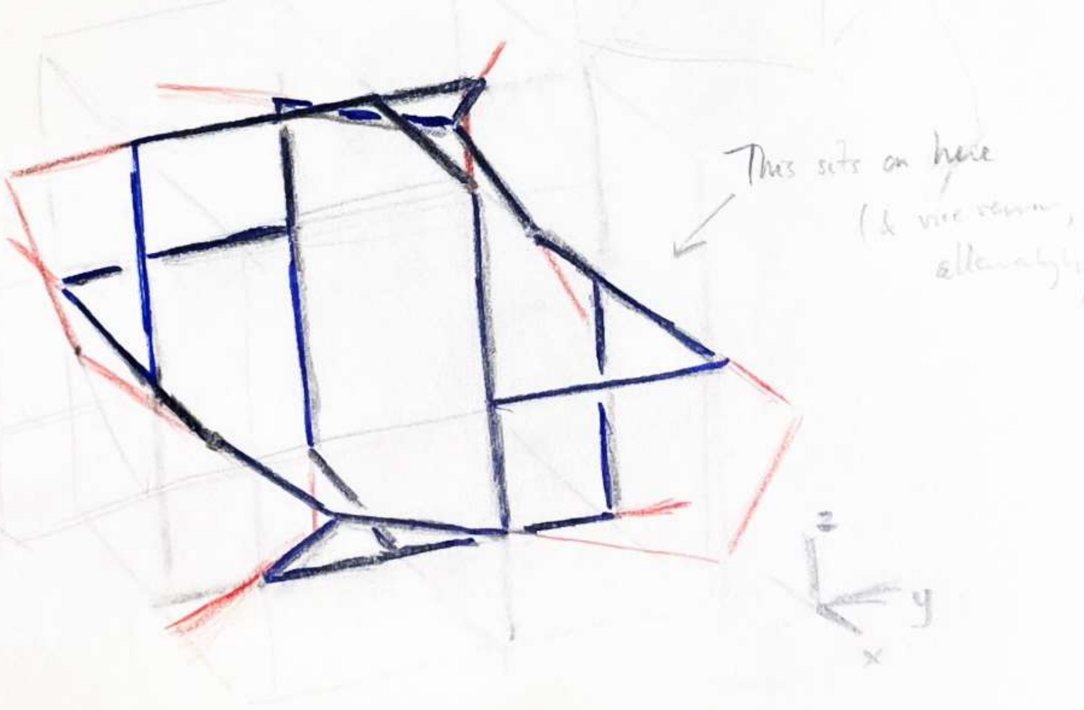
Pentagonal Minimalflächenstück

heptagonal-faced $\{4,6\}_L$

A



B



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

PROCUREMENT REQUEST (*See Instructions on reverse of part 4*)

1. DATE OF THIS REQUEST: June 25, 1969

2. TO: AAC/ERG

3. FROM:
A. NAME OF PERSON INITIATING REQUEST: DOMINIC J. DELPIDIO
B. OFFICE CODE: ABR
C. OFFICE PHONE: x2709

4A. PROGRAM AND PROJECT TITLE:

B. DATE APPROVED BY ADMINISTRATOR:

5. WILL CONTRACT INVOLVE ACCESS TO CLASSIFIED MATTER? ☐ NO ☐ YES (*If "Yes," attach NASA Form 446.*)

ITEM NO.	QUANTITY	DESCRIPTION (*Use additional sheets, if necessary*)	DELIVERY SCHEDULE	ESTIMATED COST
1.	1	Furnish Formal Design prototype, evaluation and bonding methods of Minimal Gyroid Helical Surfaces as defined in ABR Task 291 Work Statement, ABR Project 69-ROG-003, TITLE: "Minimal Gyroid Helical Surfaces"	Eight (8) WEEKS	$6,900.00 FIRM

I. TO BE COMPLETED BY INITIATING OFFICE

6. FUNDING DATA

A. INSTALLATION: NASA/Electronics Research Center

B. APPROPRIATION SYMBOL (*Indicate Year*) & ESTIMATED COST

(AO XXXXX 80X 0107 (69)	$6,900.00
R & D 80X0108 ( )	
TOTAL	$6,900.00

C. PROGRAM ACCOUNT CODE: G-0381

COST ELEM.:

7A. ACCEPTANCE POINT
☐ CONTRACTOR'S PLANT ☒ DELIVERY DESTINATION

B. DELIVER TO:
NASA/ERC
575 Tech. Sq., Cambridge, MA
Rm 829A

C. INSPECTION POINT
☒ CONTRACTOR'S PLANT ☒ DELIVERY DESTINATION

8. PREVIOUS PROCUREMENTS OF THIS ITEM
A. PR NO (S).
B. CONTRACT NO(S).

9. TYPED NAME OF ALTERNATE TECHNICAL CONTACT: Alexander Rabasco
A. OFFICE CODE: ABR
B. OFFICE PHONE: 2535

10. ATTACHMENTS HERETO

YES	NO	
	X	A. LIST OF RECOMMENDED SOURCES
	X	B. LIST OF GOVERNMENT FURNISHED PROPERTY
X		C. SOLE SOURCE JUSTIFICATION
X		D. DETAILED WORK STATEMENT
	X	E. UNSOLICITED TECH/COST PROPOSALS

11. SIGNATURE OF INITIATOR: Dominic J. Delpidio

12. APPROVED
A. DATE
B. TYPED NAME OF APPROVING OFFICIAL: ABR/R. W. Ebacher
C. SIGNATURE
D. DATE
E. TYPED NAME OF APPROVING OFFICIAL: AB/James M. Bayne
F. SIGNATURE

II. FOR FINANCIAL MANAGEMENT OFFICE USE

13. I CERTIFY that funds are available.
A. DATE
B. SIGNATURE AND PHONE EXTENSION

14. ☐ FUNDS ARE NOT AVAILABLE (*Return PR to Initiator.*)

15. STATION NO.-PR NO. ABR-0026

III. FOR PROCUREMENT & SUPPLY OFFICE USE

16. ASSIGNED
A. TO
B. BY
C. DATE

17. DATE COPY FWD TO INITIATOR

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

PROCUREMENT REQUEST *(See Instructions on reverse of part 4)*

2. TO: AAC/ERG	3. FROM: A. NAME OF PERSON INITIATING REQUEST: DOMINIC J. DELPIDIO	B. OFFICE CODE: ABR	C. OFFICE PHONE: x2709	1. DATE OF THIS REQUEST: June 25, 1969
4A. PROGRAM AND PROJECT TITLE			B. DATE APPROVED BY ADMINISTRATOR	

5. WILL CONTRACT INVOLVE ACCESS TO CLASSIFIED MATTER? ☐ NO ☐ YES (If "Yes," attach NASA Form 446.)

ITEM NO.	QUANTITY	DESCRIPTION (Use additional sheets, if necessary)	DELIVERY SCHEDULE	ESTIMATED COST
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I. TO BE COMPLETED BY INITIATING OFFICE

6. FUNDING DATA

A. INSTALLATION: NASA/Electronics Research Center

B. APPROPRIATION SYMBOL (Indicate Year) & ESTIMATED COST

(AO XXXXX 80X 0107 (69)	$6,900.00
R & D 80X0108 ( )	
TOTAL	$6,900.00

C. PROGRAM ACCOUNT CODE: G-0381 — COST ELEM.

7A. ACCEPTANCE POINT

☐ CONTRACTOR'S PLANT ☐ DELIVERY DESTINATION

B. DELIVER TO: NASA/ERC, 575 Tech. Sq., Cambridge, MA, Rm 829A

C. INSPECTION POINT

☒ CONTRACTOR'S PLANT ☒ DELIVERY DESTINATION

8. PREVIOUS PROCUREMENTS OF THIS ITEM — A. PR NO (S). — B. CONTRACT NO(S).

9. TYPED NAME OF ALTERNATE TECHNICAL CONTACT: Alexander Rabasco

A. OFFICE CODE: ABR — B. OFFICE PHONE: 2535

11. SIGNATURE OF INITIATOR: Dominic J. Delpidio

YES	NO	10. ATTACHMENTS HERETO
	X	A. LIST OF RECOMMENDED SOURCES
	X	B. LIST OF GOVERNMENT FURNISHED PROPERTY
X		C. SOLE SOURCE JUSTIFICATION
X		D. DETAILED WORK STATEMENT
	X	E. UNSOLICITED TECH/COST PROPOSALS

12. APPROVED	A. DATE	B. TYPED NAME OF APPROVING OFFICIAL: ABR/R. W. Ebacher	C. SIGNATURE
	D. DATE	E. TYPED NAME OF APPROVING OFFICIAL: AB/James M. Bayne	F. SIGNATURE

II. FOR FINANCIAL MANAGEMENT OFFICE USE

13. I CERTIFY that funds are available.

A. DATE	B. SIGNATURE AND PHONE EXTENSION	14. ☐ FUNDS ARE NOT AVAILABLE (Return PR to Initiator.)	15. STATION NO.-PR NO. ABR-0026

III. FOR PROCUREMENT & SUPPLY OFFICE USE

16. ASSIGNED A. TO	B. BY	C. DATE	17. DATE COPY FWD TO INITIATOR

analysis — ... the "Madison ...

ques...

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

PROCUREMENT REQUEST *(See Instructions on reverse of part 1)*

1. DATE OF THIS REQUEST: June 25, 1969

2. TO: AAC/ERG

FROM:

3. A. NAME OF PERSON INITIATING REQUEST: DOMINIC J. DELPIDIO

B. OFFICE CODE: ABR

C. OFFICE PHONE: x2709

4A. PROGRAM AND PROJECT TITLE

B. DATE APPROVED BY ADMINISTRATOR

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I. TO BE COMPLETED BY INITIATING OFFICE

6. FUNDING DATA

A. INSTALLATION: NASA/Electronics Research Center

B. APPROPRIATION SYMBOL (Indicate Year) & ESTIMATED COST

(AO XXXXX 80X 0107 (69)	$6,900.00
R & D 80X0108 ( )	
TOTAL	$6,900.00

C. PROGRAM ACCOUNT CODE: G-0381

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A. OFFICE CODE: ABR

B. OFFICE PHONE: 2535

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	X	A. LIST OF RECOMMENDED SOURCES
	X	B. LIST OF GOVERNMENT FURNISHED PROPERTY
X		C. SOLE SOURCE JUSTIFICATION
X		D. DETAILED WORK STATEMENT
	X	E. UNSOLICITED TECH/COST PROPOSALS

11. SIGNATURE OF INITIATOR: Dominic J Delpidio

12. APPROVED

A. DATE

B. TYPED NAME OF APPROVING OFFICIAL: ABR/R. W. Ebacher

C. SIGNATURE

D. DATE

E. TYPED NAME OF APPROVING OFFICIAL: AB/James M. Bayne

F. SIGNATURE

II. FOR FINANCIAL MANAGEMENT OFFICE USE

13. I CERTIFY that funds are available.

A. DATE

B. SIGNATURE AND PHONE EXTENSION

14. ☐ FUNDS ARE NOT AVAILABLE (Return PR to Initiator.)

15. STATION NO.-PR NO.: ABR-0026

III. FOR PROCUREMENT & SUPPLY OFFICE USE

16. ASSIGNED

A. TO

B. BY

C. DATE

17. DATE COPY FWD TO INITIATOR

7-8-F2-E730

LEHIGH Design Company, Inc.

1065 MAIN ST. • WALTHAM, MASSACHUSETTS 02154
TWX 710-324-0455

June 26, 1969

National Aeronautics and Space Administration
Electronics Research Center
565 Technology Square
Cambridge, Massachusetts 02139

Attention: Mr. James Mallen ERC/AAC Contracting Officer

Gentlemen:

This cost proposal is in response to your request for quotation in reference to ABR-Control #0026, ABR Work Statement Task 291, entitled, "Minimal Gyroid Helical Surfaces". A firm fixed price of $6,900.00 is proposed. Lehigh Design Company, Inc. agrees to perform tasks covered in your work statement to your satisfaction for this fixed price. Tasks include engineering, formal design drawings, fabrication of a prototype and evaluation of bonding methods. Lehigh's estimated cost per item (from work statement) is as follows:

Item	Estimated Cost
1	$2,060.00
2	2,543.00
3	1,670.00
4	627.00
Total	$6,900.00

The total firm fixed price of $6,900.00 covers this entire effort. Cost estimates for Items #1 through Item #4 are not submitted as unit prices. None of these items can be procured independantly of the total task.

Very truly yours,

LEHIGH DESIGN COMPANY, INC.

Richard A. Anderson
Richard A. Anderson
Vice President

RAA/rb

617-893-8730

LEHIGH Design Company, Inc.

1065 MAIN ST. • WALTHAM, MASSACHUSETTS 02154

TWX 710-324-0455

June 26, 1969

National Aeronautics and Space Administration
Electronics Research Center
565 Technology Square
Cambridge, Massachusetts 02139

Attention: Mr. James Mallen ERC/AAC Contracting Officer

Gentlemen:

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Total	$6,900.00

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Very truly yours,

LEHIGH DESIGN COMPANY, INC.

Richard A. Anderson

Richard A. Anderson
Vice President

RAA/rb

NTSA

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
NEGOTIATED CONTRACT

CONTRACT NUMBER: **NAS 12-**1550

ISSUING OFFICE

NAME	ADDRESS
ELECTRONICS RESEARCH CENTER	575 Technology Square Cambridge, Mass. 02139

CONTRACTOR

NAME	ADDRESS
Lehigh Design Company, Inc.	1065 Main Street Waltham, MA 02154

CONTRACT FOR	AMOUNT
The Formal Design of a Plastic Forming Mold for Minimal Gyroid Helical Surfaces	$ 6,900.00

APPROPRIATION AND OTHER ADMINISTRATIVE DATA

CONTROL NUMBER: ABR-0026

TYPE OF CONTRACT: Fixed Price

APPROPRIATION AND ALLOTMENT CHARGEABLE: G-0381-2520

OFFICE OF ADMINISTRATION AND PAYMENT: **NATIONAL AERONAUTICS AND SPACE ADMINISTRATION**
Electronics Research Center
575 Technology Square
Cambridge, Massachusetts 02139

MAIL INVOICES TO: Office of Administration and Payment (original plus 6 copies) marked for Code: AC

This negotiated contract is entered into pursuant to the provisions of 10 U.S.C. 2304 (a) (10) and any required Findings and Determination have been made.

THIS CONTRACT is entered into as of June 30, 19 69, by and between the United States of America, hereinafter called the Government, represented by the Contracting Officer executing this contract, and Lehigh Design Company, Inc.

(i) a corporation organized and existing under the laws of the State of Massachusetts

(ii) a partnership consisting of ______

(iii) an individual trading as ______

hereinafter called the Contractor. The parties hereto agree that the Contractor shall furnish and deliver all supplies and perform all the services set forth in the attached Schedule, for the consideration stated herein.

PPC: NJ

SCHEDULE

PART I - CONTRACT PROVISIONS

A. The General Provisions of this contract consist of Clauses 1 through 22, Standard Form 32, June 1964, and Additional General Provisions to U. S. Standard Form 32, NASA Form 250, May 1966, Clauses 23 through 34 (a through e) and the following additional clauses 35 through 37 are attached hereto and made a part hereof:

35. New Technology (May 1966)
36. Rights in Data
37. Data Requirements

PART II - PAYMENT AND CONSIDERATION: The Contractor shall be paid the contract price of $6,900.00 upon completion and acceptance of the work called for in Part III Statement of Work.

PART III - STATEMENT OF WORK

The Contractor shall supply the necessary personnel, facilities, services and materials to accomplish the work set forth below:

A. This task is for the formal design, engineering, prototype, evaluation and bonding methods of Minimal Gyroid Helical Surfaces as defined in this Work Statement. The Plastic Forming Mold and Edge Trimming Fixture is to be used in the research and construction of a Minimal Gyroid Helical Surface. In generalization, this task is essentially phase two of the advancement of the state-of-the-art in the fabrication and construction techniques of scaled-up sizes of Minimal Gyroid Helical Surfaces. The design approach for this project will follow the Conceptual Design of a Plastic Mold for Gyroid Surfaces as submitted to NASA/ERC-ABR under Purchase Order ER-21,236.

B. DESIGN CRITERIA

1. Furnish formal engineering design of a Plastic Forming Mold for Minimal Gyroid Helical Surfaces.

   a. The six (6) helical edges comprising the gyroid forming mold are best defined as having three (3) identical convex helical edges joined at their respective high and low points by three (3) identical concave helical edges.

   b. The helical dimensional requirements for both the convex and concave edges are as stated below:

   Helix for each edge to be 1/4 of the distance in an axial direction of one complete turn about a cylindrical surface (real or imaginary) whose diameter is 4.500 inches and the lead of helix is 18.000 inches.

c. A 12-inch diameter x 1/2 inch thick aluminum jig plate is to be used as a base, centrally located relative to the center point of the mold.

d. Tolerances on critical areas are to be ± .005 or better if feasible.

e. A 15° to 20° positive rake angle to within 1/32 inch of the helical forming edge is required on the periphery of the forming mold.

f. Vertical backup supports should be a consideration at the high points of the helical edges.

g. Clamping shoes with exact machined curvatures are to be considered in all cases to eliminate welding stresses and distortion of the true helical edges.

h. All support structures are to be pinned and bolted and/or shoulder screws are to be used instead of welding. All locating shoulder screws and/or pins are to be off symmetry to insure all parts can only be assembled one way.

NOTE: The design of the Gyroid Helical Mold will be reviewed for concurrence with ERC/Technical Monitor and ROG-Researcher prior to the undertaking of Paragraph B. 2.

2. Construct prototype of Gyroid Helical Mold in accordance with formal design drawings established in Paragraph B.1. The formal design drawings are to be modified to agree with the finished prototype mold if required.

3. Furnish conceptual design and formal engineering design drawings of a motorized plastic edge cutting fixture. This fixture will be required to locate the plastic Gyroid Helical Surface and also have the capability of positive 6 positioning indexing of each edge relative to the cutting tool. The possible use of a bench-type drill press or a lathe grinder should be considered for driving the cutting tool.

4. Evaluate prototype mold for possible use as a plastic vacuum forming mold. Explore and recommend possible methods of fixturing and bonding techniques of the finished plastic Helical Gyroid Surfaces.

NOTE: A typed report of Paragraph B. 4 is required.

PART IV - DRAWING REQUIREMENTS

A. Complete assembly and/or assemblies as required. Main assembly will be on Sheet #1.

B. Complete details.

C. Complete Bill of Materials, if room allows, will start on Sheet #1 including manufacturer and/or trade name of item, also stipulating any such commercial items as or equal at the end of its description.

D. All drawings to conform to NASA/ERC-ABR methods of presentation in conjunction with MIL-STD-100.

E. Scale layouts of undefined areas are to be submitted for concurrence before detailing.

F. Calculations supporting design criteria where needed should be made available to the ERC Technical Monitor.

G. Scale shall not be less than full size.

NOTE: Calculations need no formal format for presentation. Work sheets, providing they are legible, are permitted. Any information which the contractor feels would assist in the fabrication of the final hardware should be indicated on the drawing where applicable.

H. NASA/ERC-ABR Drawing Formats will be furnished to the successful bidder. Formats will be of the same size for the complete design. Multiple details are allowed on the same sheet.

PART V - ENVIRONMENT

This equipment will be operated in a laboratory environment with standard human comfort conditions.

PART VI - TIME OF PERFORMANCE AND DELIVERY SCHEDULE

A. Commencement Date: June 30, 1969

B. Completion Date: August 25, 1969

PART VII - DELIVERY, INSPECTION AND ACCEPTANCE

The National Aeronautics and Space Administration, Electronics Research Center, Room 829A, 575 Technology Square, Cambridge, Massachusetts 02139, Attention: Mr. Dominic J. Delpidio, ABR/Research Engineering Branch, is designated as the point for final inspection and acceptance for drawings and/or sketches with sufficient explanation and information called for in Part III,Statement of Work. A copy of the letter transmitting the drawings and/or sketches to the ERC Technical Monitor will be sent to the attention of the Contracting Officer.

ghts and obligations of the parties to this contract shall be subject to and governed by the dule and the General Provisions. To the extent of any inconsistency between the Schedule or the eral Provisions, and any specifications or other provisions which are made a part of this contract by reference or otherwise, the Schedule and the General Provisions shall control. To the extent of any inconsistency between the Schedule and the General Provisions, the Schedule shall control.

The Contractor represents that aggregate number of employees of the Contractor and its affiliates is: ☐ 500 or more, ☐ less than 500.

IN WITNESS WHEREOF, the parties hereto have executed this Agreement as of the day and year first above written.

THE UNITED STATES OF AMERICA

Date ______________________ By ______________________

(Signature)

____

(Contracting Officer)

CONTRACTOR

____

(Name of Company or Individual)

Date ______________________ By ______________________

(Signature)

____

(Typed Name)

____

(Title)

____

(Address)

WORK STATEMENT

ABR Task 291

ABR Project #69-ROG-003

"MINIMAL GYROID HELICAL SURFACES"

SCOPE:

This task is for the formal design, engineering, prototype, evaluation and bonding methods of Minimal Gyroid Helical Surfaces as defined in this Work Statement in conjunction with verbal instructions.

The Plastic Forming Mold and Edge Trimming Fixture is to be used in the research and construction of a Minimal Gyroid Helical Surface. In generalization, this task is essentially phase two of the advancement of the state-of-the-art in the fabrication and construction techniques of scaled-up sizes of Minimal Gyroid Helical Surfaces.

REFERENCE:

The design approach for this project will follow the Conceptual Design of a Plastic Mold for Gyroid Surfaces as submitted to NASA/ERC-ABR under Contract #ER-21,236 - Sections 1 thru 4, by Lehigh Design Co., Inc. 1065 Main St., Waltham, MA. This work statement and verbal instructions will define the formal mold design and the additional requirements of this project.

This project will consist of four (4) tasks identified as Item 1, Item 2, Item 3 and Item 4.

DESIGN CRITERIA:

Item #1 - Furnish formal engineering design of a Plastic Forming Mold for Minimal Gyroid Helical Surfaces.

A. The six (6) helical edges comprising the gyroid forming mold are best defined as having three (3) identical convex helical edges joined at their respective high and low points by three (3) identical concave helical edges.

B. The helical dimensional requirements for both the convex and concave edges are as stated below:

Helix for each edge to be 1/4 of the distance in an axial direction of one complete turn about a cylindrical surface (real or imaginary) whose diameter is 4.500 inches and the lead of helix is 18.000 inches.

C. A 12-inch diameter x 1/2-inch thick aluminum jig plate is to be used as a base, centrally located relative to the center point of the mold.

D. Tolerances on critical areas are to be $\pm$.005 or better if feasible.

E. A 15° to 20° positive rake angle to within 1/32 inch of the helical forming edge is required on the periphery of the forming mold.

F. Vertical backup supports should be a consideration at the high points of the helical edges.

G. Clamping shoes with exact machined curvatures are to be considered in all cases to eliminate welding stresses and distortion of the true helical edges.

H. All support structures are to be pinned and bolted and/or shoulder screws are to be used instead of welding. All locating shoulder screws and/or pins are to be off symmetry to insure all parts can only be assembled one way.

NOTE:

The design of the Gyroid Helical Mold will be reviewed for concurrence with ERC/Technical Monitor and ROG-Researcher prior to the undertaking of Item #2.

Item #2 - Construct prototype of Gyroid Helical Mold in accordance with formal design drawings established in Item #1. The formal design drawings are to be modified to agree with the finished prototype mold if required.

Item #3 - Furnish conceptual design and formal engineering design drawings of a motorized plastic edge cutting fixture. This fixture will be required to locate the plastic Gyroid Helical Surface and also have the capability of positive 6 positioning indexing of each edge relative to the cutting tool. The possible use of a bench-type drill press or a lathe grinder should be considered for driving the cutting tool.

Item #4 - Evaluate prototype mold for possible use as a plastic vacuum forming mold. Explore and recommend possible methods of fixturing and bonding techniques of the finished plastic Helical Gyroid Surfaces.

NOTE: A typed report of Item #4 is required.

DRAWING REQUIREMENTS:

A. Complete assembly and/or assemblies as required. Main assembly will be on Sheet #1.

B. Complete details.

C. Complete Bill of Materials, if room allows, will start on Sheet #1 including manufacturer and/or trade name of item, also stipulating any such commercial items as or equal at the end of its description.

D. All drawings to conform to NASA/ERC-ABR methods of presentation in conjunction with MIL-STD-100.

E. Scale layouts of undefined areas are to be submitted for concurrence before detailing.

F. Calculations supporting design criteria where needed should be made available to ABR personnel on request.

G. Scale shall not be less than full size.

NOTE: Calculations need no formal format for presentation. Work sheets, providing they are legible, are permitted. Any information which the vendor feels would assist in the fabrication of the final hardware should be indicated on the drawing where applicable.

Sample representative NASA/ERC-ABR drawings will be available to the bidder on request to be used as guides for presentation.

Drawing requirements which will be used for the fabrication of hardware are as indicated in this Work Statement.

NASA/ERC-ABR Drawing Formats will be furnished to the successful bidder. Formats will be of the same size for the complete design. Multiple details are allowed on the same sheet.

ENVIRONMENT:

This equipment will be operated in a laboratory environment with standard human comfort conditions.

DELIVERY SCHEDULE:

Eight (8) weeks from the date of award of contract.

Place of Delivery: NASA/Electronics Research Center
575 Technology Square
Room 829A
Cambridge, MA

Contractor will notify the ABR/ERC Technical Monitor or the ERC/ABR Design Specialist in advance of any visits required of his personnel to ERC in the performance of this task.

ABR/ERC Technical Monitor	-	Dominic J. Delpidio	494-2709
ABR/Design Specialist	-	Alexander Rabasco	494-2535

NOTE: The above personnel may visit the contractor's facility to personally check the progress and status of this project. Contractor will be notified in advance of such visits.

OPTIONAL FORM NO. 10
MAY 1962 EDITION
GSA GEN. REG. NO. 27
5010-104

UNITED STATES GOVERNMENT

Memorandum

TO : AAC/James Mallen

DATE: June 25, 1969

FROM : ABR/D. J. Delpidio

SUBJECT: Sole Source Justification for the Formal Engineering Design of a Plastic Forming Mold for Minimal Gyroid Helical Surfaces - ABR Project 69-ROG-003

Reference Information:

Four (4) Conceptual Design contracts of a Plastic Mold to form Helical Gyroid Surfaces were awarded to the following firms:

1.	Dynamac, Inc.	ER-21,235
2.	Lehigh Design Co., Inc.	ER-21,236
3.	Warren Associates	ER-21,237
4.	Cambridge Eng'rg., Inc.	ER-21,238

In accordance with Part III, Statement of Work, Note B, paragraph #3, which stipulates, "NASA/ERC, however, will have the option to allow the contractor of a given design concept of his to be the sole bidder to provide formal engineering drawings."

The four (4) concepts were received and evaluated by the laboratory researcher who is the expert in the development of minimal surfaces for NASA/ERC. After consultation with the researcher and a joint evaluation of the concept chosen, I am in complete agreement and fully concur that the conceptual design approach, reference sections 1 thru 4 as submitted by Lehigh Design Company, Inc., of 1065 Main Street, Waltham, Mass, is the best design approach to follow for this project. Additional engineering design, a mold prototype coupled with evaluation and recommendation for fixturing and bonding of the Helical Gyroid Surfaces are essential to this project. Work Statement ABR Task 291, ABR Project 69-ROG-003, defines work to be performed as a total package.

As ERC/Technical Monitor and Chief of the Design Unit, I feel that Lehigh Design Co., Inc. as the sole source for this project would be in the best interest of the Government, and the cost is fair and reasonable.

Dominic J. Delpidio
Dominic J. Delpidio

Concurrence:

____
AB/James M. Bayne, Chief
Engineering & Const. Div.

I have performed a considerable amount of analysis — still unpublished — on the "practical" question of when mechanical interferences occur, thereby terminating the collapse of an initially fully expanded (i.e., fully deployed) graph. No mention is made in PM-58 of such mechanical interferences, but they occur in all of the examples 5 through 17. These interferences are distinct from the "pair-wise vertex fusions" mentioned in the last column of Table 2, on p. 13 of PM-58; they involve the collision of edges, not vertices.

In the notation of PM-58, in which α is the angle of rotation of an edge, with respect to its initial position, about either incident vertex, it is found that for the Laves graph, for example (# 11 of Table 2), the collapse terminates for $\alpha_c \cong 76.716°$. If the finite portion of the graph which is under consideration (finite in order to be an actually realizable structure) is sufficiently large in overall diameter, in the fully deployed state, with respect to the length of an edge of the graph, then to good approximation the ratio of the overall diameter d of the collapsed graph ($\alpha \equiv \alpha_c$) to the diameter D of the fully deployed graph is

$$\frac{d}{D} \cong \cos \alpha_c . \qquad (1)$$

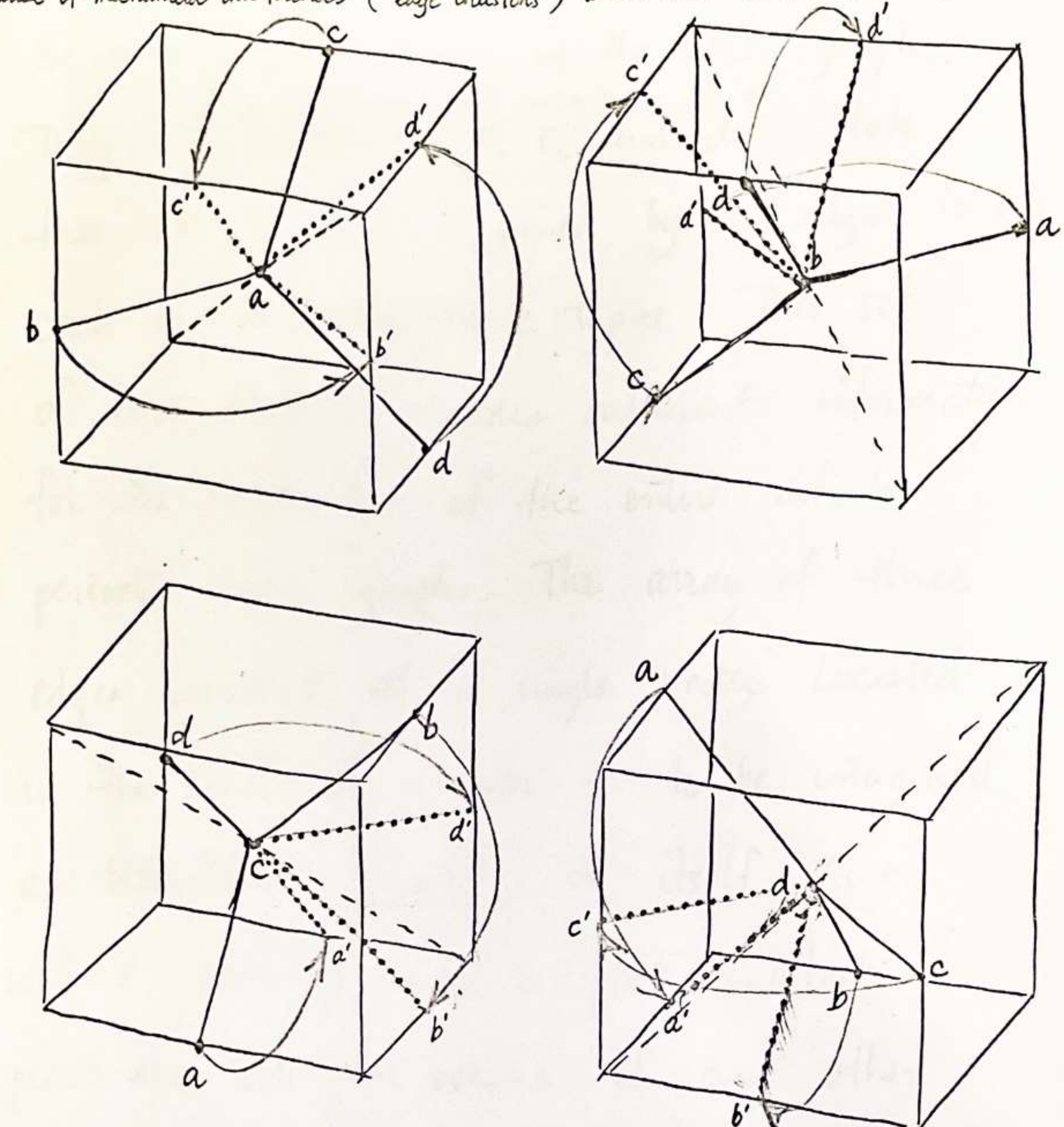
(This can be seen from an examination of Eq. 24 in PM-58.)

Thus, substituting $\alpha_c = 76.716°$ in Eq. 1, we find that for the Laves graph, $\frac{d}{D} \cong \frac{1}{4.35}$, corresponding to a volume ratio of approximately 1 : 82. (The volume ratio = $(d/D)^3$.)

Further analysis, (unpublished) has shown that the effect of providing for finite separation of the hinge connections for the several edges incident at a given vertex (as described in the analysis beginning on p. 19 of PM-58) initially leads to an increase in the volume expansion ratio, in the case of the Laves graph! This is a small effect, but at least it is in pleasant contrast to my naive expectation that a penalty would be exacted as soon as this realistic modification of the geometry was made.

The four differently oriented vertex environments in the Laves graph.

The solid lines ab, ac, ad, bc, bd, and cd show the positions of these edges in the fully expanded configuration of the Laves graph. The dotted lines ab′, ac′, ad′; ba′, bc′, bd′; ca′, cb′, cd′; and da′, db′, dc′ show the positions of these same six edges in the fully collapsed configuration of the Laves graph, a configuration which cannot actually be realized in practice because of mechanical interferences ("edge collisions") which occur at $\alpha = \alpha_c$ ($\cong 76°$).



The enclosing cubes are shown only for the purpose of illustrating how the three edges incident at a vertex are oriented. The cubes are not part of the Laves graph.

There are only six distinctly oriented edges in the Laves graph: ab, ac, ad, bc, bd, and cd. The curved arcs in the sketches indicated the quarter-circle trajectory described by each vertex with respect to any of the three vertices to which it is attached by an edge, during the transformation between fully expanded and fully collapsed states.

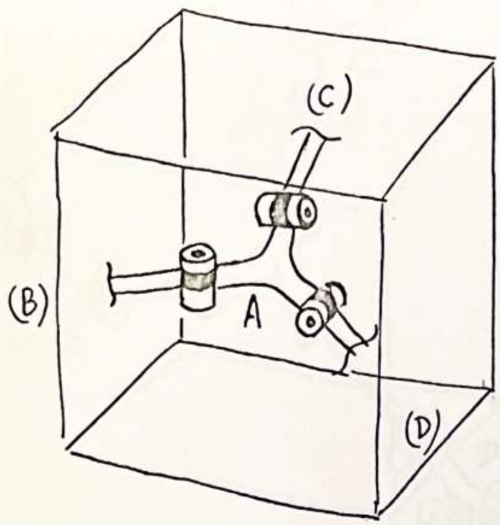
The dashed line in each cube is the axis of symmetry passing through the vertex in the center of the cube.

(Refer these comments to the sketches on p. 6.)

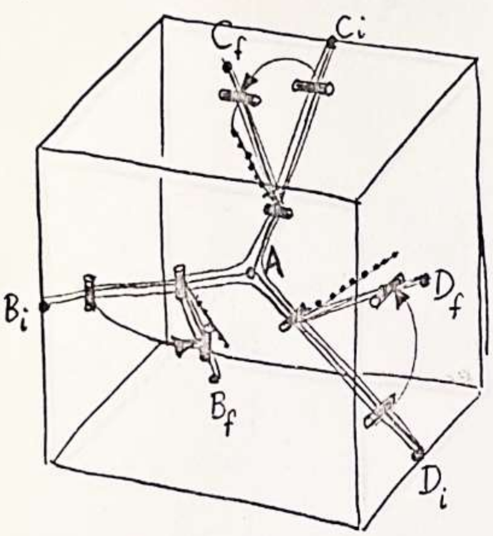
These are the four distinct orientations of the vertices which occur in the Laves graph. They are labelled "a", "b", "c", and "d". Note that each of them is joined by an edge to one each of the other three types. This set of four sketches provides sufficient information for the construction of the entire infinite periodic Laves graph. The array of three edges incident at a single vertex located in the center of a cube is to be imagined as translated parallel to itself (i.e., without rotation) into a position which penetrates into the volume of any other of the three cubes, as a representation of the connections among edges in the infinite graph. For example, the edge ac, in the cube containing the vertex <u>a</u> at its center, is to be imagined as superimposed on the edge <u>ac</u> in the cube containing the vertex <u>c</u> at its center. Such a superposition illustrates the arrangement of the five edges <u>ac</u>, <u>ba</u>, <u>ad</u>, <u>dc</u>, and <u>bc</u> (i.e., all of the edges appearing in either of the two aforementioned cubes).

The curved arrows indicate the circular trajectories, with respect to the central vertex of a given cube, of the three neighboring vertices. In the idealized case, ignoring mechanical interferences between pairs of edges, these trajectories comprise 90° central angle arcs of a circle. In fact, as already mentioned, the actual mechanical interferences limit this circular motion to about 76°.

Actual arrangement of vertices and edges in modified Laves graph, in which vertices are "extended" so as to allow space for physical hinge connections of edges.

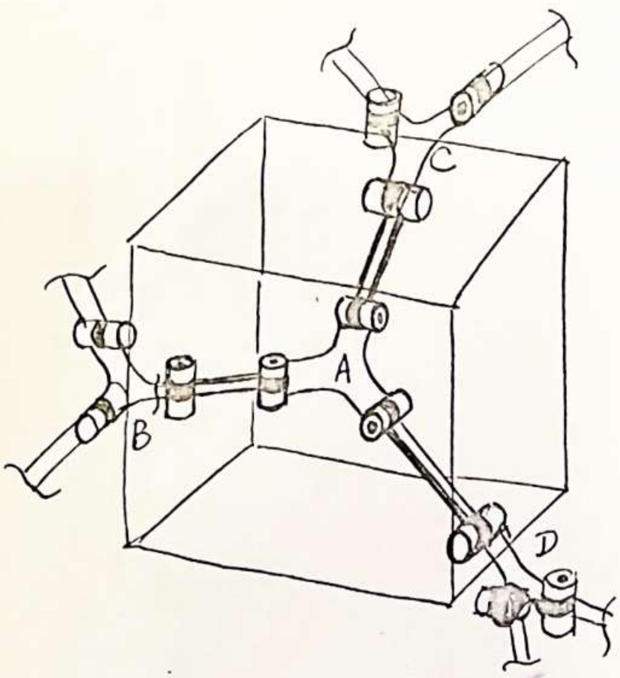


This sketch shows how the hinge pins are arranged in the hinges which connect vertex A to the three vertices B, C, and D. The hinge pins are oriented parallel to the edges of the enclosing (imaginary) cube, so as to be perpendicular to the respective planes of rotation of the three struts (edges) which are attached to vertex A.



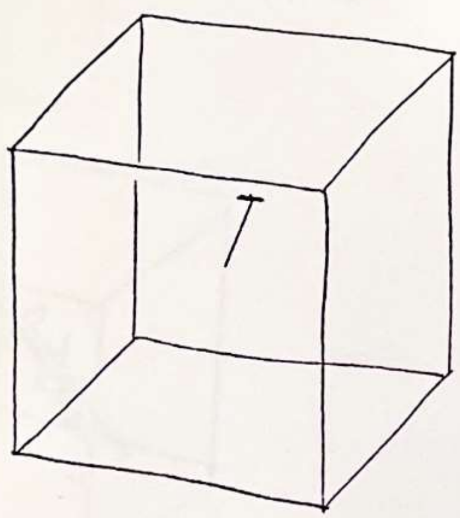
Here we see the initial and final positions of the edges joined to the "extended" vertex A. For example, C_i is the center of the extended vertex C in the fully deployed configuration of the graph; C_f is the center of the extended vertex C in the fully collapsed configuration of the graph ($\alpha = \alpha_c \cong 76°$). The dotted lines show the unrealizable position of edges for $\alpha = 90°$.

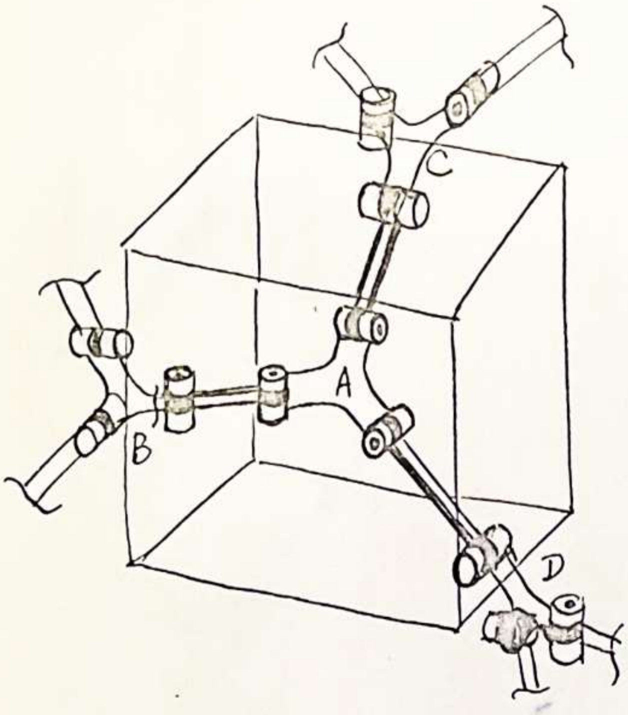
The centers of the extended vertices of types A, B, C, and D are — in the fully deployed configuration of this modified Laves graph — again situated at the midpoints of edges of the imaginary cubes with respect to which the vertices and edges of the graph are symmetrically positioned.



This rough sketch shows the arrangement of extended vertices and edges which are joined to the extended vertex A, in the fully deployed configuration of the graph.

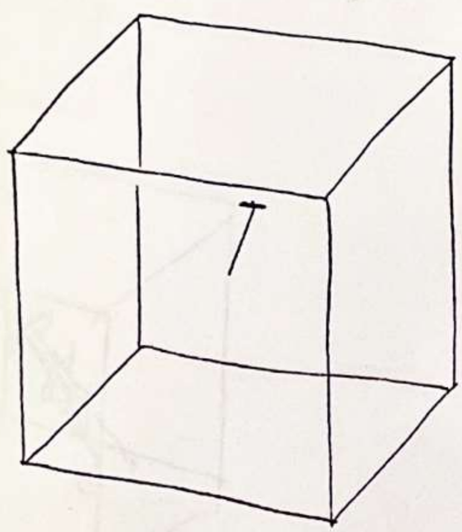
The actual relative size of the extended vertices and the edge lengths is not intended to be interpreted literally. In practice, the hinge pins would be located much closer to the center of each extended vertex than shown here, in order to avoid excessive reduction in the ratio $\left(\frac{V_e}{V_c}\right)$ of fully expanded volume to fully collapsed volume.

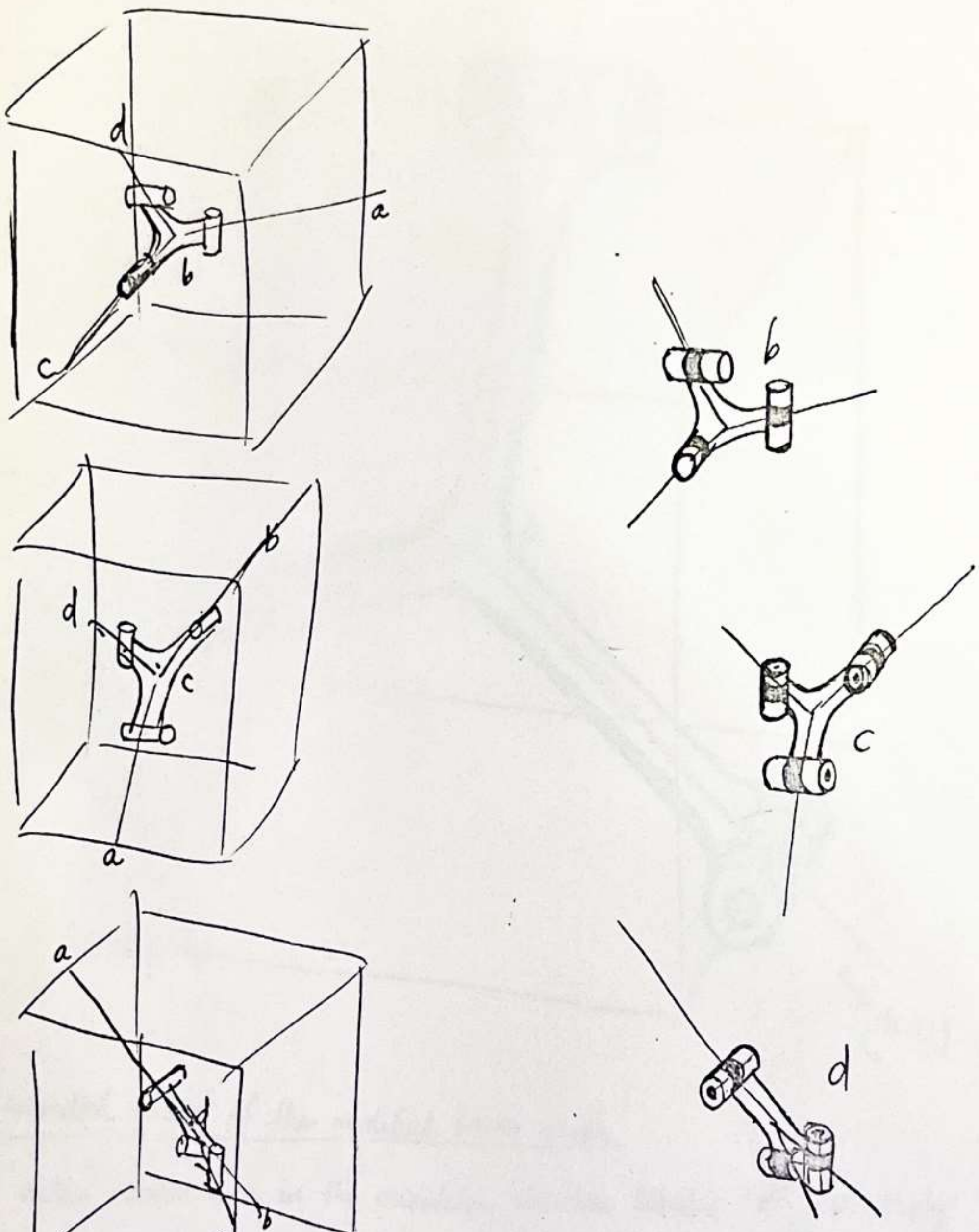




This rough sketch shows the arrangement of extended vertices and edges which are joined to the extended vertex A, in the fully deployed configuration of the graph.

The actual relative size of the extended vertices and the edge lengths is not intended to be interpreted literally. In practice, the hinge pins would be located much closer to the center of each extended vertex than shown here, in order to avoid excessive reduction in the ratio $\left(\frac{V_e}{V_c}\right)$ of fully expanded volume to fully collapsed volume.




d
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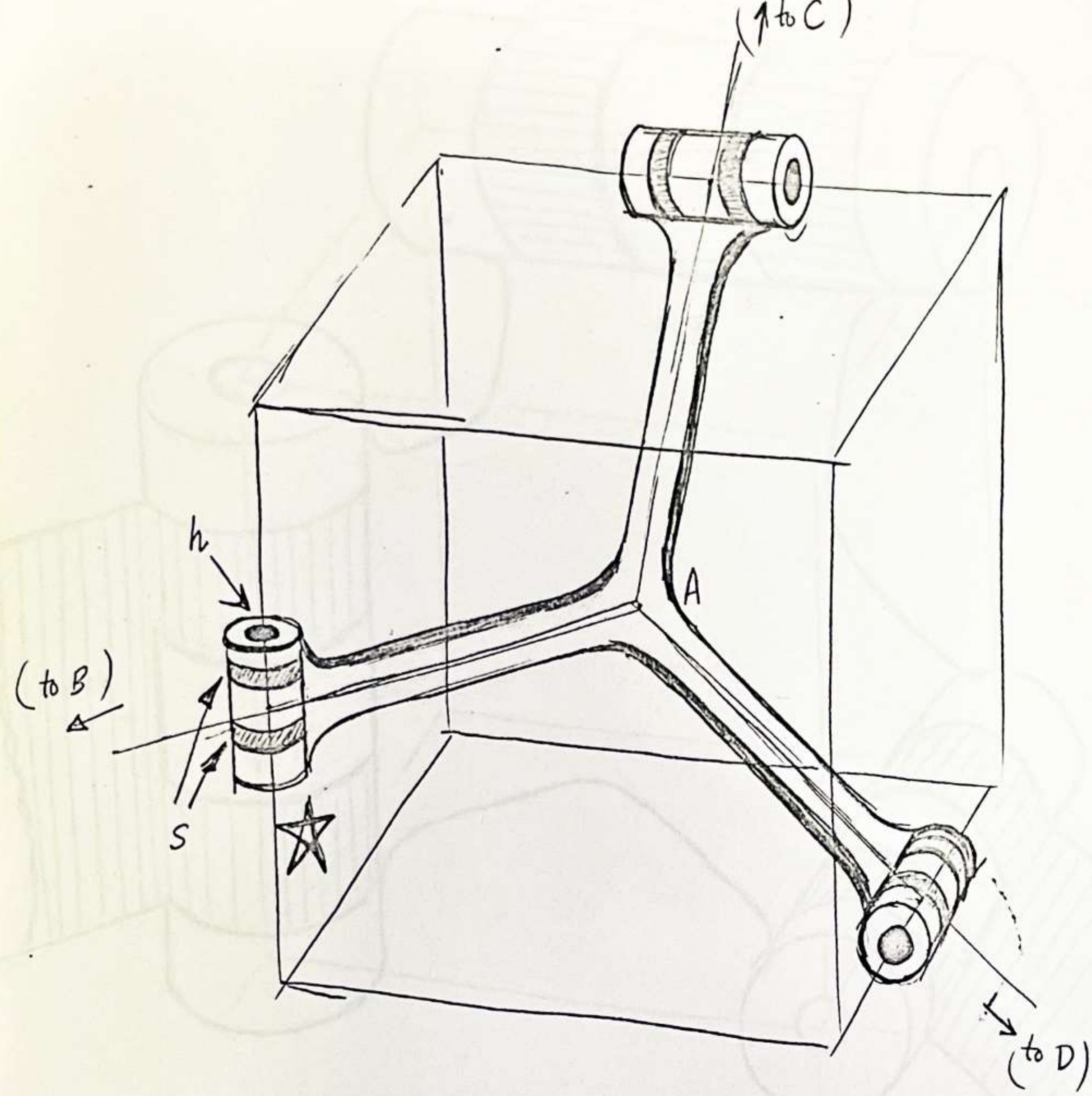
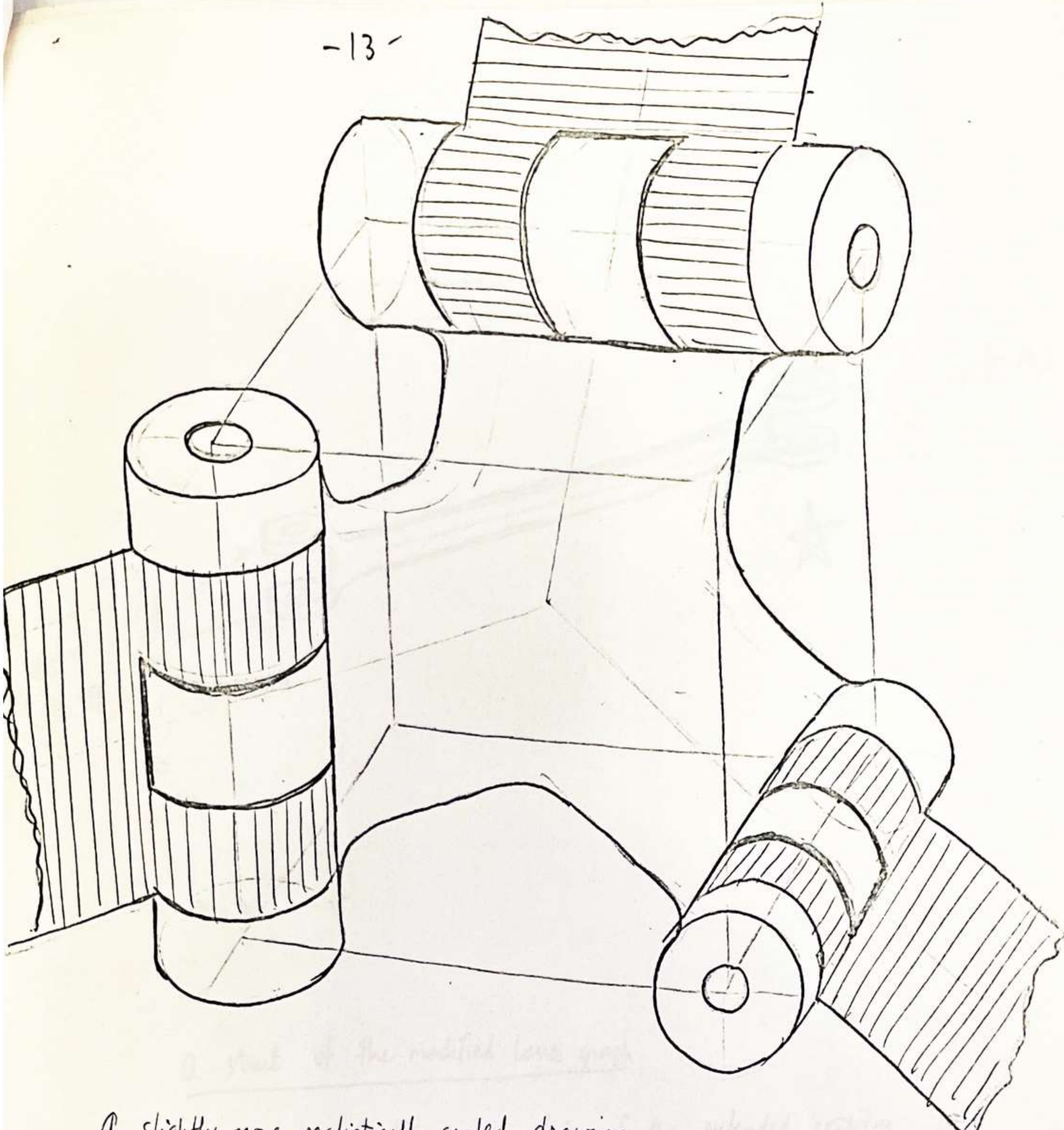


An "extended vertex" of the modified Laves graph.

This "vertex", shown here in the orientation elsewhere labelled "A", is simply rotated into different orientations to occupy the positions elsewhere labelled "B", "C", and "D". Thus, it is a standard part which is interchangeable with all the other similar parts.

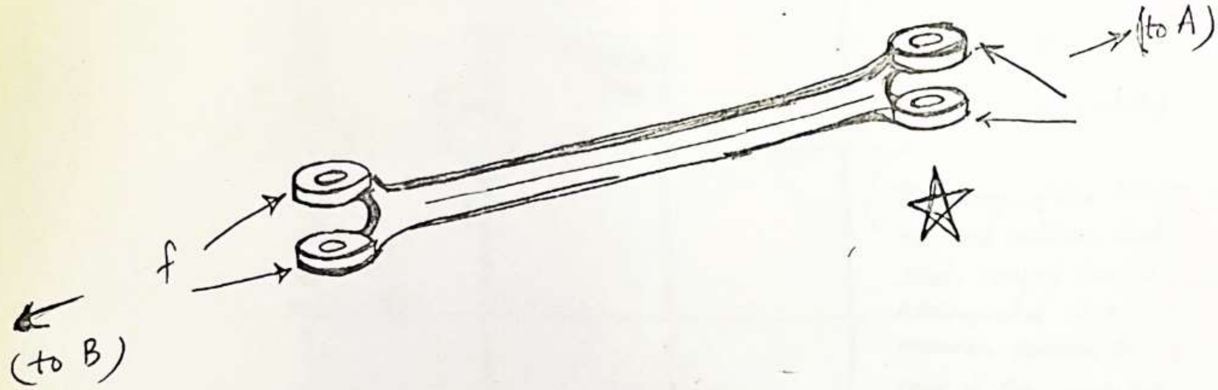
The holes "h" are for the hinge pins which fasten the edges or struts to these extended vertex parts.

The slots "s" are arranged to accommodate the flanges "f" of the struts, of which an example is sketched on the following page.



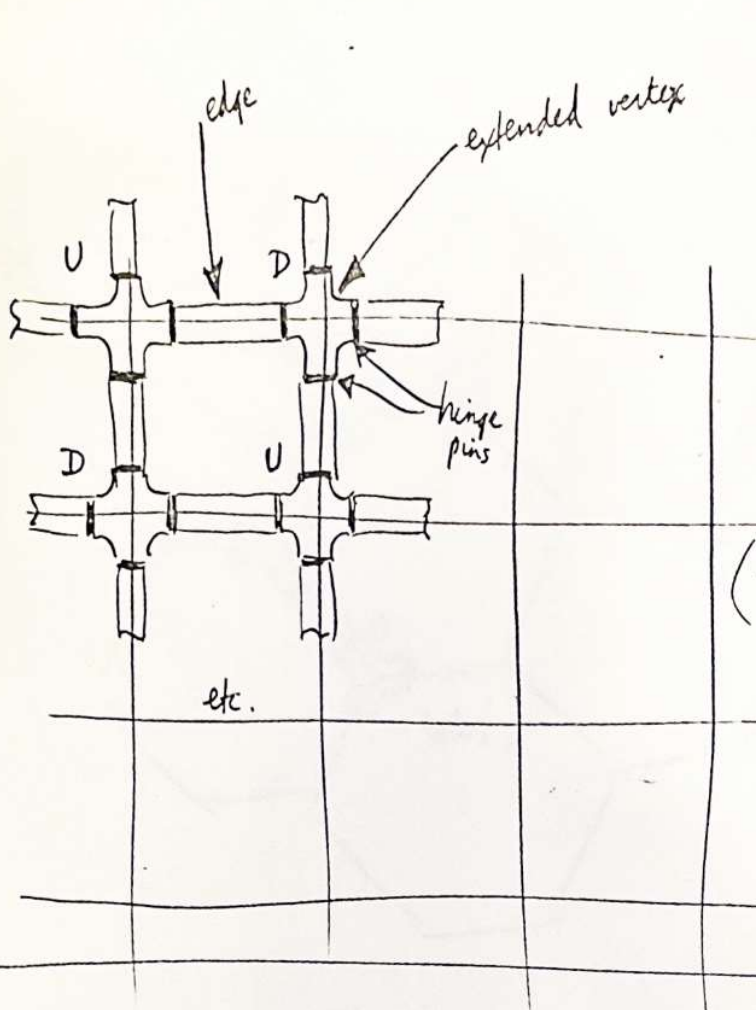
A slightly more realistically scaled drawing
than the one on the previous page of the hinge-joint assembly
for the expandable space-frame based on the modified Laves graph.

The hinges are to be provided with spring-loading or other mechanisms for automatic deployment, so that when the assembled space-frame is stored in the collapsed state initially, it will synchronously



a strut of the modified Laves graph.

The flanges f fit in the slots of the extended vertices, of which an example is illustrated on the preceding page.

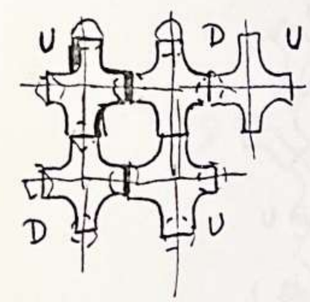


Plan view

(fully expanded)

(The design of the hinges, extended vertices, and struts (edges) can be accomplished in a manner similar to the case of the modified Laves graph.)

Plan view

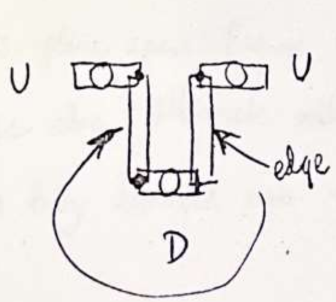


This sketch shows the fully expanded configuration (above) and the fully collapsed configuration (below) of a plane space-frame based on the square two-dimensional lattice

The principles are the same as discussed for the Laves graph in some detail.

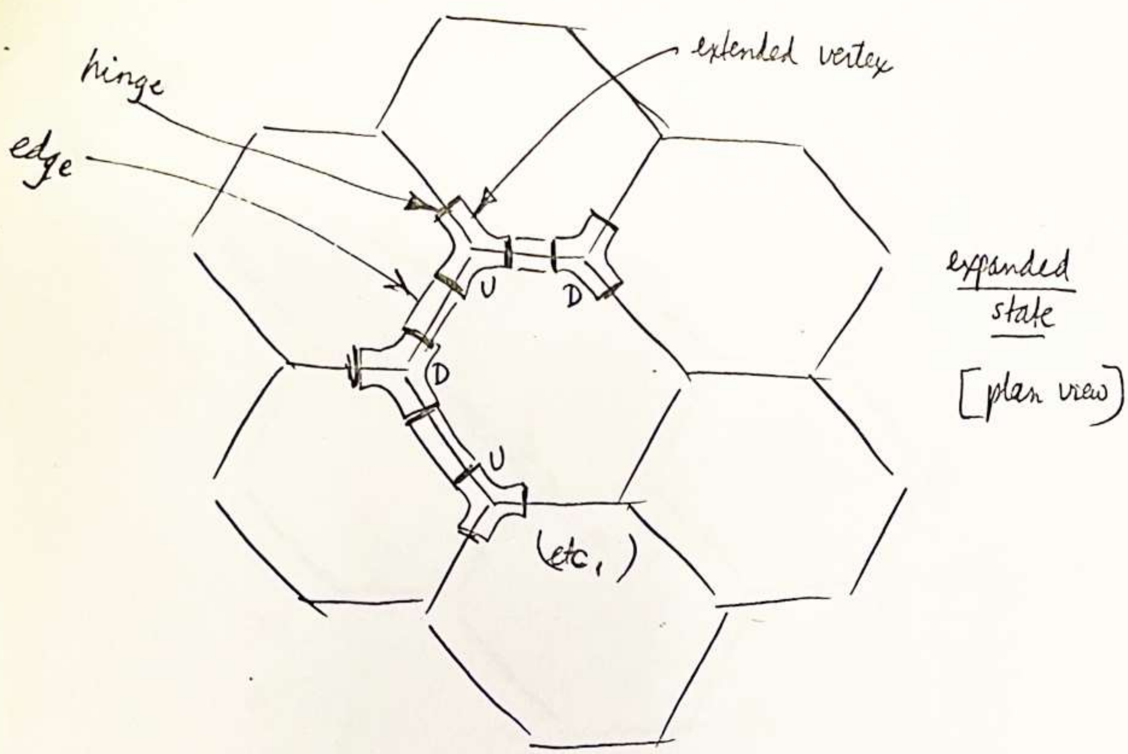
Alternate vertices, in a checkerboard pattern, move up (U) and down (D), the edges joining them being thereby constrained to rotate with respect to either vertex in a circular motion.

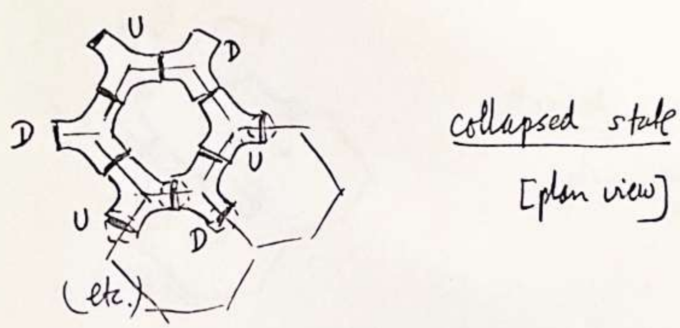
Side view



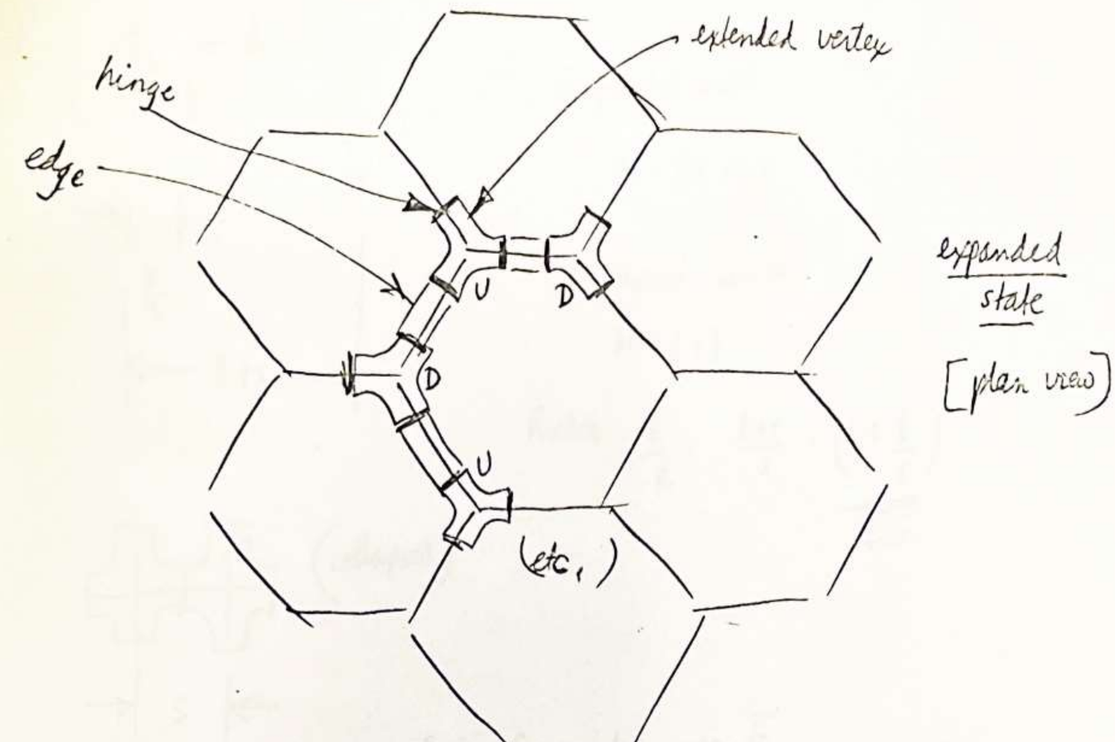
(fully collapsed configuration)

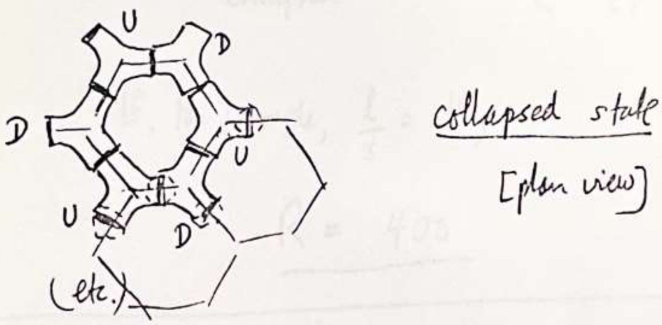
(The sketches are not to scale. The edges should be much longer, in order to obtain a usefully large ratio of expanded area to collapsed area of the space-frame.)





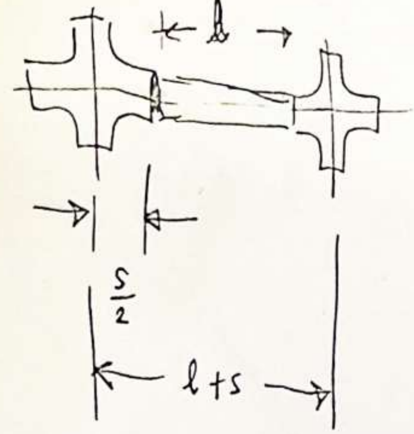
This plane space-frame is based on the regular hexagonal graph in the plane. Here also, alternate vertices move up (U) and down (D) in order for the fully deployed state to be transformed into the fully collapsed state.





This plane space-frame is based on the regular hexagonal graph in the plane. Here also, alternate vertices move up (U) and down (D) in order for the fully deployed state to be transformed into the fully collapsed state.

(expanded)



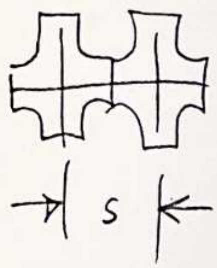
Expanded width:

$N \cdot (l + s)$

Collapsed width:

$N \cdot (s)$

Ratio $\frac{E}{C} = \frac{l+s}{s} = \left(1 + \frac{l}{s}\right)$

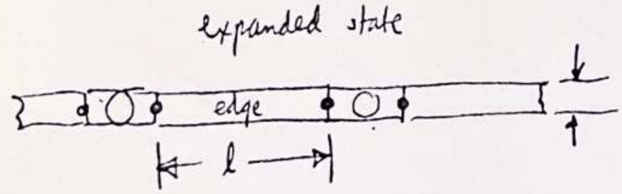


(collapsed)

Ratio of expanded area to collapsed area $= \left(1 + \frac{l}{s}\right)^2 \equiv R$

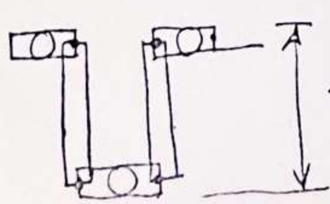
If, for example, $\frac{l}{s} = 19$,

$$\underline{R = 400}$$



$t_e = \Delta =$ thickness of edges and expanded vertices

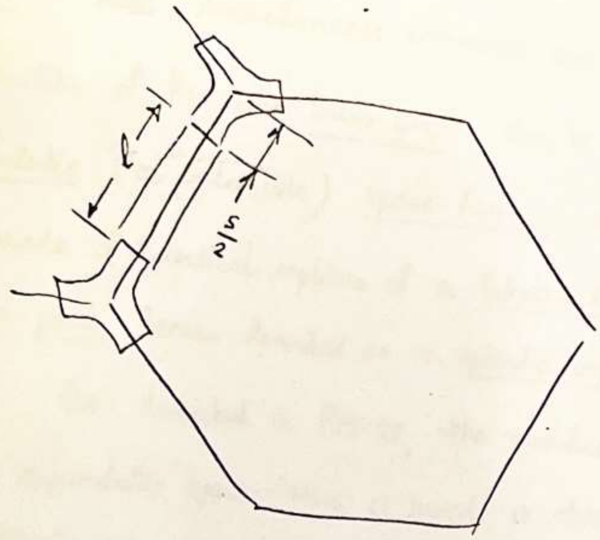
(expanded thickness)



$t_c = \Delta + l =$ thickness of expanded vertices PLUS length of edges

(collapsed thickness)

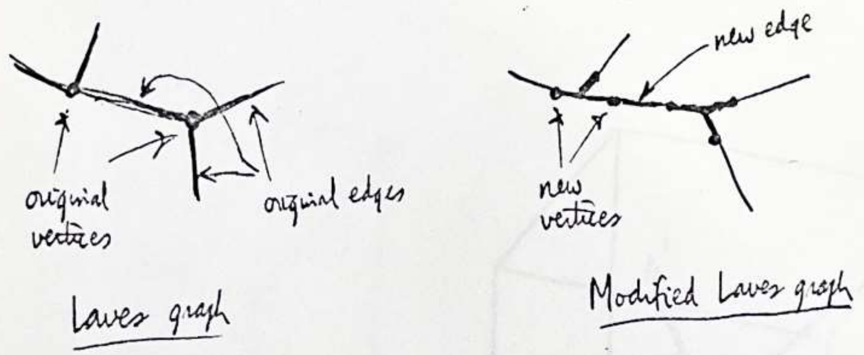
r = ratio of collapsed thickness to expanded thickness $= \frac{\Delta + l}{\Delta} = 1 + \frac{l}{\Delta}$



R = ratio of expanded area to collapsed area = $\left(1 + \frac{l}{s}\right)^2$

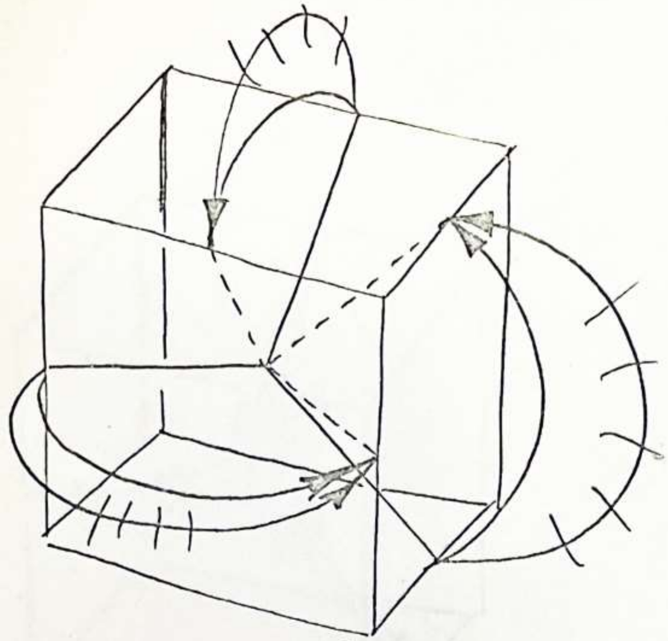
<u>Note:</u> These miscellaneous comments are intended to provide an explanation of how the <u>Laves graph</u> can be used as the basis for an <u>expandable</u> (or collapsible) <u>space-frame</u>, a structural framework which is made of identical replicas of a <u>tubular strut</u> and identical replicas of a <u>joint</u>, herein described as an <u>extended vertex</u>.

As described in PM-58, the modified (Laves) graph on which this expandable space-frame is based is obtained by symmetrically introducing <u>new vertices</u> a fixed distance — along each edge — away from



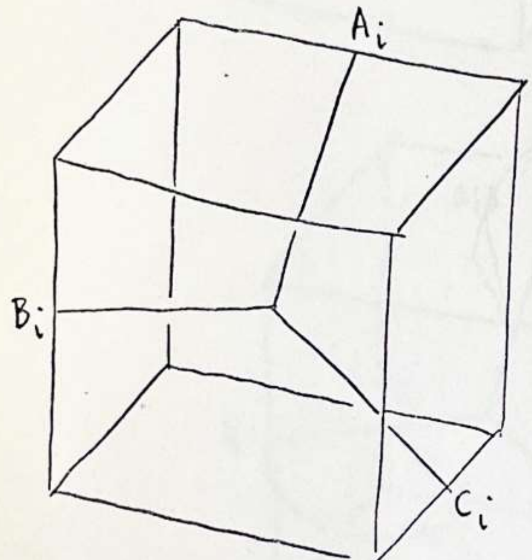
the original vertices. The old edges are thereby reduced in length, as the above rough sketch shows. The rigid structure

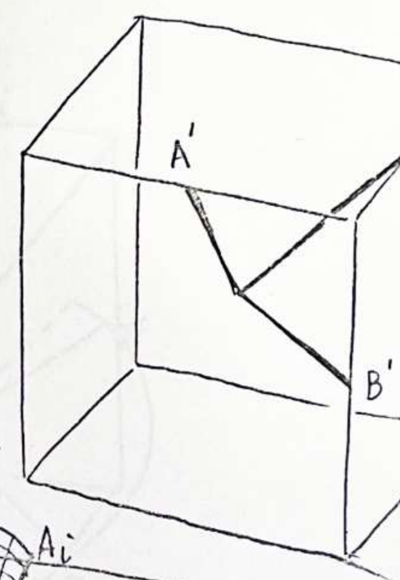
produced in this transformation is the object called an extended vertex. It is connected by ordinary hinges to the new edges.

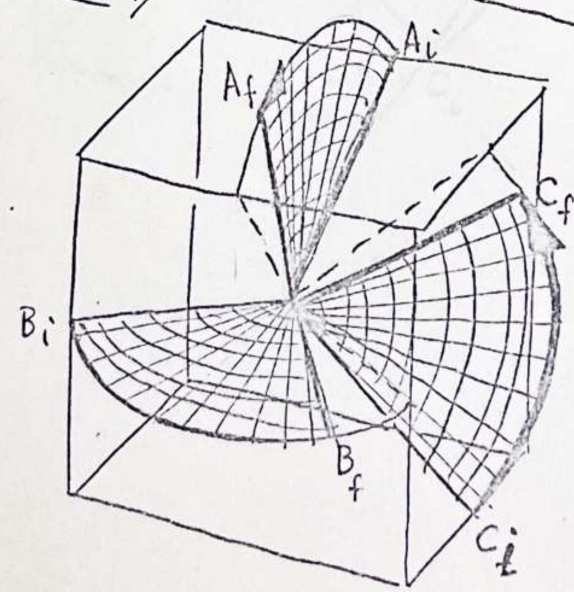


laves graph

(idealized geometry, with zero separation of pivots for the three struts incident at each vertex)

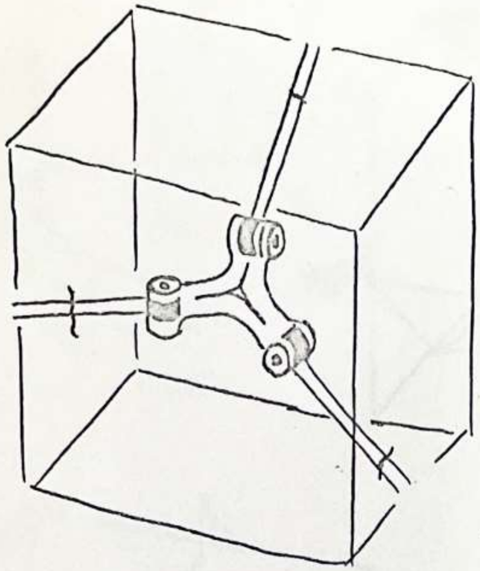


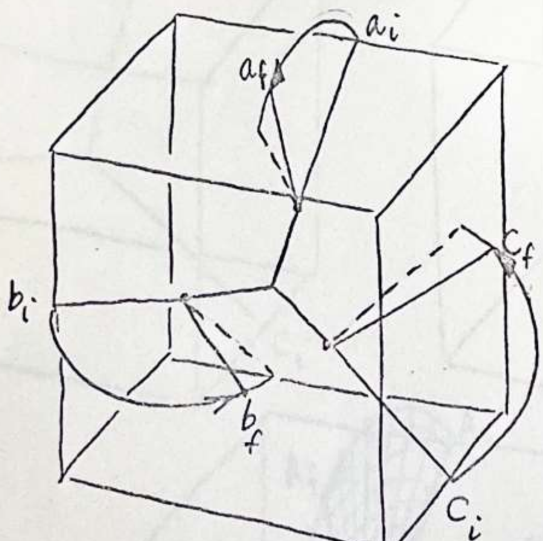


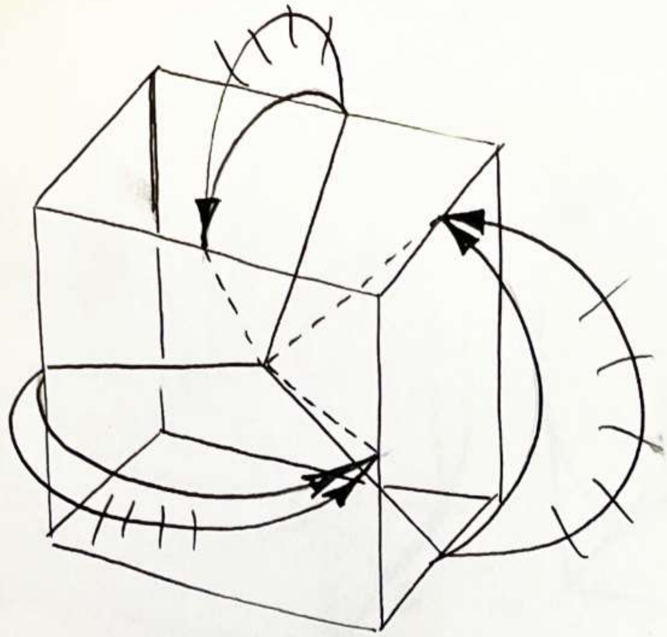


Laves graph

(actual geometry, with non-zero separation of the pivots for the three struts incident at each vertex)

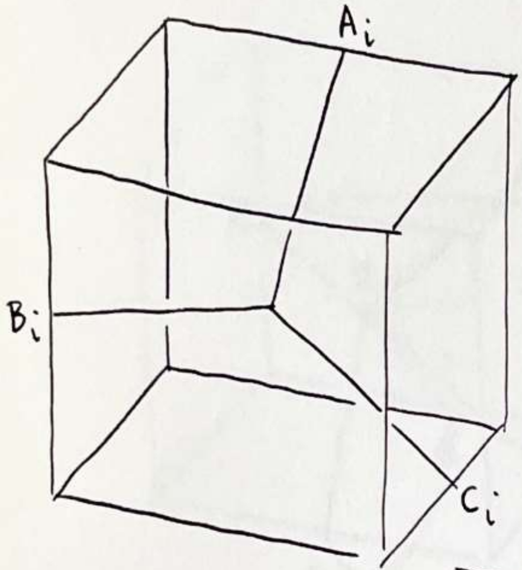


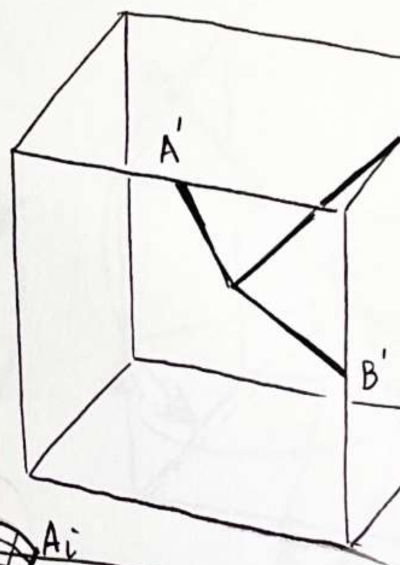


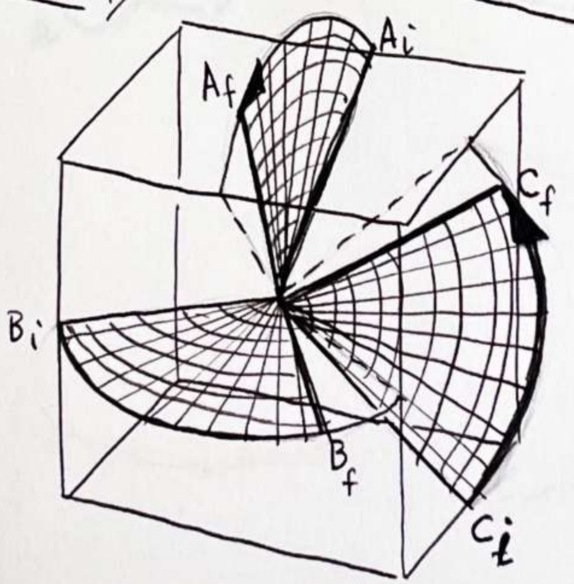


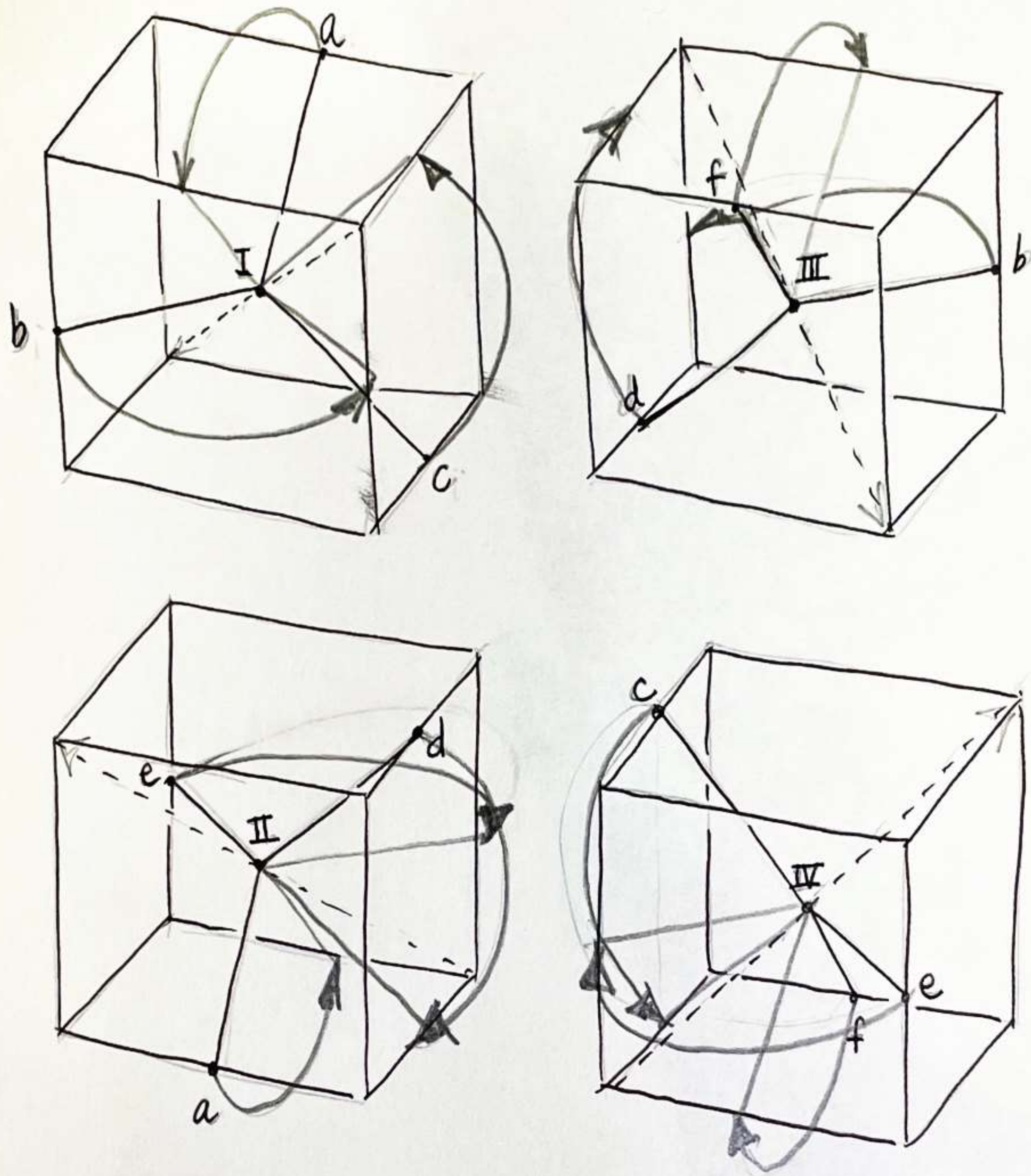
laves graph

(idealized geometry, with zero separation of pivots for the three struts incident at each vertex)








a
I
b
c
f
III
b
d
d
e
II
a
c
IV
e
f