12/15/68

Pre-stressed IPMS structures

Since P, D, and C are "Biegungsfläche" of one another, and since there is an infinite continuum of such surfaces — so long as one is considering a portion as small as a fundamental region or smaller — it would be possible to build, say, a finite structure in the shape of a multi-celled portion of P, using hexagonal modules of the regular map $\{\tilde{6}, \tilde{4} | 4\}$, ~~but~~ each prepared in the form of a nearby Biegungsfläche hexagon relative of $\{\tilde{6}, \tilde{4} | 4\}_P$. By bending such hexagons in the opposite sense and bonding them back-to-back, so as to produce double-layer modules, ~~they would~~ one could make them "equilibrate" to the desired shape (provided they had to be subjected to equal amounts of strain energy to bend into the desired shape). With a sufficiently strong bonding agent (e.g., epoxy, etc.), one would have a double-layer labyrinthine structure with complementary stress patterns from the two layers. Such a structure might be very well suited to supporting arbitrary loads. Resolved shear stresses in the surface which would tend to lead to plastic deformation in one layer might be successfully opposed by the properly pre-stressed configuration of the other layer. (Modules from dual maps could be bonded back-to-back, as in the "Quincunx" model I made in Los Angeles of D.) This idea might also be useful when applied to other configurations. For example: plane layers which have been prestressed by elongation (or equivalent treatment, via phase transformation in alloy of preferred orientation, etc.) could be bonded (e.g., by diffusion bonding, adhesive-bonded lamination, etc.) in alternating orientations, so as to achieve a net strength, with respect to some kinds of loads, which might exceed the strength of an equivalent thickness of material not prepared by this recipe.

2/3/2013

Made stereo plots with Mathematica of this polyhedron (p. 1)

6 viewpoints:

1 1 -1

-1 -1 1

1 0 0

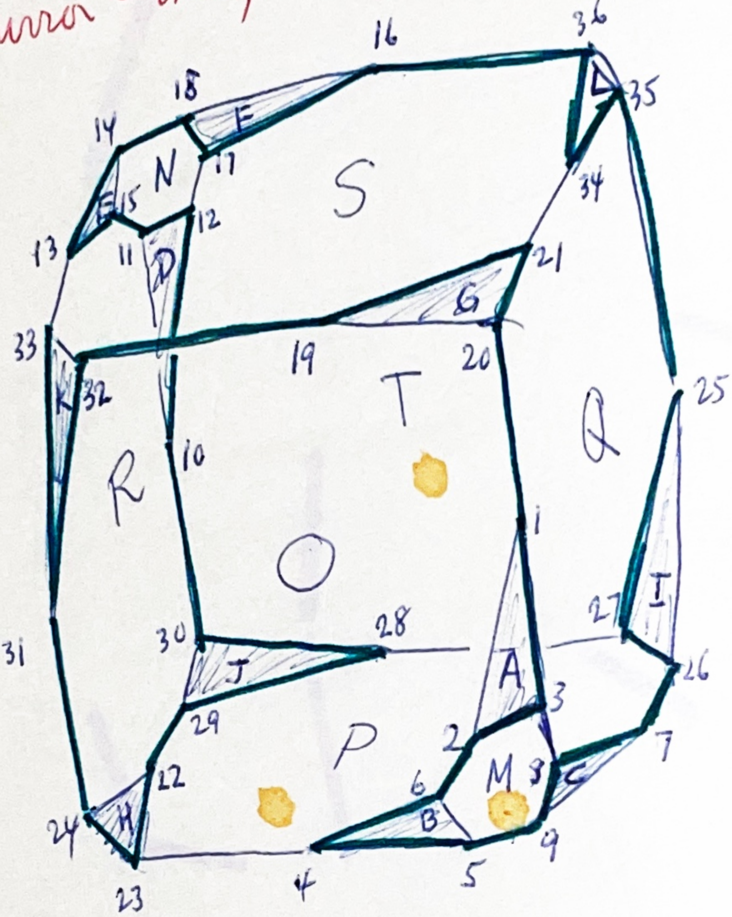
0 1 0

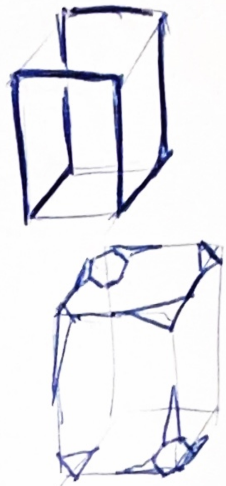
-1 0 0

0 -1 0

1.0

(A mirror-image cell)

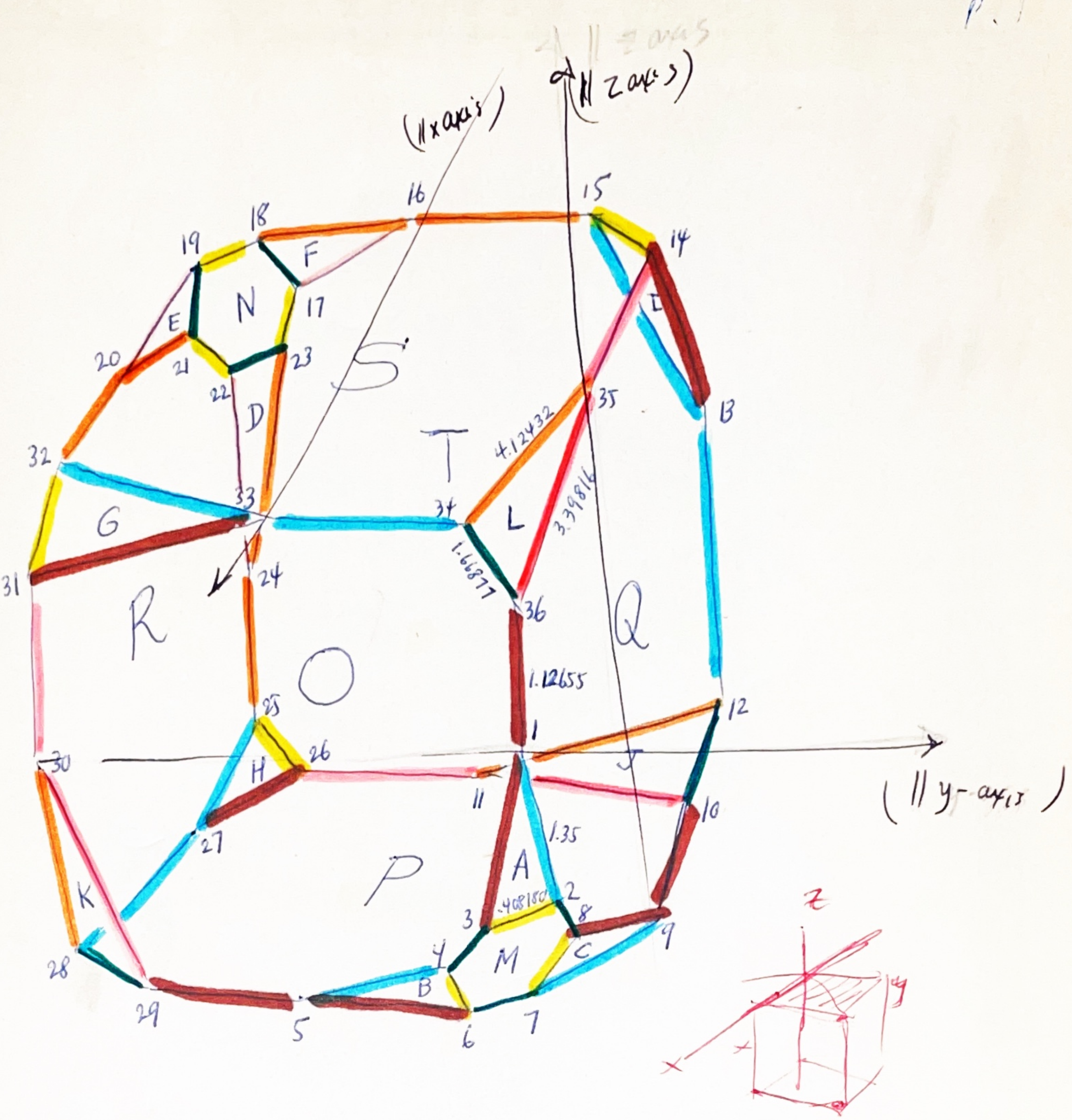




$$\left\{\begin{aligned} V &= 36 \\ E &= 54 \\ F &= 20 \end{aligned}\right)$$

A
B
C
D
E
F
G
H
I
J

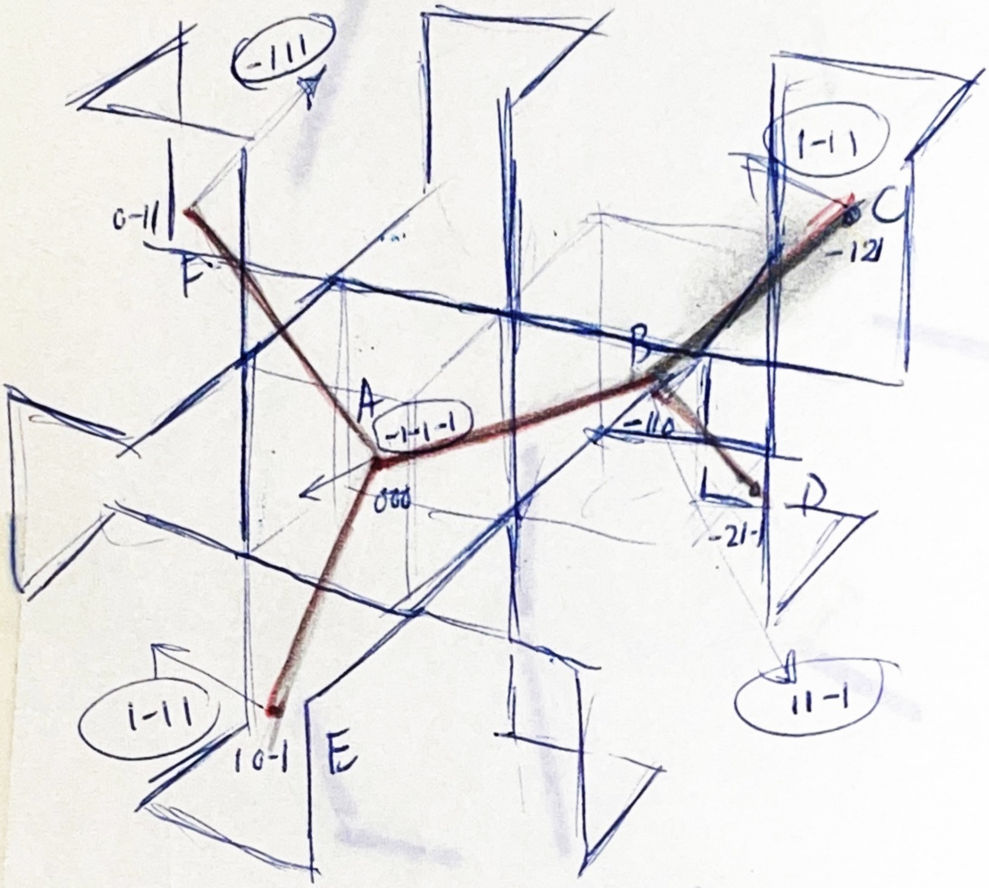
K
L
M
N
O
P
Q
R
S
T

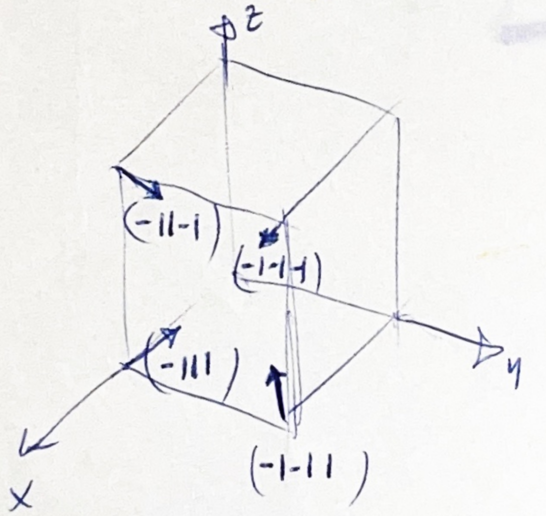


- $|1-2| = 1.35$
- $|2-3| = .408180$
- $|3-4| = 1.66877$
- $|5-6| = 1.12655$
- $|10-11| = 3.39816$
- $|11-12|$ 4.12432

The 2-fold axes lying in the 6 principal faces are invariants under the whole collapsing transformation.

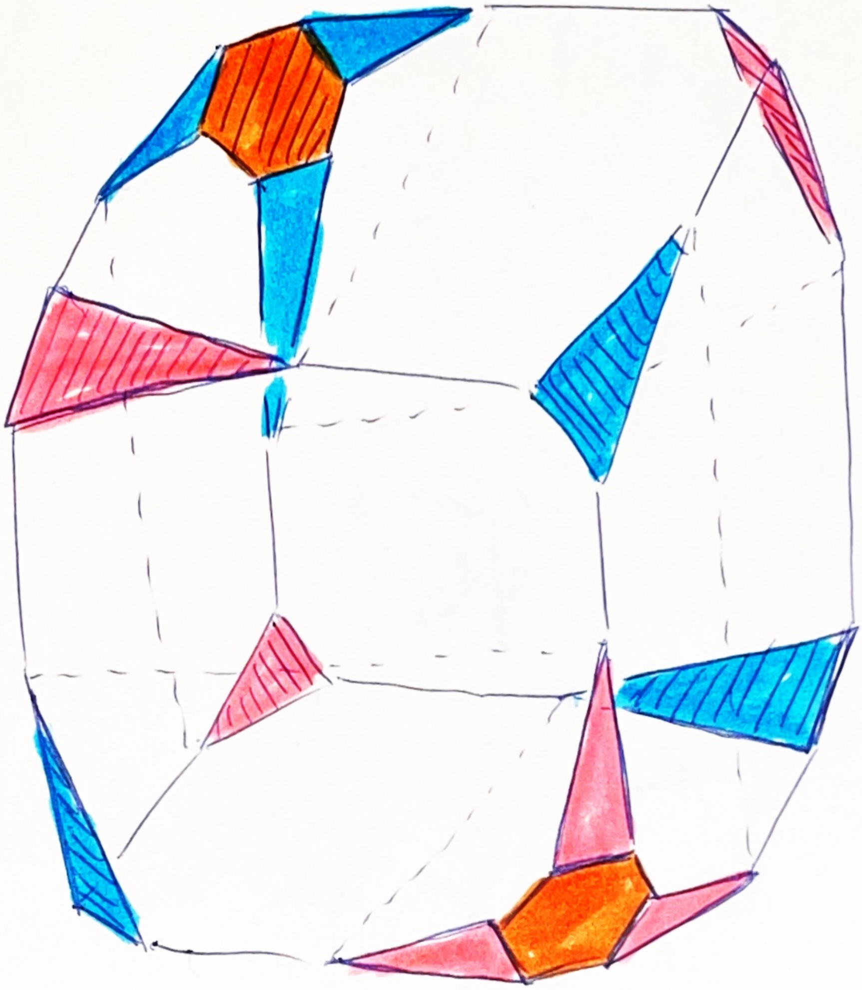
Laves graph   Only 4 $\vec{\Lambda}_i$





Shaded triangles are congruent to triangles of same color, but enantiomorphic.

There are 5 distinct shapes of faces

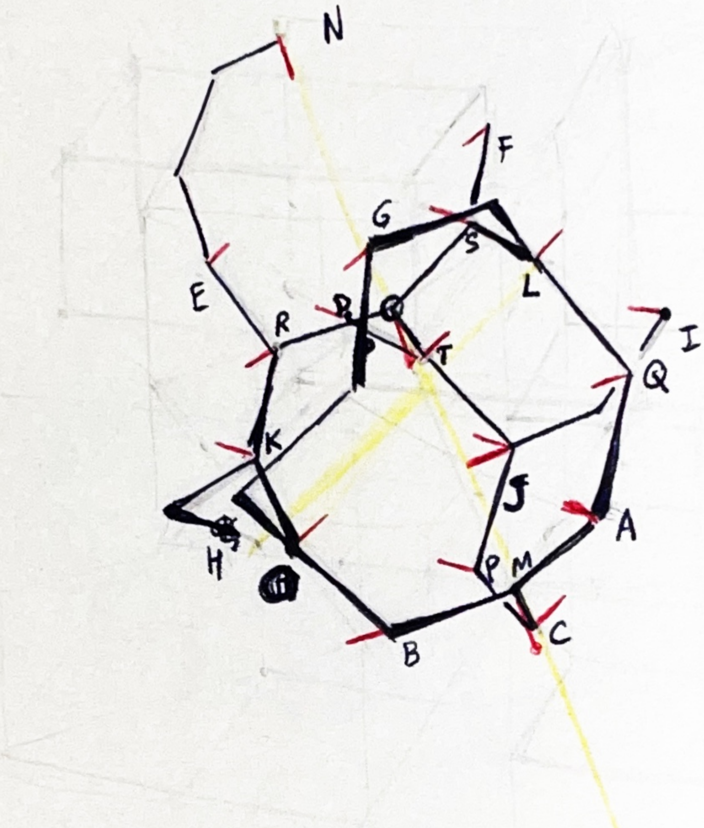


These faces can be thought of as slightly mis-oriented (100) faces [6]

+ (110) faces [12]

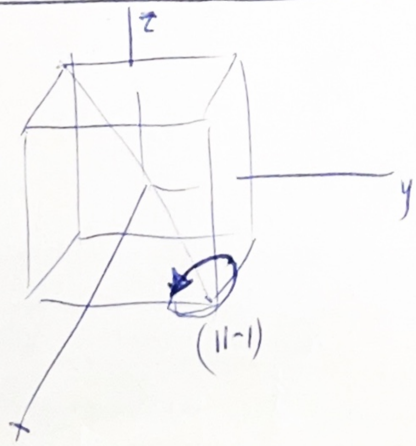
+ 2 (1-1-1) faces [2]

$\Sigma = 20$

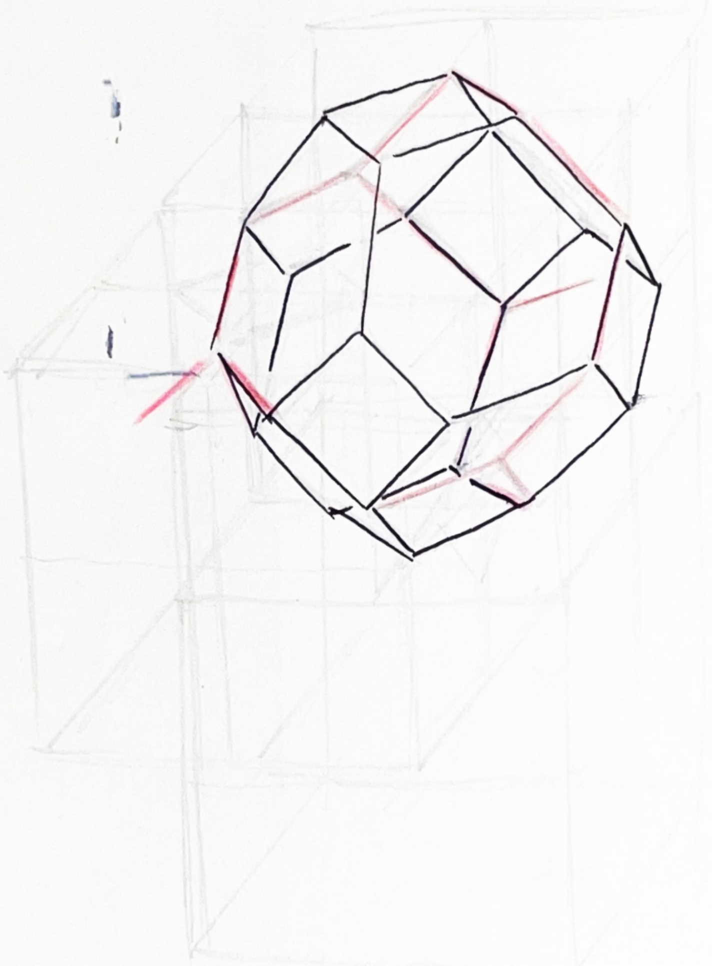


N
F
G
S
L
E
R
Q
T
I
K
J
A
H
P
M
C
B

Name of face	origin to "face" (normal to plane of face	Position P of displaced vertex (vertex is vertex of collapsing Laves graph).	Normalized P_o (original vertex position)	Normalized P ("collapsed" vertex position)	$\vec{P} - \vec{r}_{origin}$ (after displacement)	
A	3 4 -1	2 3 -1 + $\frac{1}{6}$ (1 -1 1)	12, 18, -6	13, 17, -5	12, 16, -4	1
B	4 1 -3	3 1 -2 -1 -1 -1	18, 6, -12	17, 5, -13	16, 4, -12	2
C	1 3 -4	1 2 -3 -1 1 1	6, 12, -18	5, 13, -17	4, 12, -16	3
D	-3 -2 -1	-2 -1 -1 1 -1 1	-12, -6, -6	-11, -7, -5	-12, -8, -4	4
E	1 -3 2	1 -2 1 -1 1 1	6, -12, 6	5, -11, 7	4, -12, 8	5
F	-2 1 3	-1 1 2 -1 -1 -1	-6, 6, 12	-7, 5, 11	-8, 4, 12	6
G	4 1 3	3 1 2 -1 -1 -1	18, 6, 12	17, 5, 11	16, 4, 12	7
H	1 -3 -4	1 -2 -3 -1 1 1	6, -12, -18	5, -11, -17	4, -12, -16	8
I	-3 4 -1	-2 3 -1 1 -1 1	-12, 18, -6	-11, 17, -5	-12, 16, -4	9
J	-2 1 -3	-1 1 -2 -1 -1 -1	-6, 6, -12	-7, 5, -13	-8, 4, -12	10
K	3 -2 -1	2 -1 -1 1 -1 1	12, -6, -6	13, -7, -5	12, -8, -4	11
L	1 3 2	1 2 1 -1 1 1	6, 12, 6	5, 13, 7	4, 12, 8	12
M	3 3 -3	2 2 -2 1 1 -1	6, 6, -6	13, 13, -13	12, 12, -12	13
N	-3 -3 3	-2 -2 2 1 1 -1	-6, -6, 6	-11, -11, 11	-12, -12, 12	14
O	4 0 -1	3 0 -1 + $\frac{1}{6}$ (-1 1 1)	18, 0, -6	17, 1, -5	16, 0, -4	15
P	0 1 -4	0 1 -3 + $\frac{1}{6}$ (1 -1 1)	0, 6, -18	1, 5, -17	0, 4, -16	16
Q	1 4 0	1 3 0 + $\frac{1}{6}$ (-1 -1 -1)	6, 18, 0	5, 17, -1	4, 16, 0	17
R	1 -2 0	1 -1 0 + $\frac{1}{6}$ (-1 -1 -1)	6, -6, 0	5, -7, -1	4, -8, 0	18
S	0 1 2	0 1 1 + $\frac{1}{6}$ (1 -1 1)	0, 6, 6	1, 5, 7	0, 4, 8	19
T	-2 0 -1	-1 0 -1 + $\frac{1}{6}$ (-1 1 1)	-6, 0, -6	-7, 1, -5	-8, 0, -4	20



The matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$ rotates the cube CCW through 120° about the 11-1 axis (looking toward the origin)



//FB7272

sim q

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & -1 \end{pmatrix}$$

$$\begin{matrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{matrix}$$

//FB7272 JOB E1,

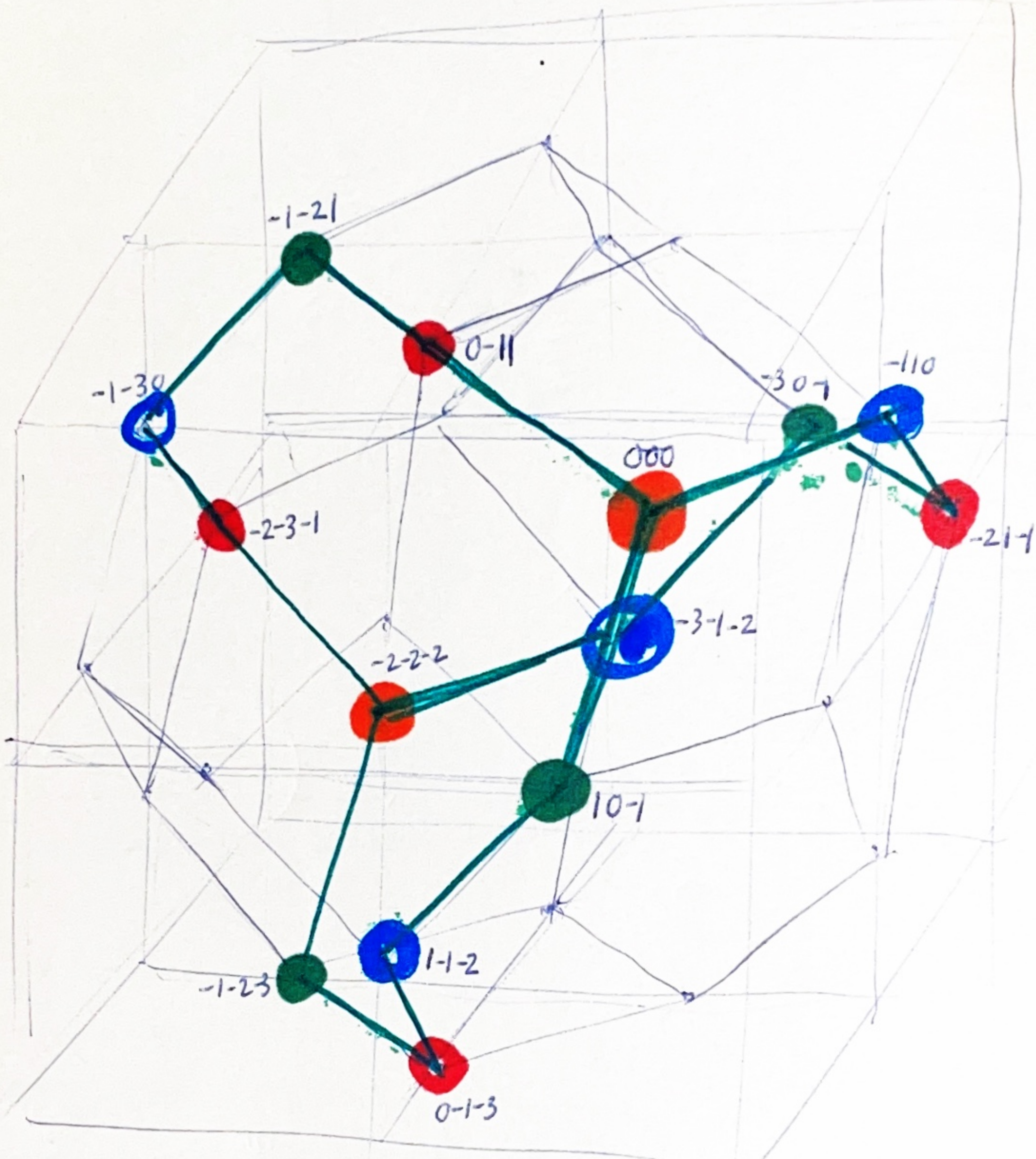
Computed vertices of 20-faced Dirichlet cell. from orig…

name	vertex	coordinates of vertex			
1	AOQ	4	3.25	-1	1/4(16, 13, -4)
2	AMQ	3.4	3.4	-2.2	1/5(17, 17, -11)
3	AOM	3.727272	3.181816	-2.090909	1/11(41, 35, -23)
4	BMO	3.4	2.2	-3.4	1/5(17, 11, -17)
5	BOP	3.25	1	-4	1/4(13, 4, -16)
6	BMP	3.181818	2.090909	-3.727272	1/11(35, 23, -41)
7	CMP	2.2	3.4	-3.4	1/5(11, 17, -17)
8	CMQ	2.090909	3.727272	-3.181818	1/11(23, 41, -35)
9	CPQ	1	4	-3.25	1/4(4 16 -13)
10	JPQ	-.090909	4.272727	-3.181818	1/11(-1, 47, -35) (2-6-2013 verified)
11	JPT	-.5	1	-4	1/2(-1, 2, -8)
12	JQT	-1.4	4.6	-2.2	1/5(-7, 23, -11)
13	IQT	-2	4.75	-1	1/4(-8, 19, -4)
14	IGS	-2.272727	4.818181	.090909	1/11(-25, 53, 1)
15	IST	-2.6	4.6	.2	1/5(-13, 23, 1)
16	FST	-3.5	1	2	1/2(-7, 2, 4)
17	FNT	-3.9090909	-2.272727	2.818181	1/11(-43, -25, 31)
18	FNS	-2.6	-2.6	3.8	1/5(-13, -13, 19)
19	ENS	-2.272727	-2.81818	3.9090909	1/11(-25, -31, 43)
20	ERS	1	-2	7/2 = 3.5	1/2(2, -4, 7)
21	ENR	-2.6	-3.8	2.6	1/5(-13, -19, 13)
22	DNR	-2.818181	-3.9090909	2.272727	1/11(-31, -43, 25)
23	DNT	-3.8	-2.6	2.6	1/5(-19, -13, 13)
24	DRT	-2	-3.5	-1	1/2(-4, -7, -2)
25	HRT	-.2	-2.6	-4.6	1/5(-1, -13, -23)
26	HPT	-.090909	-2.272727	-4.818181	1/11(-1, -25, -53)
27	HPR	1	-2	-4.75	1/4(4, -8, -19)
28	KPR	2.2	-1.4	-4.6	1/5(11, -7, -23)
29	KOP	3.181818	-.090909	-4.272727	1/11(35, -1, -47)
30	KOR	4	-.5	-1	1/2(8, -1, -2)
31	GOR	4.818181	-.090909	2.272727	1/11(53, -1, 25)
32	GRS	4.6	-.2	2.6	1/5(23, -1, 13)
33	GOS	4.75	1	2	1/4(19, 4, 8)
34	LOS	4.6	2.2	1.4	1/5(23, 11, 7)
35	LQS	1	4	.5	1/2(2, 8, 1)
36	LOQ	4.272727	3.181818	.090909	1/11(47, 35, 1)

Computed vertices of 20-faced Dirichlet cell.

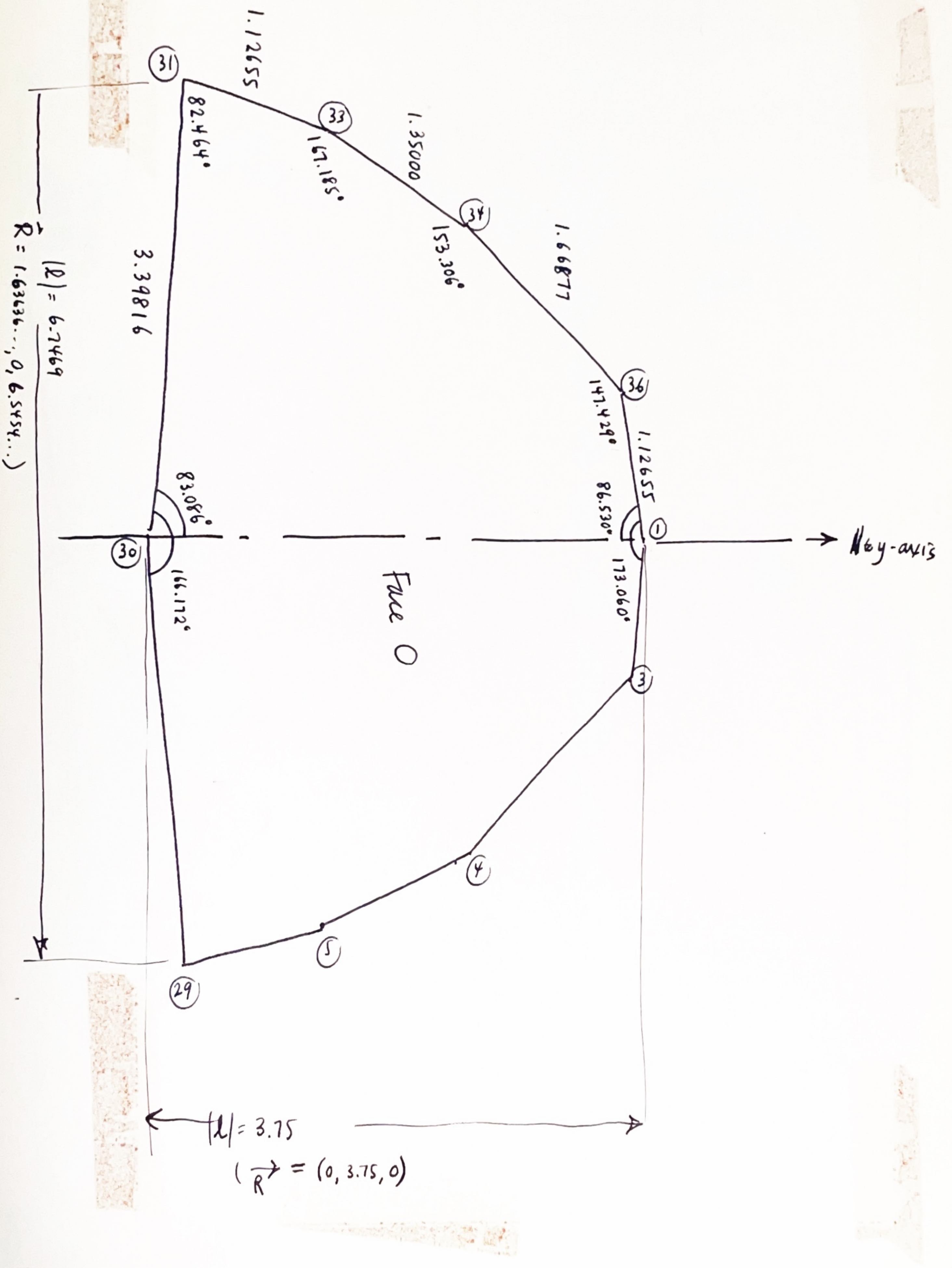
distance of farthest vertex from origin = 5.328079

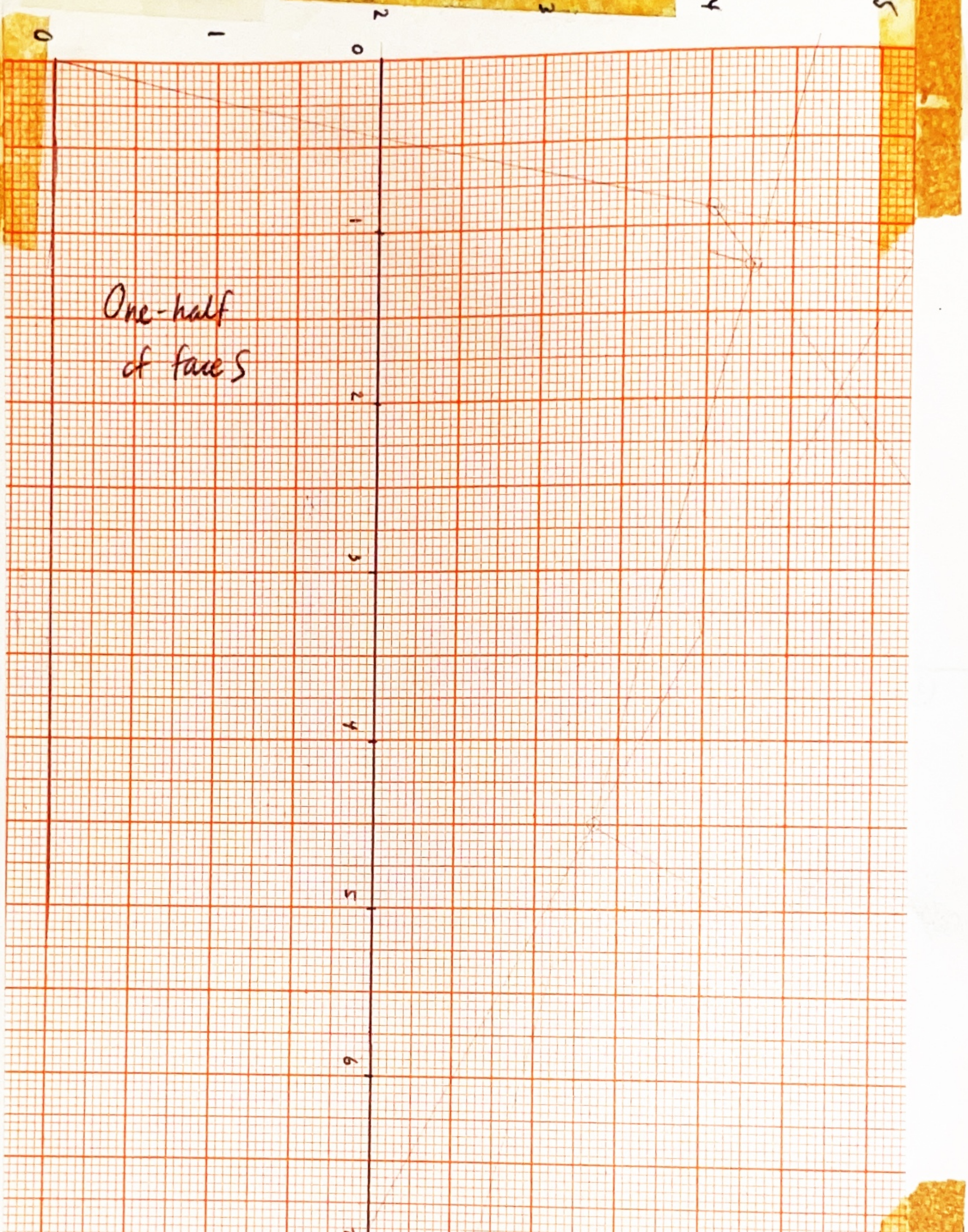
name	vertex	coordinates of vertex				
1	AOQ	4	3.25	-1	1/4(16, 13, -4)	
2	AMQ	3.4	3.4	-2.2	1/5(17, 17, -11)	
3	AOM	3.727272	3.181816	-2.090909	1/11(41, 35, -23)	
4	BMO	3.4	2.2	-3.4	1/5(17, 11, -17)	
5	BOP	3.25	1	-4	1/4(13, 4, -16)	
6	BMP	3.181818	2.090909	-3.727272	1/11(35, 23, -41)	
7	CMP	2.2	3.4	-3.4	1/5(11, 17, -17)	
8	CMQ	2.090909	3.727272	-3.181818	1/11(23, 41, -35)	
9	CPG	1	4	-3.25	1/4(4 16 -13)	
10	JPQ	-.090909	4.272727	-3.181818	1/11(-1, 47, -35)	(2-6-2013 verified)
11	JPT	-.5	1	-4	1/2(-1, 2, -8)	
12	JGT	-1.4	4.6	-2.2	1/5(-7, 23, -11)	
13	IGT	-2	4.75	-1	1/4(-8, 19, -4)	
14	IGS	-2.272727	4.818181	.090909	1/11(-25, 53, 1)	
15	IST	-2.6	4.6	.2	1/5(-13, 23, 1)	
16	FST	-3.5	1	2	1/2(-7, 2, 4)	
17	FNT	-3.9090909	-2.272727	2.818181	1/11(-43, -25, 31)	
18	FNS	-2.6	-2.6	3.8	1/5(-13, -13, 19)	
19	ENS	-2.272727	-2.81818	3.9090909	1/11(-25, -31, 43)	
20	ERS	1	-2	7/2 = 3.5	1/2(2, -4, 7)	
21	ENR	-2.6	-3.8	2.6	1/5(-13, -19, 13)	
22	DNR	-2.818181	-3.9090909	2.272727	1/11(-31, -43, 25)	
23	DNT	-3.8	-2.6	2.6	1/5(-19, -13, 13)	
24	DRT	-2	-3.5	-1	1/2(-4, -7, -2)	
25	HRT	-.2	-2.6	-4.6	1/5(-1, -13, -23)	
26	HPT	-.090909	-2.272727	-4.818181	1/11(-1, -25, -53)	
27	HPR	1	-2	-4.75	1/4(4, -8, -19)	
28	KPR	2.2	-1.4	-4.6	1/5(11, -7, -23)	
29	KOP	3.181818	-.090909	-4.272727	1/11(35, -1, -47)	
30	KOR	4	-.5	-1	1/2(8, -1, -2)	
31	GOR	4.818181	-.090909	2.272727	1/11(53, -1, 25)	
32	GRS	4.6	-.2	2.6	1/5(23, -1, 13)	
33	GOS	4.75	1	2	1/4(19, 4, 8)	
34	LOS	4.6	2.2	1.4	1/5(23, 11, 7)	
35	LQS	1	4	.5	1/2(2, 8, 1)	
36	LOQ	4.272727	3.1818[illegible]	.090909	[illegible]	



-1-21
0-11
-1-30
-30-1
-110
000
-2-3-1
-21-1
-3-1-2
-2-2-2
10-1
1-1-2
-1-23
0-1-3

Face 0 (congruent to P & Q)

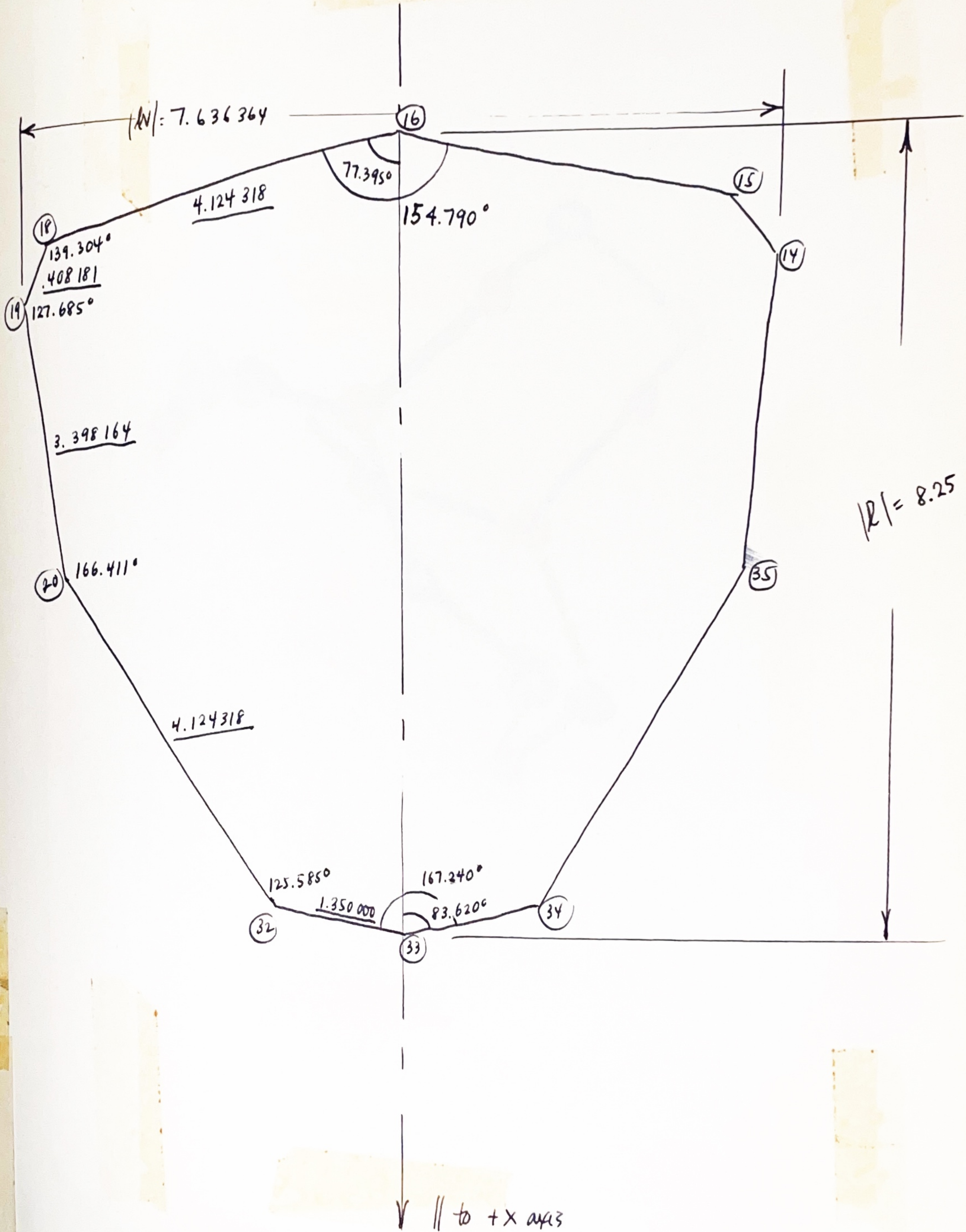


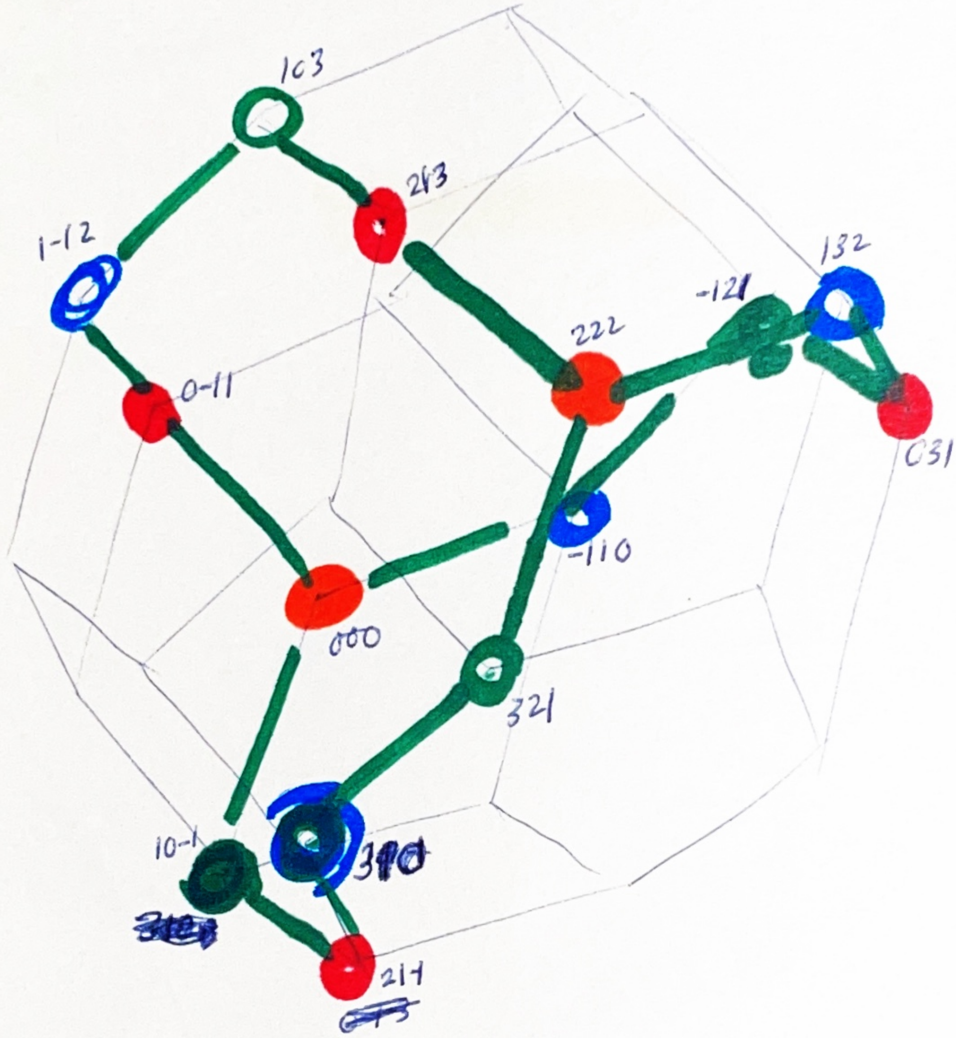


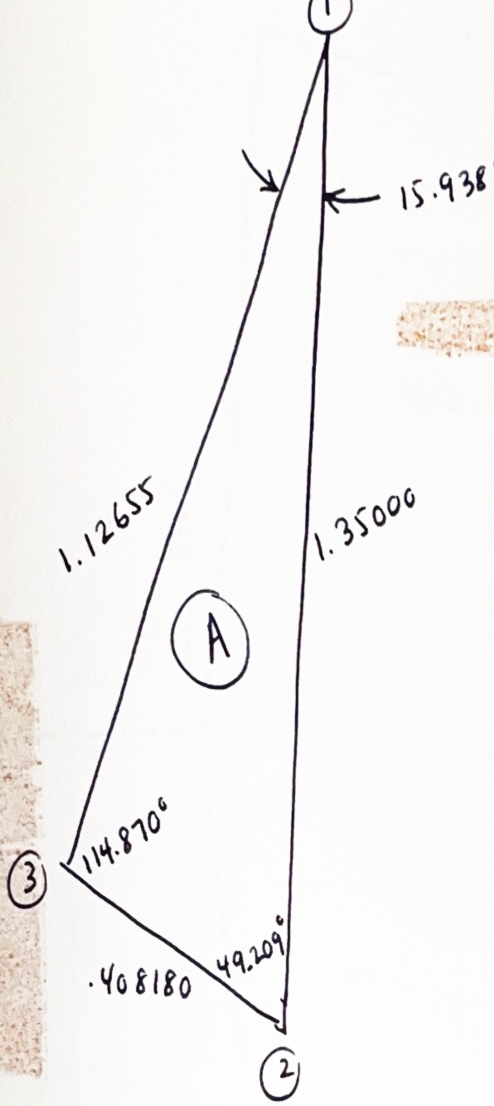
One-half of face 5

Face S

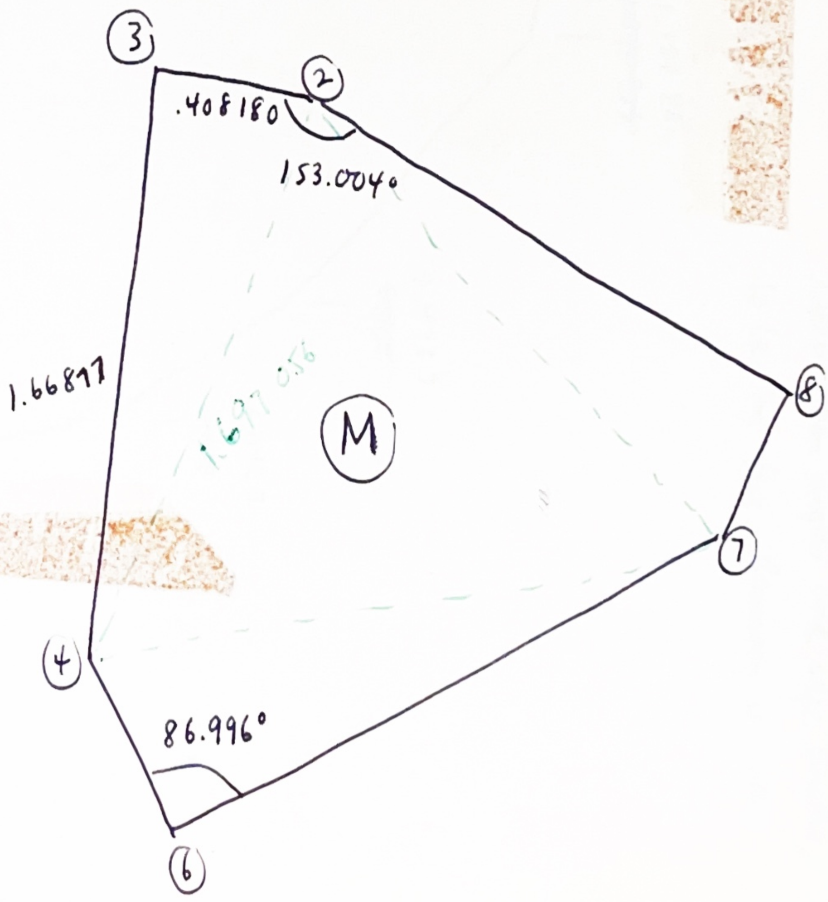
(congruent to R & T)




103
213
1-12
0-11
222
-121
132
031
-110
000
321
10-1
211

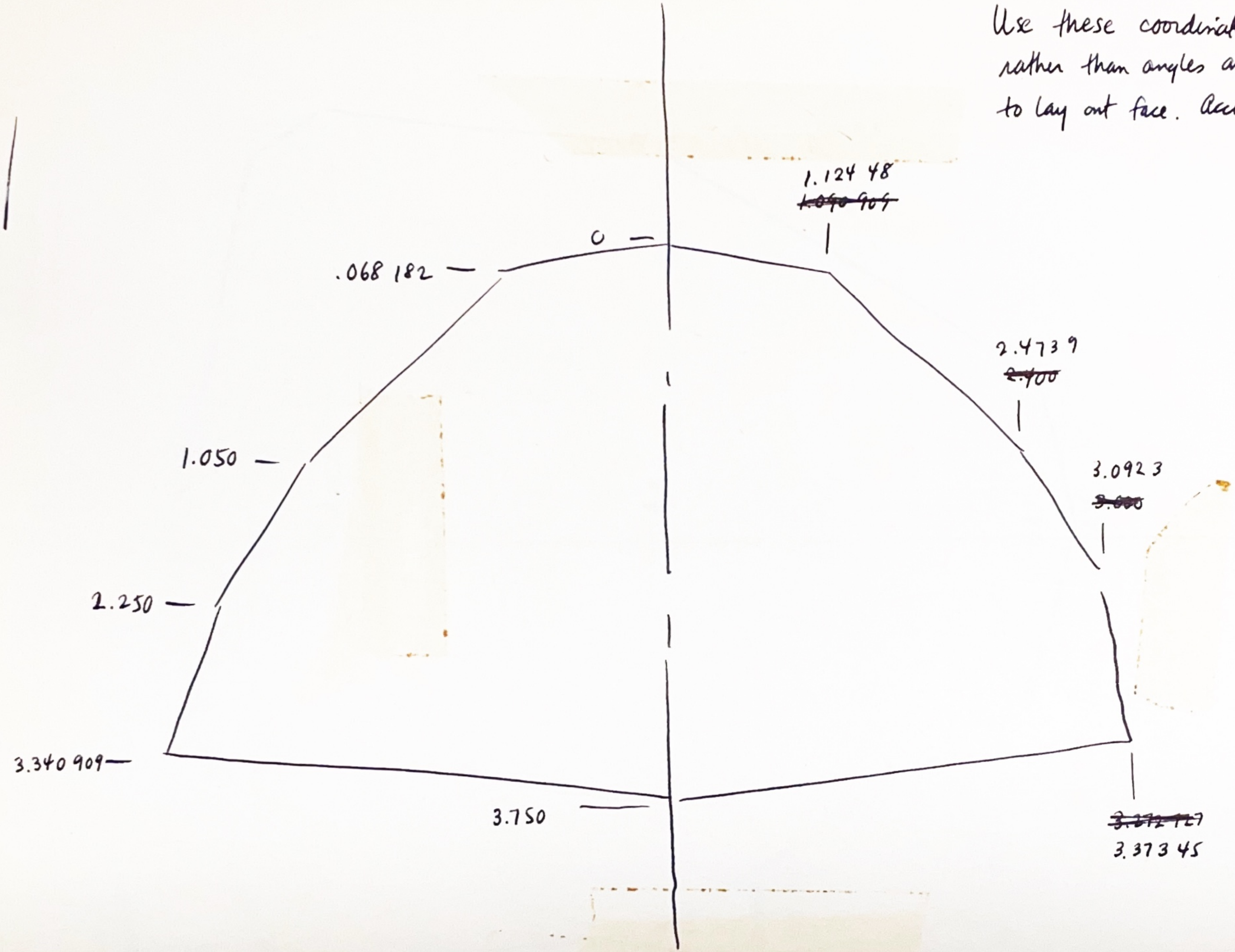

1
15.938°
1.12655
1.35000
A
3
114.870°
49.209°
.408180
2


35
23.153°
4.12432
3.39816
L
103.656°
36
53.191°
1.66877
34

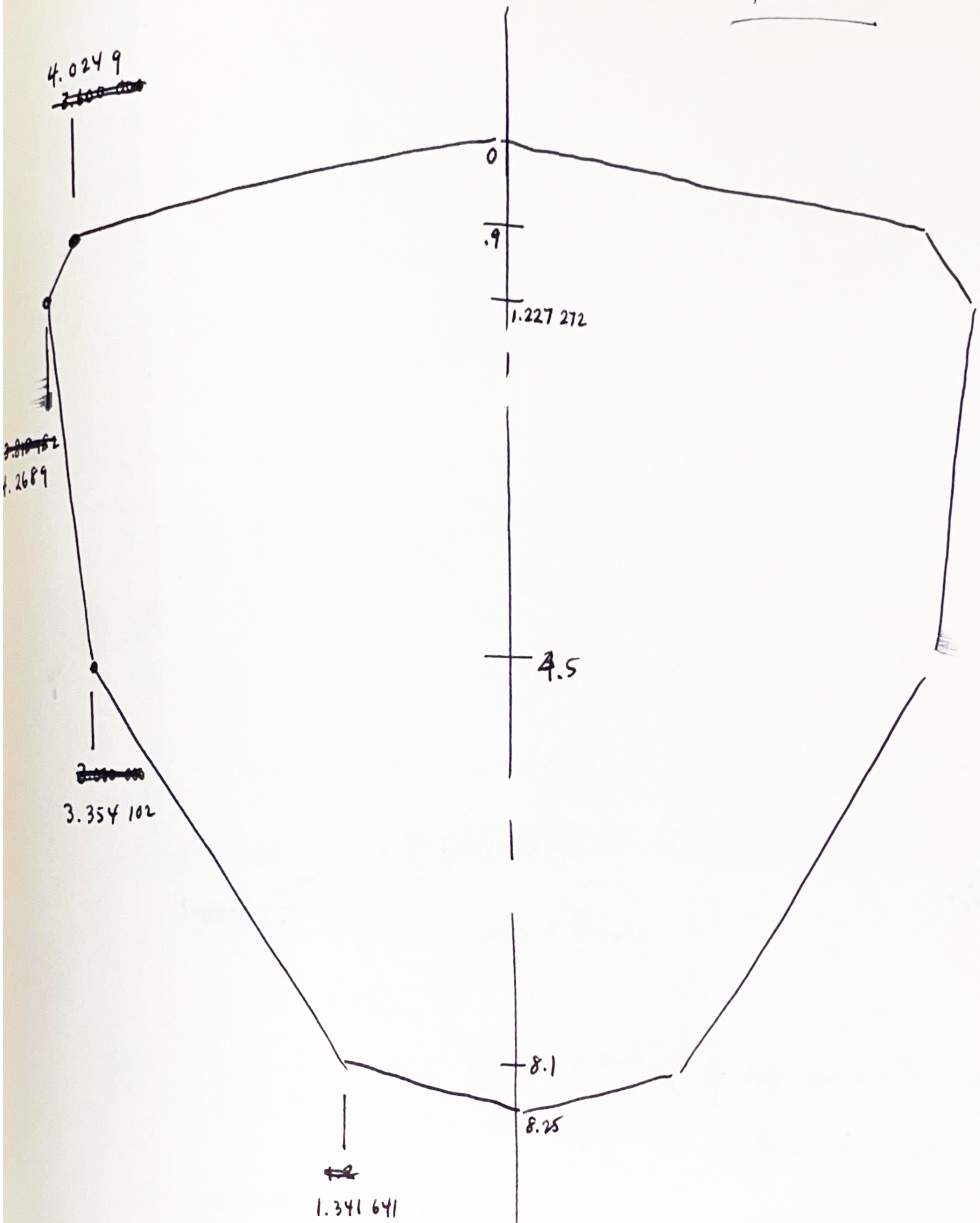

3
2
.408180
153.004°
1.66877
M
8
7
4
86.996°
6

Face O

Use these coordinates of vertices, rather than angles and edge lengths, to lay out face. Accuracy is better.



Face 5



α	$\cos\alpha$	$\sin\alpha$	$\frac{\sqrt{2}}{2}\sin\alpha$	R $-\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha$	S $\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha$	$\frac{R}{S}$
0	1	0	$\frac{\sqrt{2}}{2}$	-1.0	1.	
5°	.99619	.08716	.06163	-.93456	1.05782	
10°	.98481	.17365	.12279	-.86202	1.10760	
15°	.96593	.25882	.18301	-.78292	1.14894	
20°	.93969	.34202	.24184	-.69785	1.18153	
25°	.90631	.42262	.29884	-.60747	1.20515	
26°	.89879	.43837	.30997	-.58882	1.20876	
27°	.89101	.45399	.32102	-.56999	1.21203	
28°	.88295	.46947	.33197	-.55098	1.21743	
29°	.87462	.48481	.34281	-.53181	1.21958	
30°	.86603	.50000	.35355	-.51248		
40°	.7660	.6428	.4545	-.3115	1.2265	-.2552

40.315.7
43.3140°

~~-.766667~~

The proportions of this 20-faced polyhedron were chosen by first looking at the stereo pairs printed out at 5° intervals in the computer calculations of the collapse of the Laves graph and the transformation of the associated Dirichlet cell, and choosing a value for α (the edge rotation angle) which would yield relatively large areas for the added 14 faces.

Using the relation $\boxed{\tau_\alpha \vec{r}_{AB} = \vec{r}_{AB}\cos\alpha + \vec{\Lambda}_{AB}\sin\alpha}$, I computed the value of $\alpha\ (= \alpha')$ for which $\boxed{\tau_{\alpha'}\, \vec{r}_{-110} \propto \vec{r}_{-120}}$, in order to have as many whole number coordinates as possible for the vertices of the Dirichlet cell.

$$\tau_\alpha \vec{r}_{AB} = \vec{r}_{AB}\cos\alpha + \vec{\Lambda}_{AB}\sin\alpha$$

If $\vec{r}_A = (000)$ & $\vec{r}_B = (-110)$, then

$$\vec{\Lambda}_A = \frac{\sqrt{6}}{4}\left[\frac{1}{\sqrt{3}}(-1-1-1)\right] \text{ and } \vec{\Lambda}_B = \frac{\sqrt{6}}{4}\left[\frac{1}{\sqrt{3}}(11-1)\right].$$

$$\therefore \vec{\Lambda}_{AB} = \frac{\sqrt{2}}{4}(220) \text{ and } \vec{r}_{AB} = (-110).$$

$$\tau_\alpha \vec{r}_{AB} = (-110)\cos\alpha + \frac{\sqrt{2}}{2}(110)\sin\alpha$$

$$= \Big(\underset{(R)}{-\cos\alpha + \tfrac{\sqrt{2}}{2}\sin\alpha},\ \underset{(S)}{\cos\alpha + \tfrac{\sqrt{2}}{2}\sin\alpha},\ \underset{(T)}{0}\Big)$$

Want $\frac{R}{S} = -\frac{1}{2}$. Find $\dfrac{-\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha}{\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha} = -\frac{1}{2} \Rightarrow \tan\alpha = \frac{\sqrt{2}}{3} = .47140450$

Hence $\boxed{\alpha' \cong 25.239°}$

$\cos\alpha' = .90454$

$\sin\alpha' = .42640$

$\lambda(\alpha) = \Lambda\tan\alpha$

Hence $\lambda(\alpha') = \left(\frac{\sqrt{6}}{4}\right)\left(\frac{\sqrt{2}}{3}\right) = \frac{\sqrt{12}}{12} = \left(\frac{2\sqrt{3}}{12}\right) = \frac{\sqrt{3}}{6}$

Thus λ is $\frac{1}{12}$ of the separation $(2\sqrt{3})$ between similarly oriented vertices at (000) and (2-2-2).