(50)

σ_2 (same signs)

$$\cos\varphi_\alpha = \frac{\begin{pmatrix}-2\eta\\1-2\eta\\0\end{pmatrix}\begin{pmatrix}0\\-2\eta\\-1-2\eta\end{pmatrix}}{\underbrace{\sqrt{1-4\eta+8\eta^2}}_{(l_1)}\underbrace{\sqrt{1+4\eta+8\eta^2}}_{(l_2)}} = \frac{-2\eta+4\eta^2 = 2\eta(2\eta-1)}{l_1 l_2} = \boxed{\frac{2\eta(2\eta-1)}{\sqrt{1-4\eta+8\eta^2}\sqrt{1+4\eta+8\eta^2}}}$$

$$\cos\varphi_\beta = \frac{\begin{pmatrix}2\eta\\-1+2\eta\\0\end{pmatrix}\begin{pmatrix}1+2\eta\\0\\-2\eta\end{pmatrix}}{l_1 l_2} = \frac{2\eta(1+2\eta)}{l_1 l_2} = \boxed{\frac{2\eta(2\eta+1)}{\sqrt{1-4\eta+8\eta^2}\sqrt{1+4\eta+8\eta^2}}}$$

$(1-2\eta)^2+(2\eta)^2$
$= 1-4\eta+4\eta^2+4\eta^2$
$= 1-4\eta+8\eta^2$

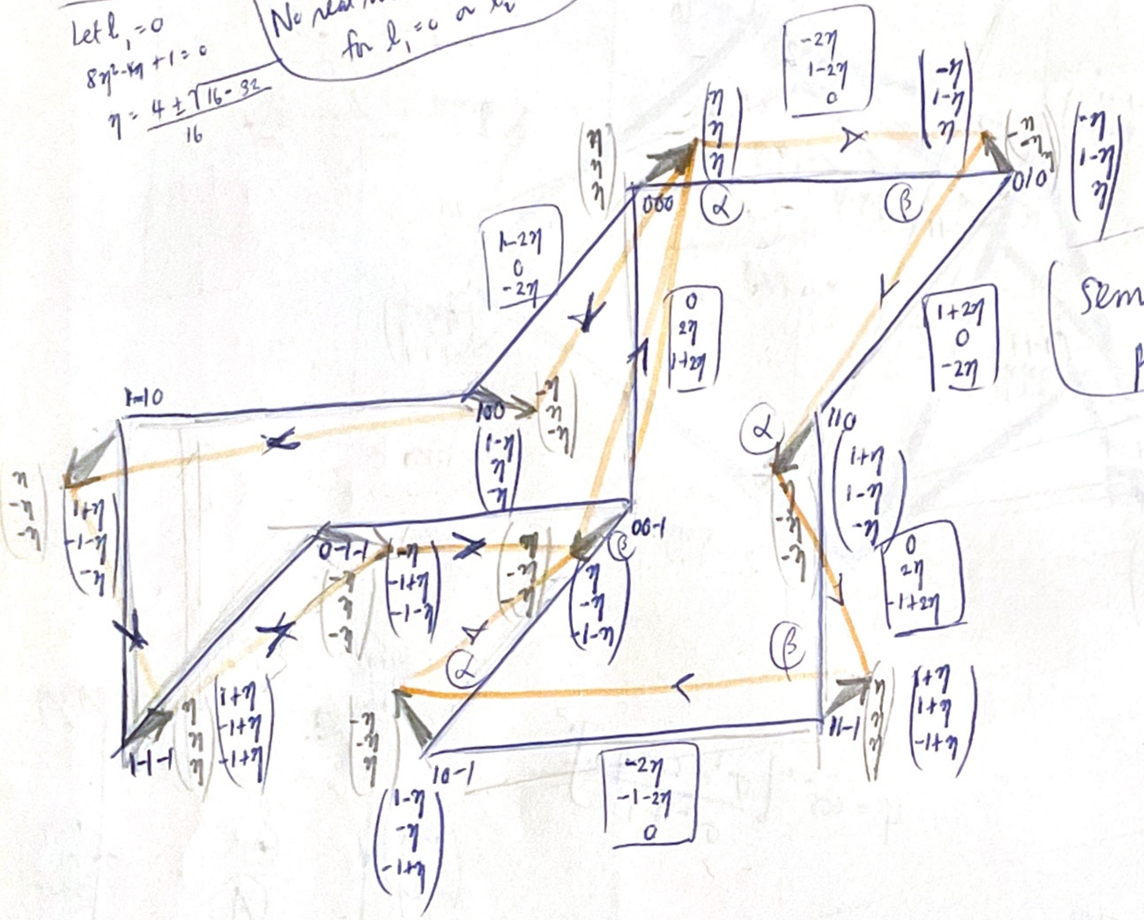
$(1+2\eta)^2+(2\eta)^2$
$= 1+4\eta+4\eta^2+4\eta^2$
$= 1+4\eta+8\eta^2$

Let $l_1 = 0$
$8\eta^2-4\eta+1=0$
$\eta = \frac{4\pm\sqrt{16-32}}{16}$

No real roots for $l_1=0$ or $l_2=0$

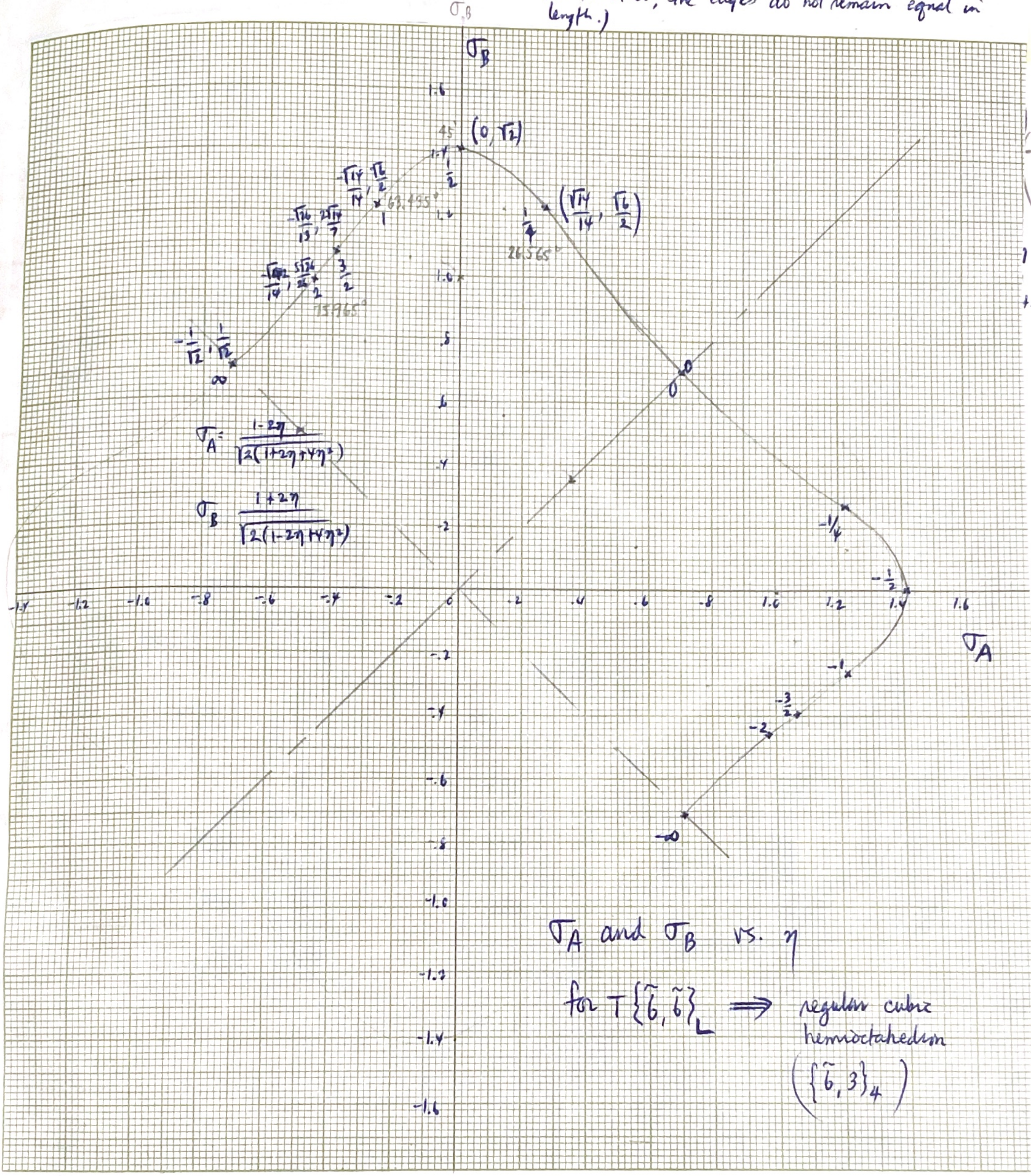
$\varphi_\alpha \uparrow (\eta)$

$\varphi_\beta \downarrow (\eta)$

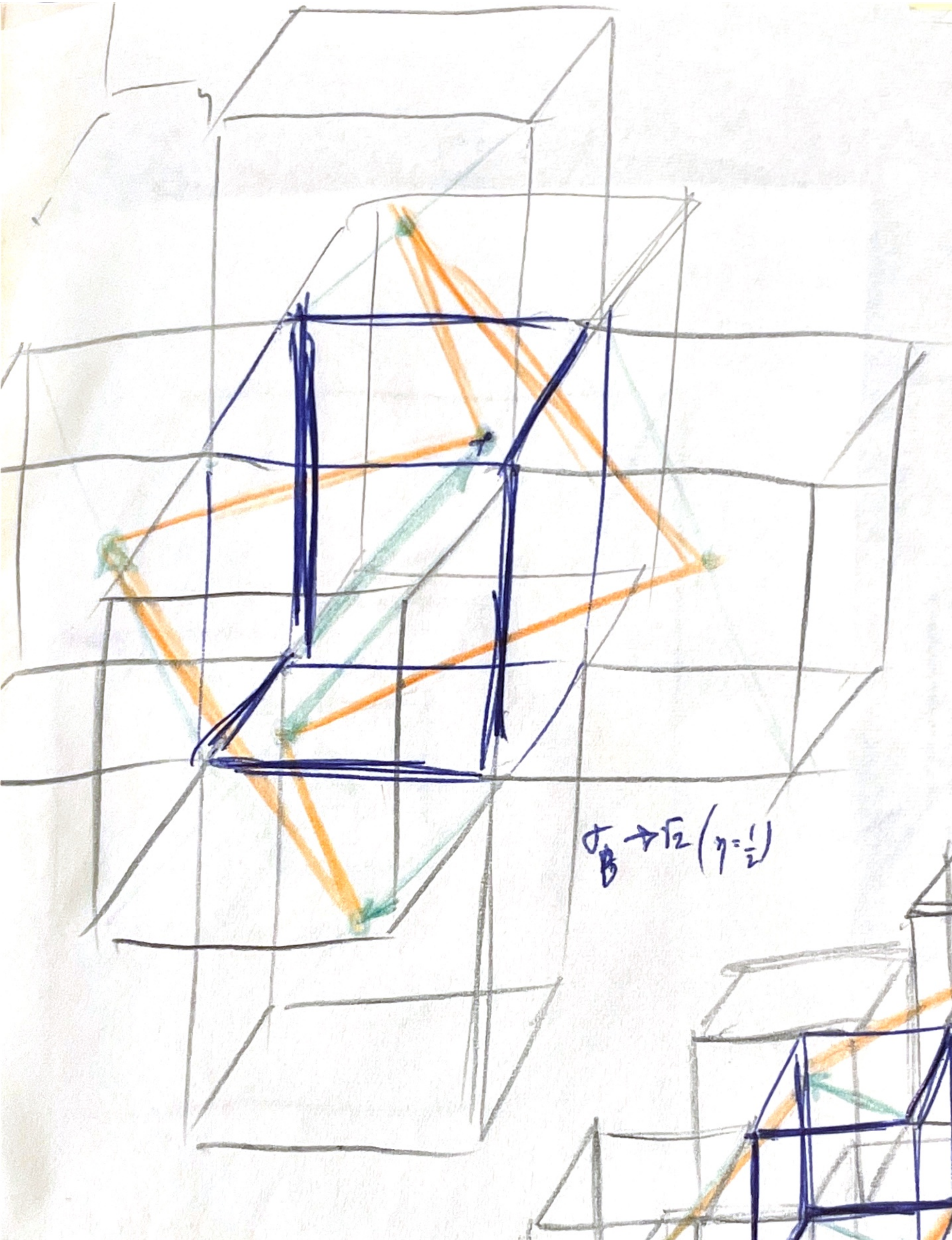


Semi-regular polyhedron

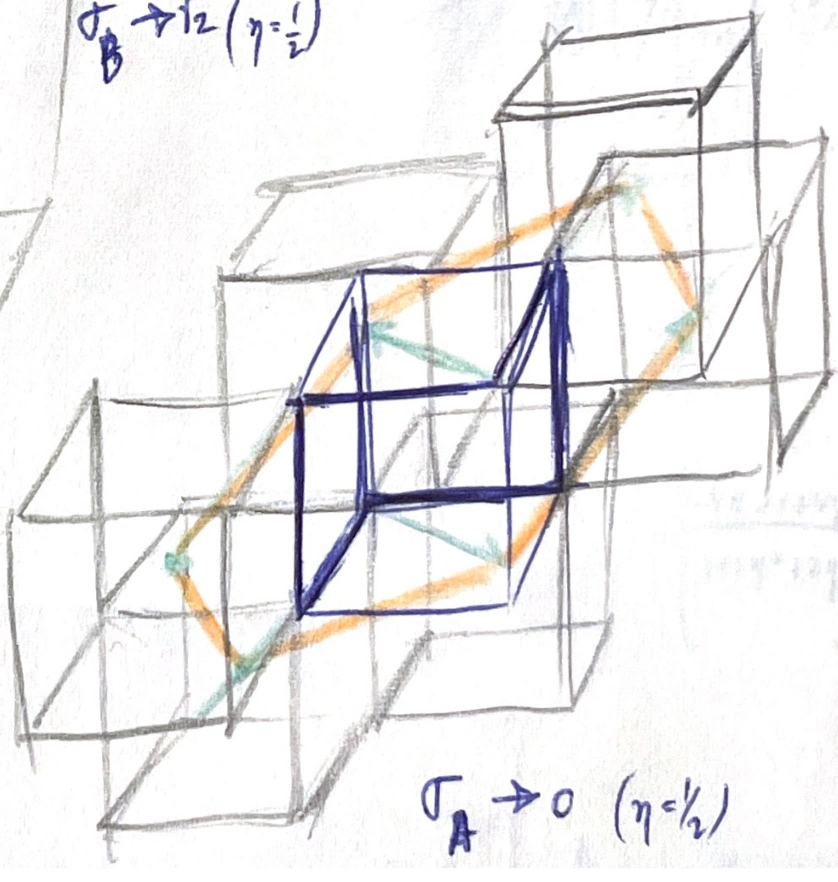
Vertex collision occurs at $\eta = \frac{1}{2}$ (or $\eta = -\frac{1}{2}$). (If all vertices are displaced from one labyrinth into the other, the edges do not remain equal in length.)



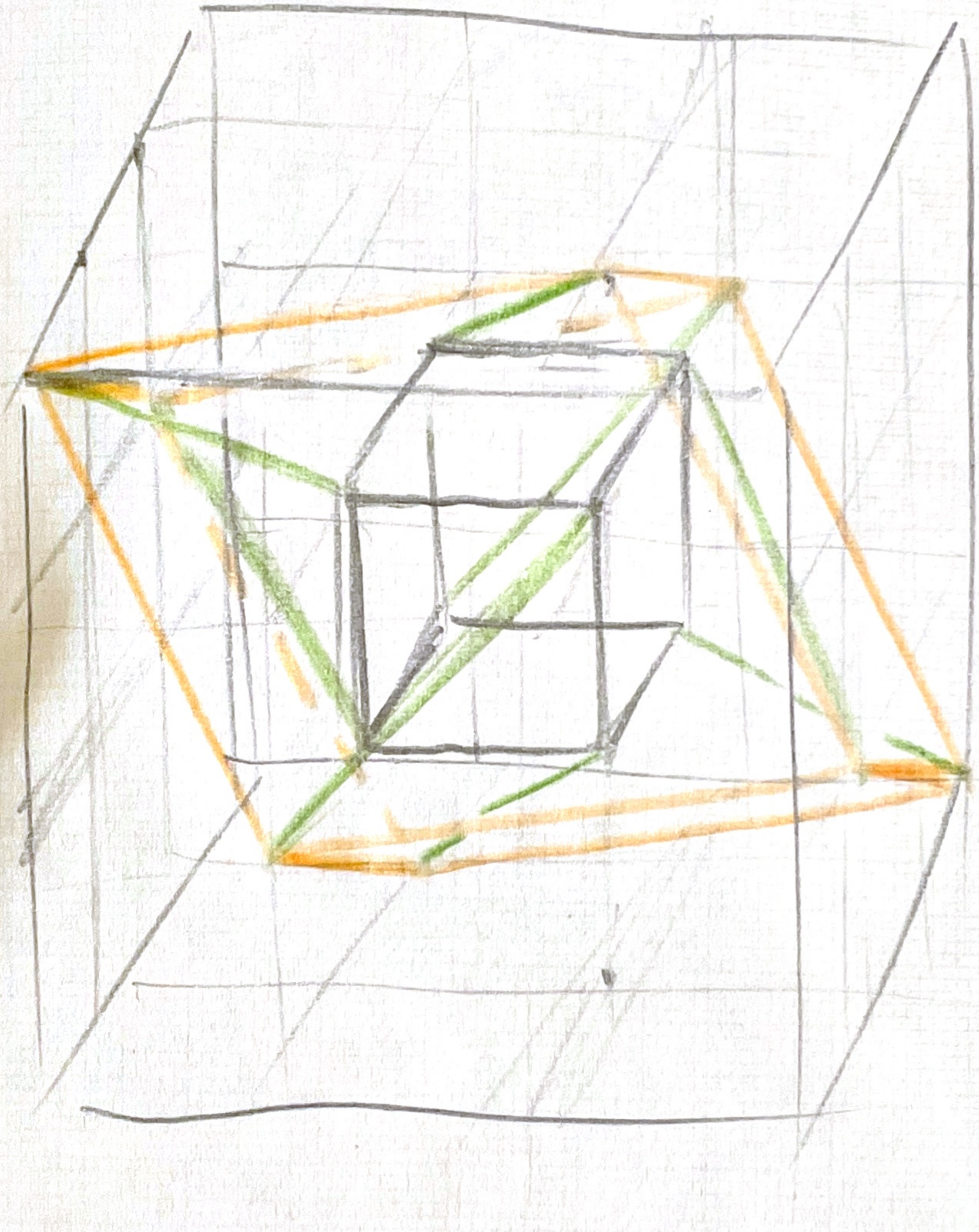
η	$\sigma_A = \frac{1-2\eta}{\sqrt{2(1+2\eta+4\eta^2)}}$	$\sigma_B = \frac{1+2\eta}{\sqrt{2}(1-2\eta+4\eta^2)}$
$-\infty$	.707	-.707
-4		
-3		
-2	$\frac{5\sqrt{26}}{26}$ .981	-.463
$-\frac{3}{2}$	$\frac{2\sqrt{14}}{7}$ 1.069	-.3923
-1	$\frac{\sqrt{6}}{2}$ 1.224	-.2673
$-\frac{1}{2}$	$\sqrt{2}$ 1.414	0
$-\frac{1}{4}$	$\frac{\sqrt{6}}{2}$ 1.224	$\frac{\sqrt{14}}{14}$ = .2676
0	$\frac{1}{\sqrt{2}}$ = .707	$\frac{1}{\sqrt{2}}$ = .707
$\frac{1}{4}$	$\frac{\sqrt{14}}{14}$ = .2676	$\frac{\sqrt{6}}{2}$ 1.224
$\frac{1}{2}$	0	$\sqrt{2}$ 1.414
1	-.2673	$\frac{\sqrt{6}}{2}$ 1.224
$3/2$	-.3923	$\frac{2\sqrt{14}}{7}$ 1.069
2	-.463	$\frac{5\sqrt{26}}{26}$ .981
3		
4		
∞		.707



$\sigma_B \to \sqrt{2}\ (\eta = \frac{1}{2})$



$\sigma_A \to 0\ \ (\eta = 1/2)$



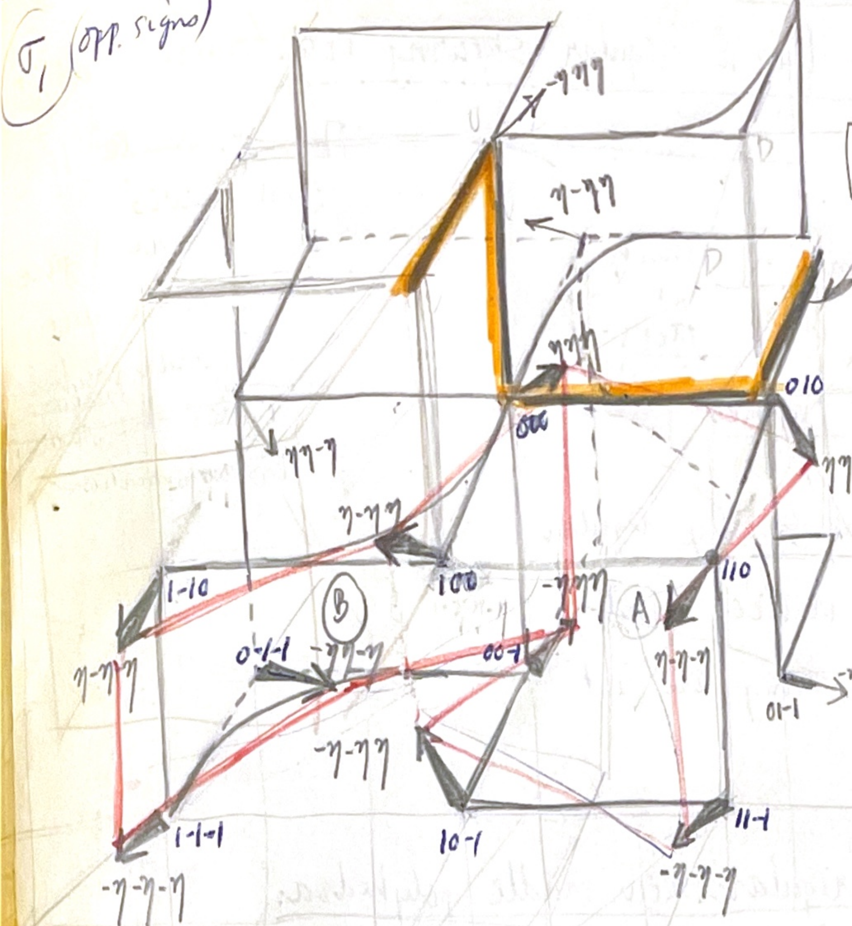
...ving displacements as shown ... A and ... B

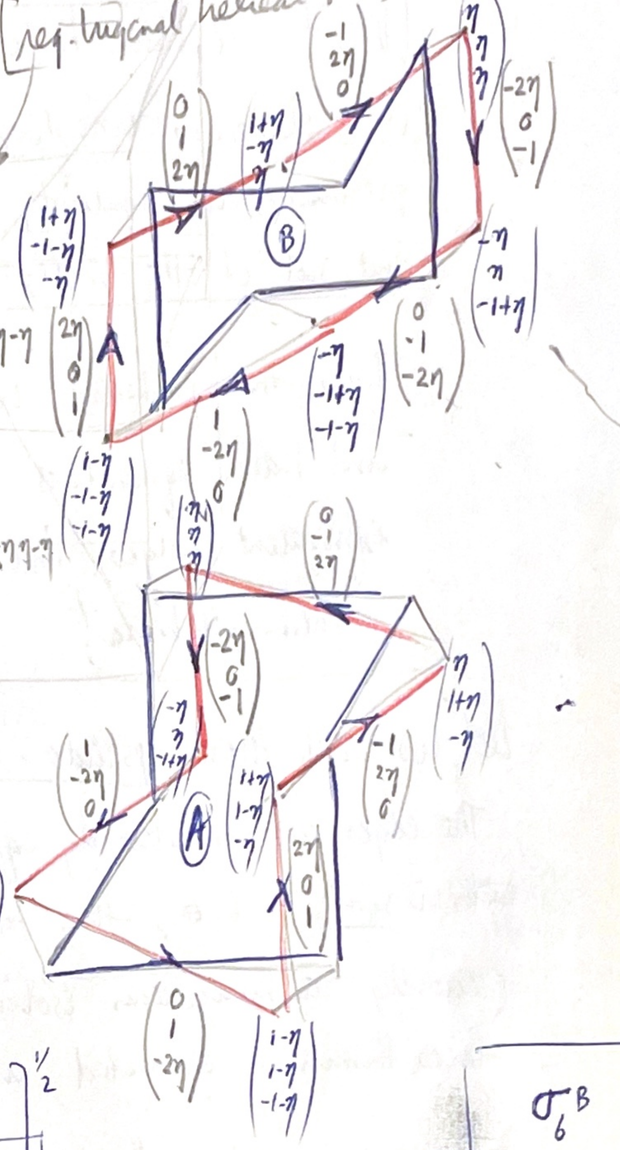
σ₁ (opp. signs)

$\frac{\sqrt{3}}{2}\tan\alpha = \frac{\sqrt{3}}{2}$, $\alpha = 45^\circ$

$|\cos 2\eta| = 1$ when $\eta = \frac{1}{2}$

[reg. trigonal helical polygon (ℓh)]





$$\cos\varphi_6^A = \frac{\begin{pmatrix}0\\-1\\2\eta\end{pmatrix}\begin{pmatrix}1\\-2\eta\\0\end{pmatrix}}{1+4\eta^2} = \frac{2\eta}{1+4\eta^2}$$

$$\cos\varphi_6^B = \frac{\begin{pmatrix}-1\\2\eta\\0\end{pmatrix}\begin{pmatrix}2\eta\\0\\1\end{pmatrix}}{1+4\eta^2} = \frac{-2\eta}{1+4\eta^2}$$

$$\sigma_6^A = \sqrt{\frac{\cos\varphi - \cos\varphi_c}{1-\cos\varphi}} = \left[\frac{\frac{2\eta}{1+4\eta^2}+\frac{1}{2}}{1-\frac{2\eta}{1+4\eta^2}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{4\eta+1+4\eta^2}{2\left[1+4\eta^2-2\eta\right]}\right]^{1/2} = \frac{(2\eta+1)}{\left[2(1-2\eta+4\eta^2)\right]^{1/2}} \Bigg|_{\eta=0} = \frac{1}{\sqrt{2}}\ \checkmark$$

i.e., $\boxed{\sigma_6^A = \frac{2\eta+1}{\left[2(1-2\eta+4\eta^2)\right]^{1/2}}}$ no real roots

$$\sigma_6^B = \left[\frac{\frac{-2\eta}{1+4\eta^2}+\frac{1}{2}}{1-\frac{(-2\eta)}{1+4\eta^2}}\right]^{\frac{1}{2}} = \left[\frac{-4\eta+1+4\eta^2}{1+4\eta^2+2\eta}\right]^{1/2} = \boxed{\sigma_6^B = \frac{1-2\eta}{\left[2(1+2\eta+4\eta^2)\right]^{1/2}}}$$

When $\eta = 1/2$, $\boxed{\sigma_6^B = 0}$, $\boxed{\sigma_6^A = \sqrt{2}}$ (60° hex.) This is not the same as $\left\{\begin{matrix}6\\ \tilde{6}(\sqrt{2})\end{matrix}\right\}_4^{60^\circ}$ $= \mathcal{M}\{\tilde{6};\tilde{6}\}$, because this is $\left\{\begin{matrix}\tilde{6}\\ \tilde{6}\end{matrix}\right\}_6$

This is extremely interesting! i.e., 60° hexes and plane hexes can be joined either by fours or sixes to make a quasi-regular polyhedron!

Variable skewness quasi-regular $\left\{\begin{matrix}\tilde{6}\\ \tilde{6}\end{matrix}\right\}_6$

(The regular skew polyhedra behave peculiarly under the regular skewing transformation. $\{6,\tilde{6}\}$ has no regular skewing transformation at all!

What we have here →

is a skewness transformation which preserves the regularity of the faces but not of the vertex figures.

The ~~pseudo~~ IPMS and Laves ∞ regular figures generate quasi-regular polyhedra under the skewing transformation.

Thus, these polyhedra are facially regular, and have equivalent vertices and equivalent edges, but they are not regular polyhedra!

Let us call these pseudo-regular skew saddle polyhedra.

The edges are symmetrically equivalent only when account is taken of their sense, i.e., they form the edges of a symmetrical directed graph. (actually, a homogeneous isotropic directed graph with regular skew faces forming a connected "surface" without self-intersections).

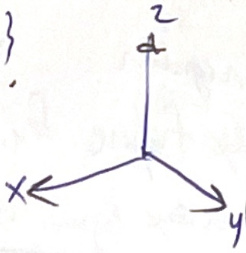
They do not have "appropriate" duals. The face-centers remain fixed during the transformation. Hence, the vertices of the duals remain fixed.

(When the transformation is complete, of course, then the symmetry-domain — interstitial domain "duality" appears.)

(This is true for all 3 cases: $\sigma\{6,\tilde{4}\}$, $\sigma\{4,\tilde{6}\}$, & $\sigma\{6,\tilde{6}\}$.)

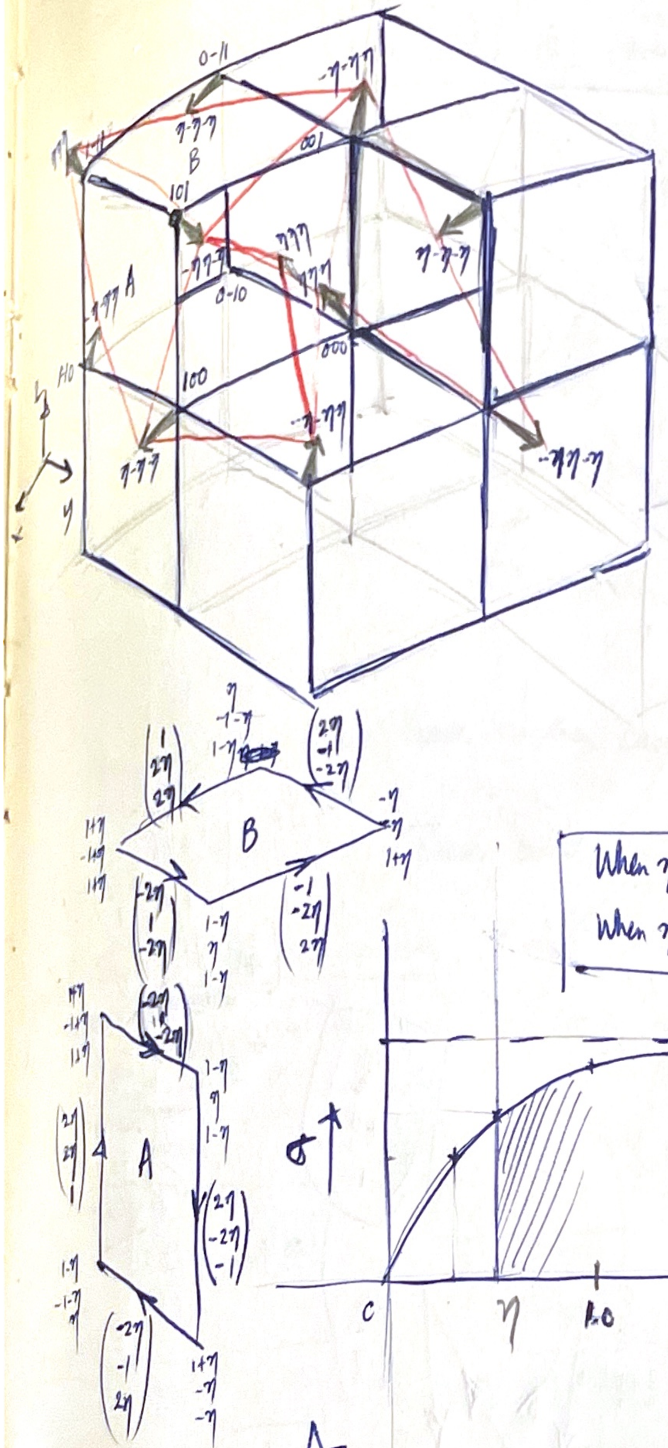
(sd / id)(IBCC / bcc) (b.c.c. / IBCC)(sd / id) (◇ sd / ◇ id)

Now apply skewing transformation to $\{4, \tilde{6}\}$



$\Lambda = \frac{\sqrt{6}}{4}$ $(\Lambda_{ij} = 1)$

$\frac{\eta}{\lambda} = \frac{1}{\sqrt{3}}$



$$\cos\varphi_4^A = \frac{\begin{pmatrix}2\eta\\2\eta\\1\end{pmatrix}\begin{pmatrix}2\eta\\-1\\2\eta\end{pmatrix}}{1+4\eta^2+4\eta^2} = \frac{4\eta^2 - 2\eta + 2\eta}{1+8\eta^2} = \frac{4\eta^2}{1+8\eta^2}$$

$$\cos\varphi_4^B = \frac{\begin{pmatrix}1\\2\eta\\2\eta\end{pmatrix}\begin{pmatrix}-2\eta\\1\\2\eta\end{pmatrix}}{1+8\eta^2} = \frac{-2\eta+2\eta+4\eta^2}{1+8\eta^2} = \frac{4\eta^2}{1+8\eta^2} = \cos\varphi_4^A$$

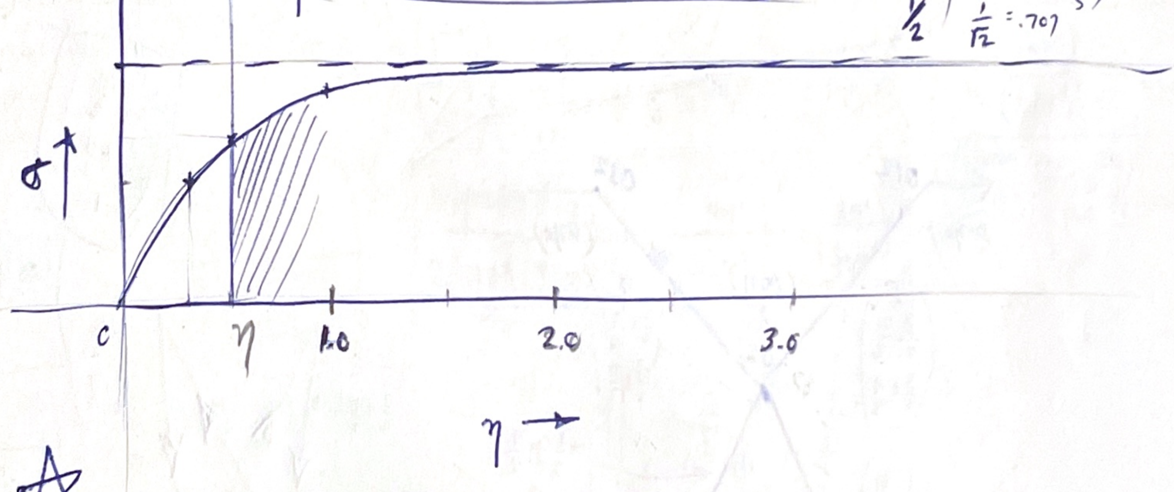
$$\sigma = \sqrt{\frac{\cos\varphi - \cos\varphi_c}{1-\cos\varphi}} = \sqrt{\frac{\frac{4\eta^2}{1+8\eta^2} - 0}{1 - \frac{4\eta^2}{1+8\eta^2}}} = \left[\frac{4\eta^2}{1+8\eta^2 - 4\eta^2}\right]^{1/2}$$

$$\therefore \boxed{\sigma = \frac{2\eta}{[1+4\eta^2]^{1/2}}}$$

η	σ
0	0
$\frac{1}{\sqrt{12}} = .2887$	$\frac{1}{2}$
1	$.894 \left(= \frac{2\sqrt{5}}{5}\right)$
$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = .707$

When $\eta = 0$, $\sigma = 0$

When $\eta = \sqrt{\frac{1}{12}} = \frac{\sqrt{3}}{6} \cong .2887$, $\sigma = \frac{1}{2}$



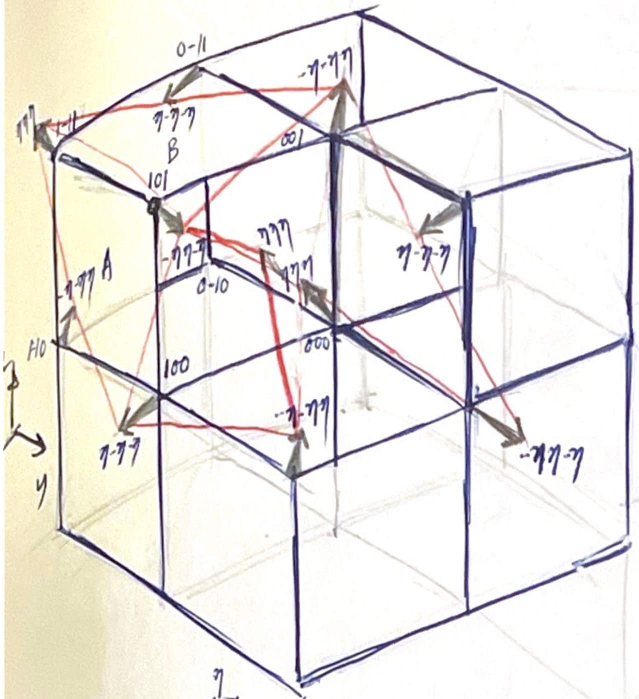
Thus is extremely interesting: we can deform $\{4, \tilde{6}\}$ continuously, until we reach a space filling of tetragonal tetrahedra. $(\eta = \frac{1}{2}; \sigma = \sqrt{\frac{1}{2}})$

Thus, we have found $\{$~~$\tilde{4}[\sigma_4], \tilde{6}[\sigma_6]\}_{sc}'$, a regular~~ skew saddle polyhedron of s.c. labyrinths, which is continuously deformable.

σ_4 goes from zero to $\frac{\sqrt{2}}{2}$

σ_6 goes from $\sqrt{2}$ to ∞

Now apply ~~skewing transformation~~ to $\{4,\tilde{6}\}$



$\Lambda = \frac{\sqrt{6}}{4}$ $(\Lambda_{ij} = 1)$

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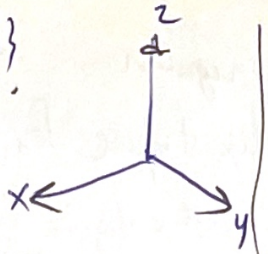

$$\cos\varphi_4^A = \frac{\begin{pmatrix}2\eta\\2\eta\\1\end{pmatrix}\begin{pmatrix}2\eta\\-1\\2\eta\end{pmatrix}}{1+4\eta^2+4\eta^2} = \frac{4\eta^2 - 2\eta + 2\eta}{1+8\eta^2} = \frac{4\eta^2}{1+8\eta^2}$$

$$\cos\varphi_4^B = \frac{\begin{pmatrix}1\\2\eta\\2\eta\end{pmatrix}\begin{pmatrix}-2\eta\\1\\2\eta\end{pmatrix}}{1+8\eta^2} = \frac{-2\eta+2\eta+4\eta^2}{1+8\eta^2} = \frac{4\eta^2}{1+8\eta^2} = \cos\varphi_4^A$$

$$\sigma = \sqrt{\frac{\cos\varphi - \cos\varphi_c}{1-\cos\varphi}} = \sqrt{\frac{\frac{4\eta^2}{1+8\eta^2} - 0}{1 - \frac{4\eta^2}{1+8\eta^2}}} = \left[\frac{4\eta^2}{1+8\eta^2-4\eta^2}\right]^{1/2}$$

$$\therefore \boxed{\sigma = \frac{2\eta}{[1+4\eta^2]^{1/2}}}$$

η	σ
0	0
$\frac{1}{\sqrt{12}} = .2887$	$\frac{1}{2}$
1	$.894\ (=\frac{2\sqrt{5}}{5})$
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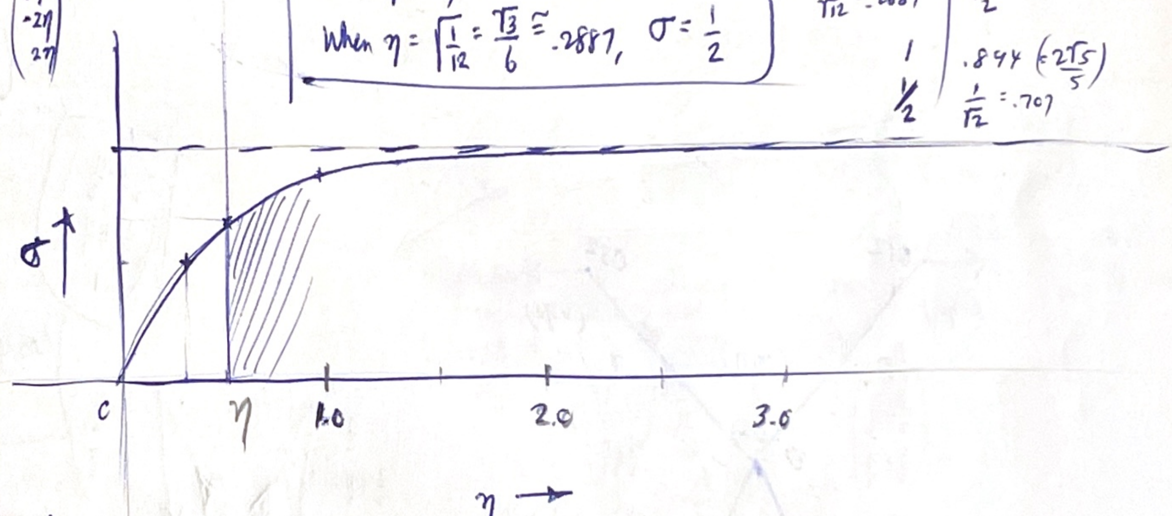
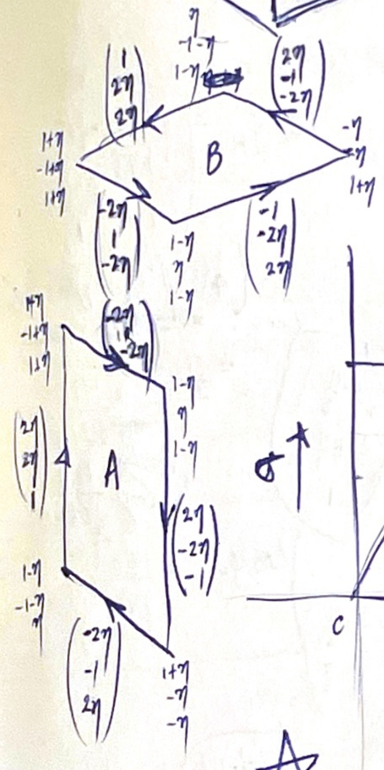
When $\eta = 0$, $\sigma = 0$
When $\eta = \sqrt{\frac{1}{12}} = \frac{\sqrt{3}}{6} \cong .2887$, $\sigma = \frac{1}{2}$





This is extremely interesting: we can deform $\{4,\tilde{6}\}$ continuously until we reach a space filling of tetragonal tetrahedra. $(\eta = \frac{1}{2};\ \sigma = \sqrt{\frac{1}{2}})$

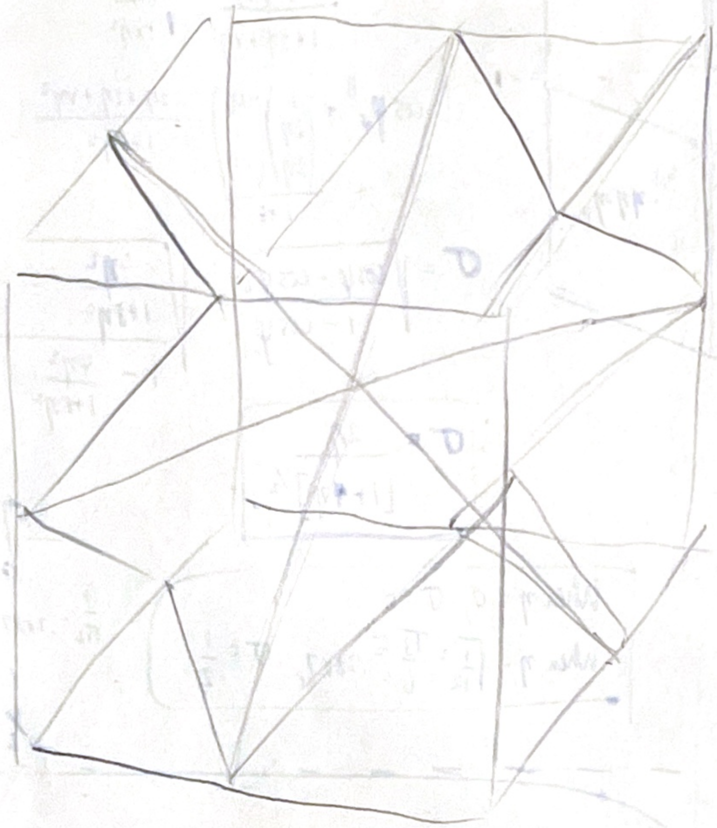
Thus, we have found $\{\tilde{4}[\sigma_4], \tilde{6}[\sigma_6]\}_{sc}$, a regular skew saddle polyhedron of s.c. labyrinths, which is continuously deformable.

σ_4 goes from zero to $\frac{\sqrt{2}}{2}$

σ_6 goes from $\sqrt{2}$ to ∞

~~Skewness of~~ Non-regular vertex figure for transf. on opp. page.

(Note that the vertex figure is regular only for $\{6, \tilde{4}\}$)



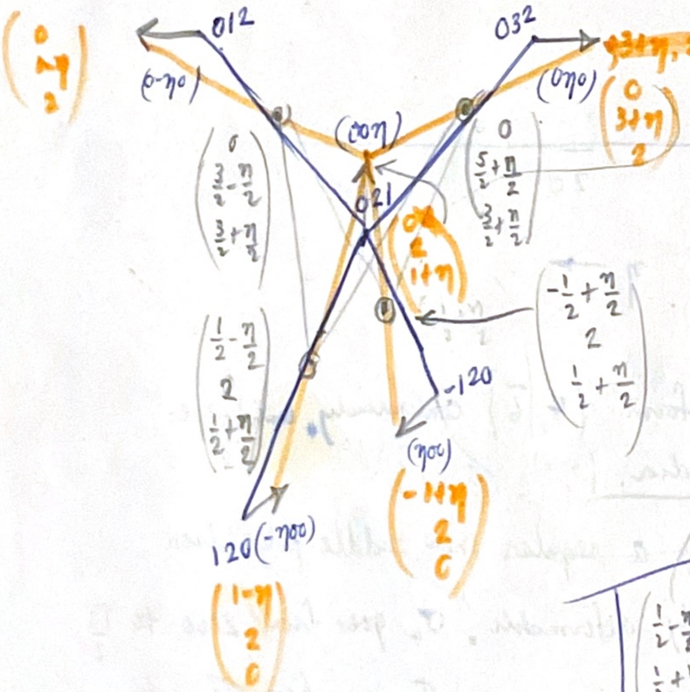
$$\cos\varphi_4 = \frac{\begin{pmatrix}-\frac{1}{2}+\frac{\eta}{2}\\ \frac{1}{2}+\frac{\eta}{2}\\ 1\end{pmatrix}\cdot\begin{pmatrix}-\frac{1}{2}+\frac{\eta}{2}\\ -\frac{1}{2}-\frac{\eta}{2}\\ 1\end{pmatrix}}{\frac{1}{4}(1-\eta)^2+\frac{1}{4}(1+\eta)^2+1=\frac{1}{2}(3+\eta^2)}$$

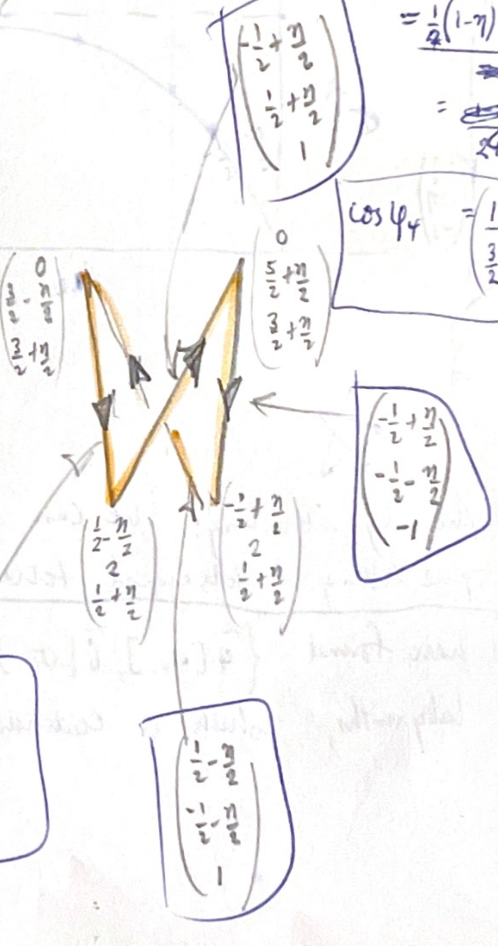
$$= \frac{\frac{1}{4}(1-\eta)^2-\frac{1}{4}(1+\eta)^2+1}{\frac{1}{2}(3+\eta^2)}$$

$$= \frac{1+\eta}{\frac{1}{2}(3+\eta^2)}$$

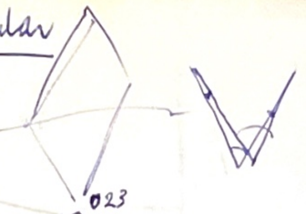
$$\boxed{\cos\varphi_4 = \frac{1+\eta}{\frac{3}{2}+\frac{\eta^2}{2}}}$$

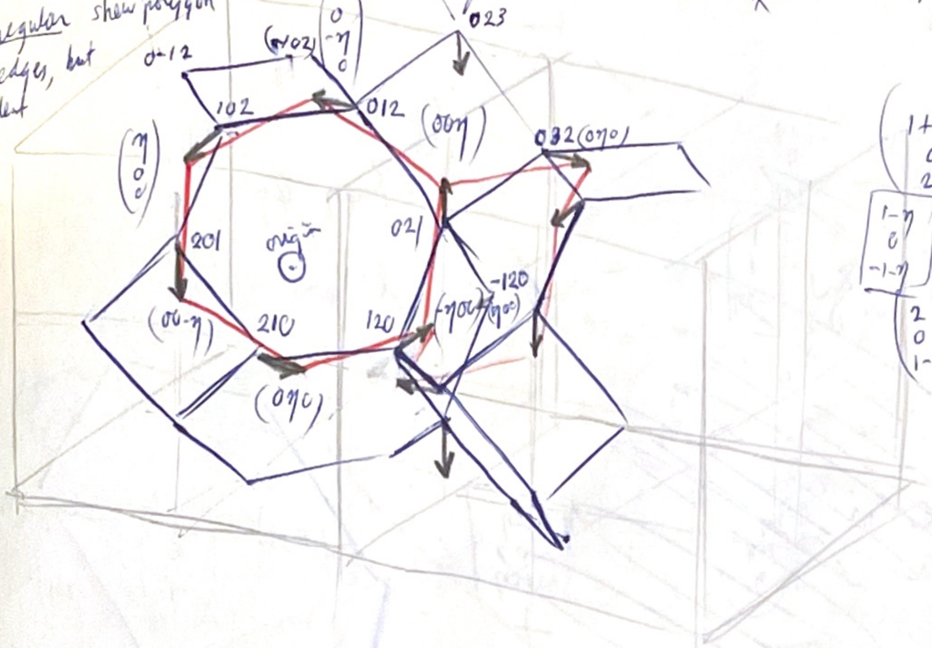
When $\eta=0$, $\varphi_4 = 48.19°$ $\boxed{\cos\varphi_4 = 2/3}$

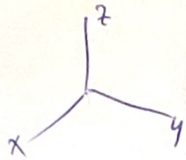


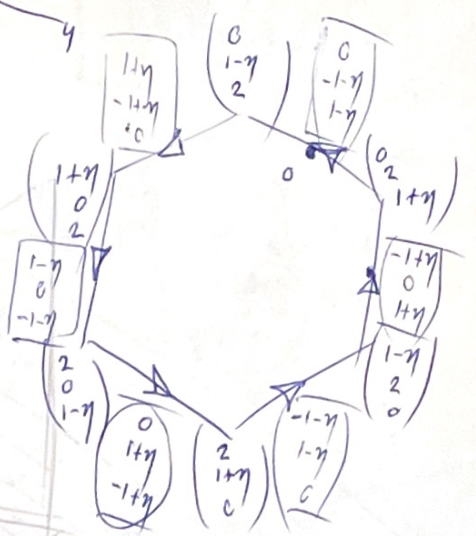


These are degenerate quasi-regular polyhedra. There is only one kind of regular face. The vertex figure is a semi-regular skew polygon (equivalent edges, but not equivalent vertices).









Here, it appears we have another case of an ∞ family of regular polyhedra: $\{\tilde{6}[\sigma_6], \tilde{4}[\sigma_4]\}_{sc}$. These eventually pass over into a space filling of expanded octahedra (the plane hexagon faces of $\{6, \tilde{4}\}$ become $90°$ skew hexagons).

$$\cos\varphi_6 = \begin{pmatrix}1+\eta\\-1+\eta\\0\end{pmatrix}\begin{pmatrix}-1+\eta\\0\\1+\eta\end{pmatrix}\Big/(1+\eta)^2+(1-\eta)^2 = \frac{-1+\eta^2}{2+2\eta^2} = \boxed{-\frac{1}{2}\frac{(1-\eta^2)}{(1+\eta^2)} = \cos\varphi_6}$$

(When $\eta = 0$, $\cos\varphi_6 = -\frac{1}{2}$, $\varphi_6 = 120°$ ✓)

$$\sigma = \sqrt{\frac{\cos\varphi - \cos\varphi_6}{1-\cos\varphi}} = \left[\frac{\frac{\eta^2-1}{2+2\eta^2}+\frac{1}{2}}{1+\frac{1-\eta^2}{2+2\eta^2}}\right]^{1/2} = \left[\frac{\frac{2\eta^2\cancel{-2}\cancel{+2}+2\eta^2}{2+2\eta^2+1-\eta^2}}{1/2}\right]^{1/2} = \left(\frac{4\eta^2}{\frac{\eta^2+3}{(1/2)}}\right)^{1/2} = \left(\frac{2\eta^2}{3+\eta^2}\right)^{1/2} = \sigma$$

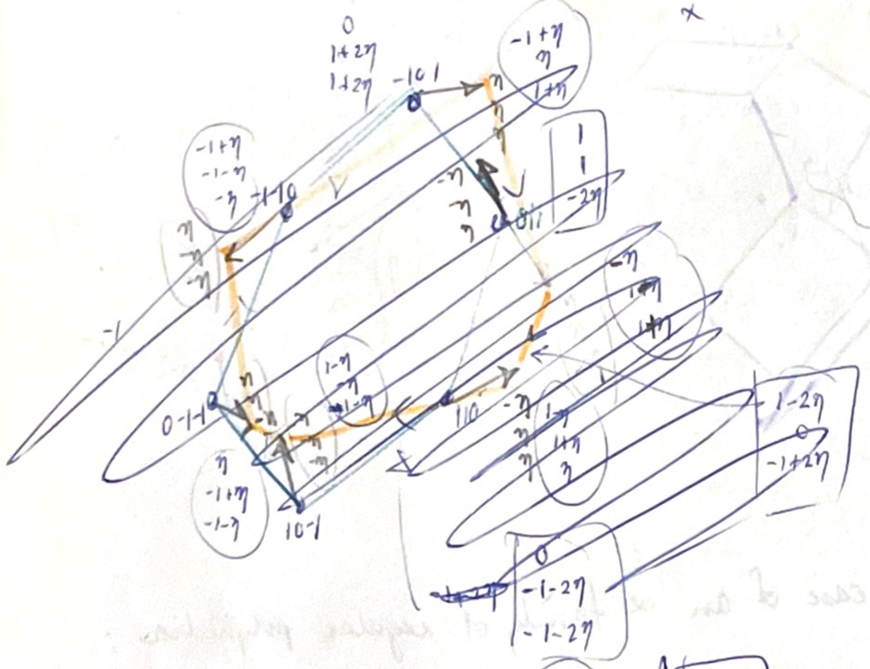
When $\eta = 1$, $\cos\varphi_6 = 0$. $\therefore$ hex $= 90°$ hex ✓ $\quad \sigma\Big|_{\eta=1} = \frac{\sqrt{2}}{(3+\eta^2)^{1/2}}\,\eta\Big|_{\eta=1} = \frac{\sqrt{2}}{2}$ ✓

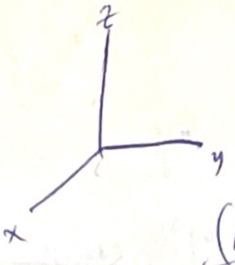
$\therefore \rightarrow$ space-filling for $\eta = 1$

The vertex figures of all quasi-regular polyhedra have the property that alternate edges are equivalent and alternate vertices are equivalent

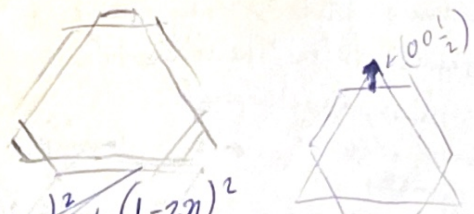
56

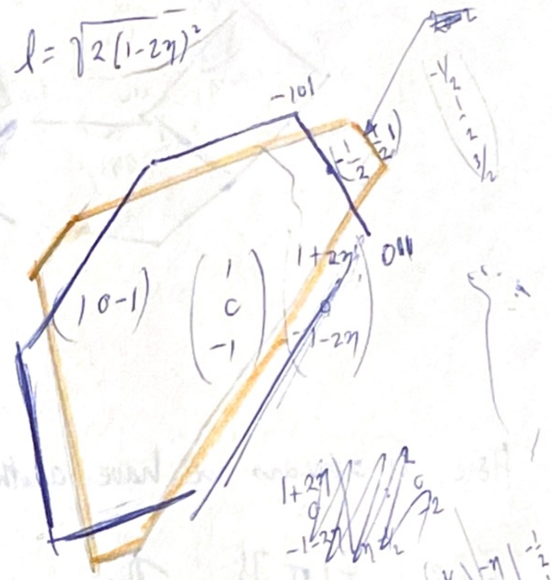
$\sigma_2[\{6,\tilde{4}\}]$

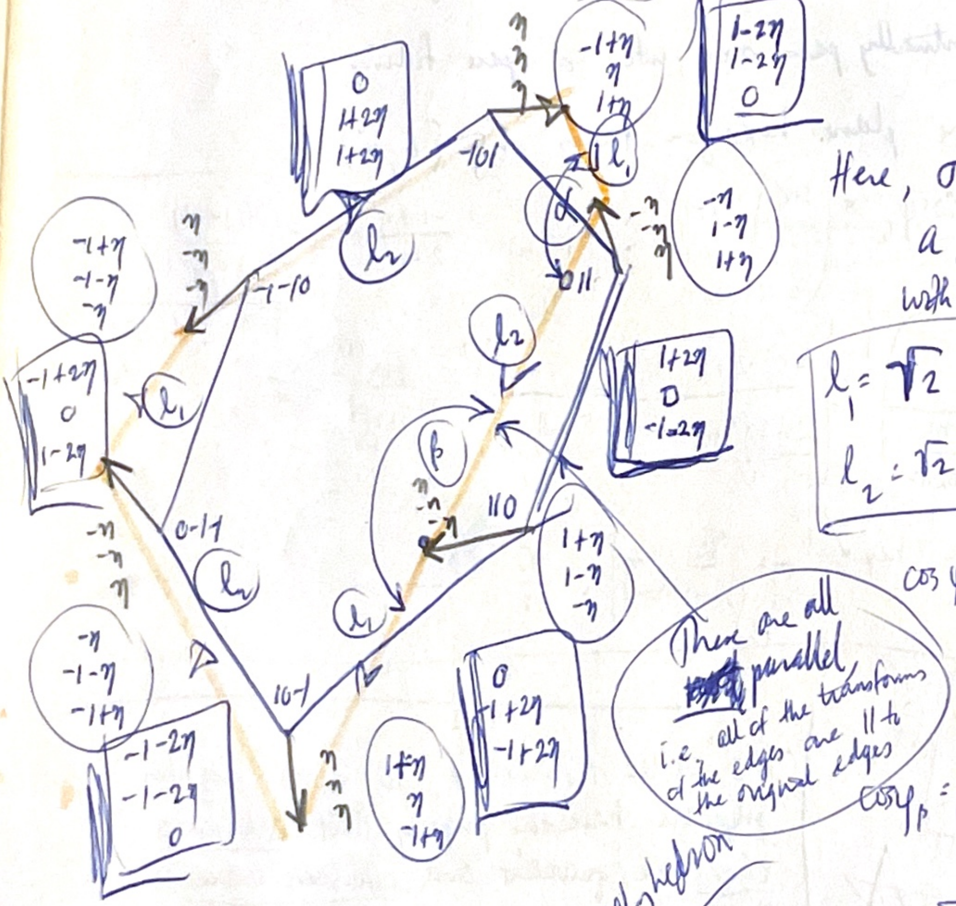




$$(1-2\eta)^2 + (1-2\eta)^2$$
$$= 1-4\eta+4\eta^2$$
$$l = \sqrt{2(1-2\eta)^2}$$







Here, $\sigma_2\{6,\tilde{4}\}$ produces a semi-regular polyhedron with edges of 2 distinct lengths:

$$l_1 = \sqrt{2}\,(1-2\eta)$$
$$l_2 = \sqrt{2}\,(1+2\eta)$$

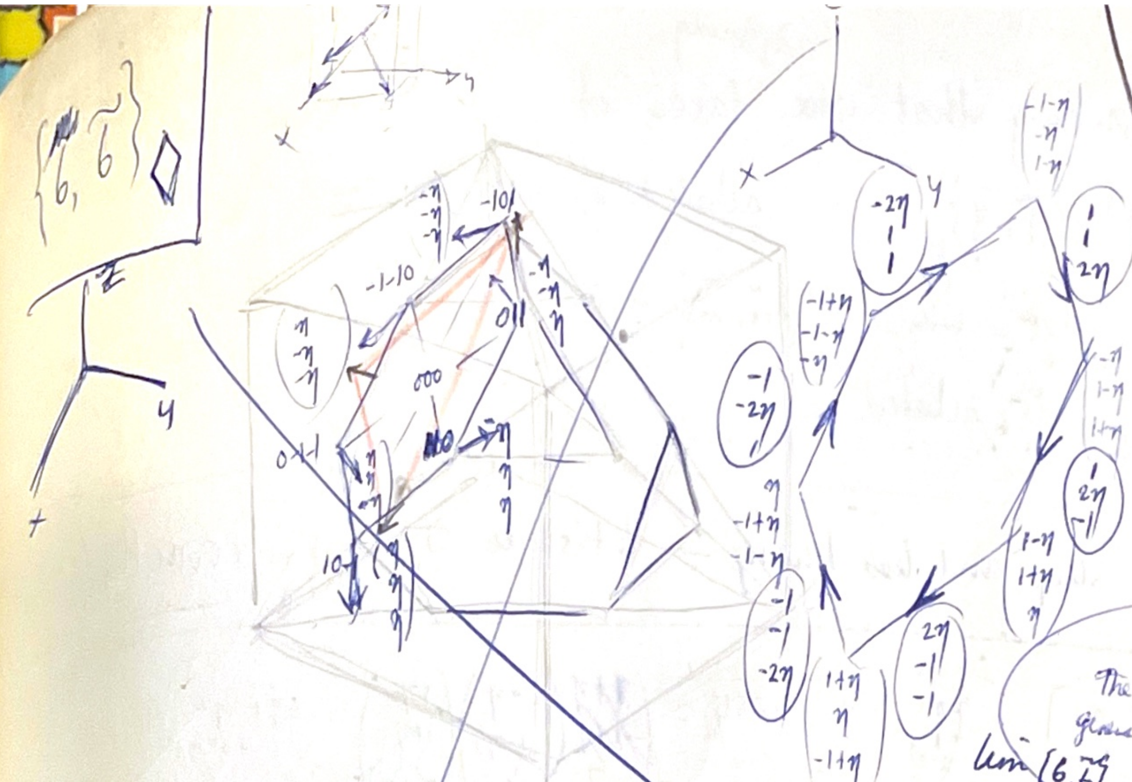
When $\eta = \frac{1}{2}$: $l_1 = 0$, $l_2 = 2\sqrt{2}$

These are all parallel, i.e. all of the transforms of the edges are ∥ to the original edges

$$\cos\varphi_\alpha = \frac{\begin{pmatrix}-1+2\eta\\-1+2\eta\\0\end{pmatrix}\begin{pmatrix}1+2\eta\\0\\-1-2\eta\end{pmatrix}}{2(1-4\eta^2)} = \frac{-1+4\eta^2}{2(1-4\eta^2)} = \frac{-1(1-4\eta^2)}{2(1-4\eta^2)}$$
$$= -\frac{1}{2} = \cos\varphi_\alpha$$
$$\boxed{\varphi_\alpha = 120^\circ}$$

$$\cos\varphi_\beta = \frac{\begin{pmatrix}-1-2\eta\\0\\1+2\eta\end{pmatrix}\begin{pmatrix}0\\-1+2\eta\\-1+2\eta\end{pmatrix}}{2(1-4\eta^2)}$$
$$= \frac{-1+4\eta^2}{2(1-4\eta^2)} \therefore \varphi_\beta = 120^\circ$$

semi-regular polyhedron



This must be wrong. Because of the triangular circuits, there is no symmetrical arrangement of alternating displacements.

It is wrong

No, it isn't "wrong"! The double vertex displacements generate "unconnected" "regular compound hexagons"

$\lim_{\eta \to \infty} \{6, \tilde{6}\} \Rightarrow \{\tilde{6}, 3\}_4$ 7-25-68

$$-\cos\varphi_6 = \begin{pmatrix}-2\eta\\1\\1\end{pmatrix}\begin{pmatrix}1\\1\\2\eta\end{pmatrix} \Big/ 2+4\eta^2 = \frac{1}{2+4\eta^2}$$

$$\therefore \boxed{\cos\varphi_6 = \frac{-1}{2+4\eta^2}}$$

When $\eta = 0$, $\cos\varphi_6 = -\frac{1}{2}$ $\therefore \underline{\varphi_6 = 120°}$

Let $\cos\varphi_6 = -\frac{1}{3} = \frac{-1}{2+4\eta^2}$ $\therefore$ $2+4\eta^2 = 3$, $4\eta^2 = 1$, $\eta^2 = \frac{1}{4}$ or $\boxed{\eta = \frac{1}{2}}$

$$\sigma = \sqrt{\frac{\cos\varphi - \cos\varphi_0}{1-\cos\varphi}} = \left[\frac{\frac{-1}{2+4\eta^2} + \frac{1}{2}}{1 - \frac{(-1)}{2+4\eta^2}}\right]^{1/2} = \left[\frac{-2+2+4\eta^2}{2[2+4\eta^2+1]}\right]^{1/2}$$

$$\text{or } \boxed{\sigma = \frac{2\eta}{[2(4\eta^2+3)]^{1/2}}}$$

~~This becomes a space-filling of ◊ saddle tetrahedra when $\eta = \frac{1}{2}$~~ $\left(\sigma = \sqrt{\frac{1}{8}}\right)$

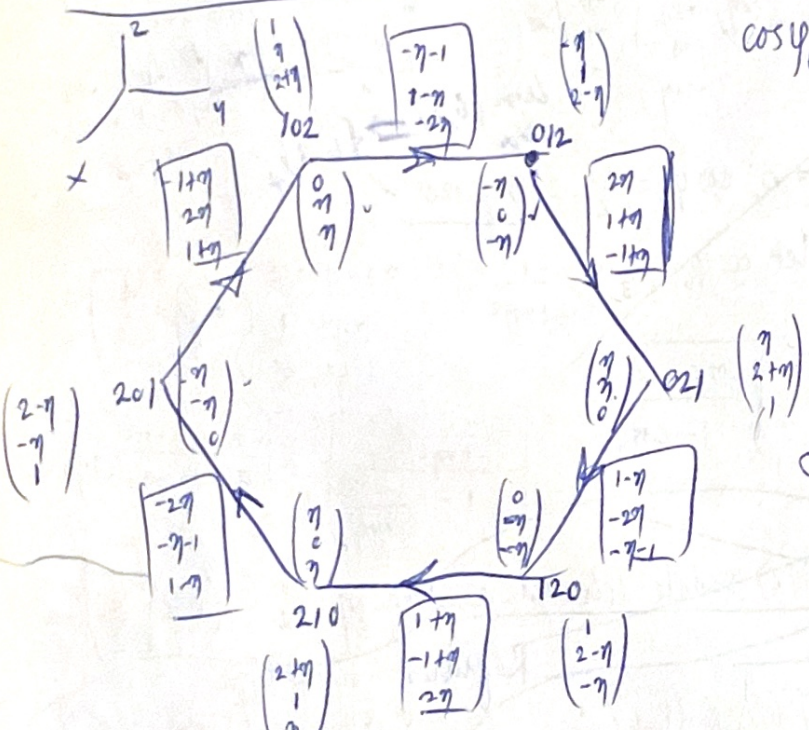
Requires new type of shearing ("split-edge") transformation

To summarize these ~~three~~ two cases, $(\tilde{6}, \tilde{4})$ $(\tilde{4}, \tilde{6})$ & $(6, \tilde{6})$:
S.C. S.C. ◊

These figures begin as an assembly of Dirichlet cells of particular homog. isotropic nets (b.c.c., IBCC, and ◊), with all faces removed except for principal faces. They remain pseudo-regular polyhedra until they reach their limiting form, at which each of them becomes a space-filling assembly of symmetry domains of the original net corresponding to the Dirichlet cells.

It is very striking that adjoining faces of $\{\tilde{p}, q\}_{\text{space}}$ and $\{\tilde{p}, \tilde{q}\}_{\text{Laves}}$ are related by rotation about their common edge, while adjoining faces of $\{\tilde{p}, \tilde{q}\}_{\text{s.c.}}$ are related by reflection.

Now — with more sleep and less hurry — let's do $\sigma_1\left\{\begin{matrix}4\\6\end{matrix}\right\}_D$ correctly!

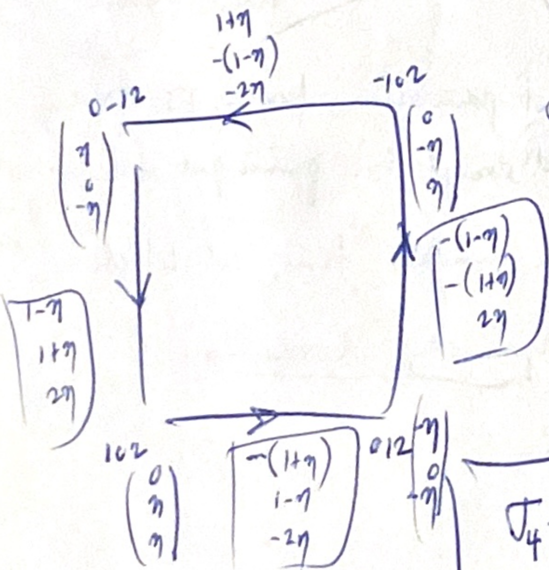


$$\cos\varphi_6 = \frac{\begin{pmatrix}1+\eta\\-(1-\eta)\\+2\eta\end{pmatrix}\begin{pmatrix}2\eta\\1+\eta\\-(1-\eta)\end{pmatrix}}{(1+\eta)^2+(1-\eta)^2+4\eta^2} = \frac{2\eta+2\eta^2+\eta^2-1}{1+2\eta+\eta^2+1-2\eta+\eta^2+4\eta^2} = \frac{5\eta^2-1}{6\eta^2+2} \quad \cos\varphi_6$$

Check: $\begin{pmatrix}2\eta\\1+\eta\\-(1-\eta)\end{pmatrix}\begin{pmatrix}-(1-\eta)\\2\eta\\1+\eta\end{pmatrix} = -2\eta+2\eta^2+2\eta+2\eta^2+\eta^2-1$ ✓ $5\eta^2-1$

$$\sigma_6 = \left[\frac{\frac{5\eta^2-1}{6\eta^2+2}+\frac{1}{2}}{1-\frac{5\eta^2-1}{6\eta^2+2}}\right]^{1/2} = \left[\frac{1}{2}\frac{10\eta^2-2+6\eta^2+2}{6\eta^2+2-5\eta^2+1}\right]^{1/2} = \left[\frac{1}{2}\frac{16\eta^2}{\eta^2+3}\right]^{1/2}$$

$$\boxed{\sigma_6 = \frac{2\sqrt{2}\,\eta}{(\eta^2+3)^{1/2}}} = \frac{2\sqrt{2}\,\eta}{(\eta^2+3)^{1/2}}$$



$$\cos\varphi_4 = \frac{\begin{pmatrix}1-\eta\\1+\eta\\-2\eta\end{pmatrix}\begin{pmatrix}-(1+\eta)\\1-\eta\\-2\eta\end{pmatrix}}{6\eta^2+2} = \frac{-1+\eta^2+1-\eta^2+4\eta^2}{6\eta^2+2} = \frac{4\eta^2}{6\eta^2+2} = \boxed{\frac{2\eta^2}{3\eta^2+1}} \quad \cos\varphi_4$$

$$\therefore \sigma_4 = \left[\frac{\frac{2\eta^2}{3\eta^2+1}}{1-\frac{2\eta^2}{1+3\eta^2}}\right]^{1/2} = \left[\frac{2\eta^2}{1+3\eta^2-2\eta^2}\right]^{1/2} = \boxed{\frac{\sqrt{2}\,\eta}{(1+\eta^2)^{1/2}}}$$

$$\boxed{\sigma_4 = \frac{\sqrt{2}\,\eta}{\sqrt{1+\eta^2}} \qquad \sigma_6 = \frac{2\sqrt{2}\,\eta}{\sqrt{3+\eta^2}}}$$

$$\boxed{\sigma_4 = \frac{\sqrt{2}\,\eta}{(1+\eta^2)^{1/2}} \qquad \sigma_6 = \frac{2\sqrt{2}\,\eta}{(3+\eta^2)^{1/2}}}$$

CORRECT (Finally!!!)

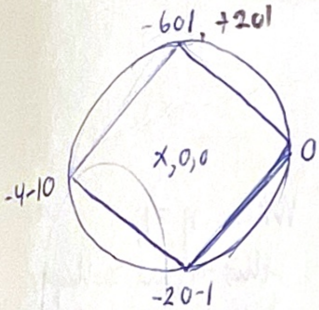
~~$\sigma_4 = \frac{2\sqrt{2}\,\eta}{(\eta^2+3)^{1/2}}$~~

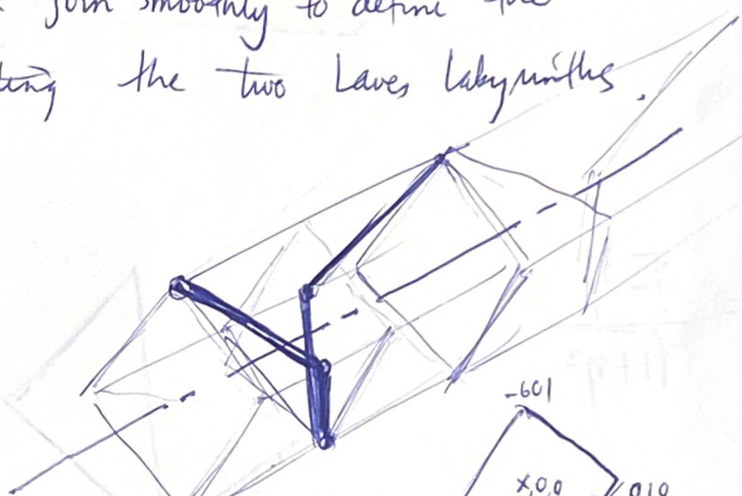
~~$\sigma_6 = \frac{2\eta}{\sqrt{3.1}}$~~

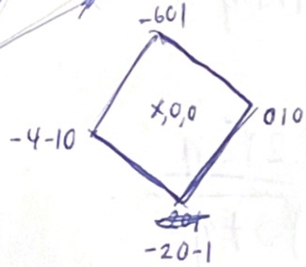
The Schwarz minimal surfaces $[\{\tilde{6},4\},\{\tilde{4},6\}]$ and $[\{\tilde{6},6\}]$ are of smaller area than any other surfaces which divide space into ◊ and s.c. labyrinths, respectively.

Of the 3 ∞ reg. skew saddle polyhedra which divide space into 2 Laves labyrinths, the case $\{\tilde{6},\tilde{4}\}$ appears to have smaller area than $\{\tilde{4},\tilde{6}\}$ and $\{\tilde{6},\tilde{6}\}$. The dihedral angle ($> 135°$) is closer to 180° than in the other two cases (120° and 90°).

It appears quite possible that if the edges of the $\{\tilde{6},\tilde{4}\}$ net are replaced by l.h. and r.h. <u>helices</u> which intersect at the vertices of $\{\tilde{6},\tilde{4}\}$, the modified skew hexagons (with helical edges) may provide minimal surface modules which join smoothly to define the surface of least area separating the two Laves labyrinths.





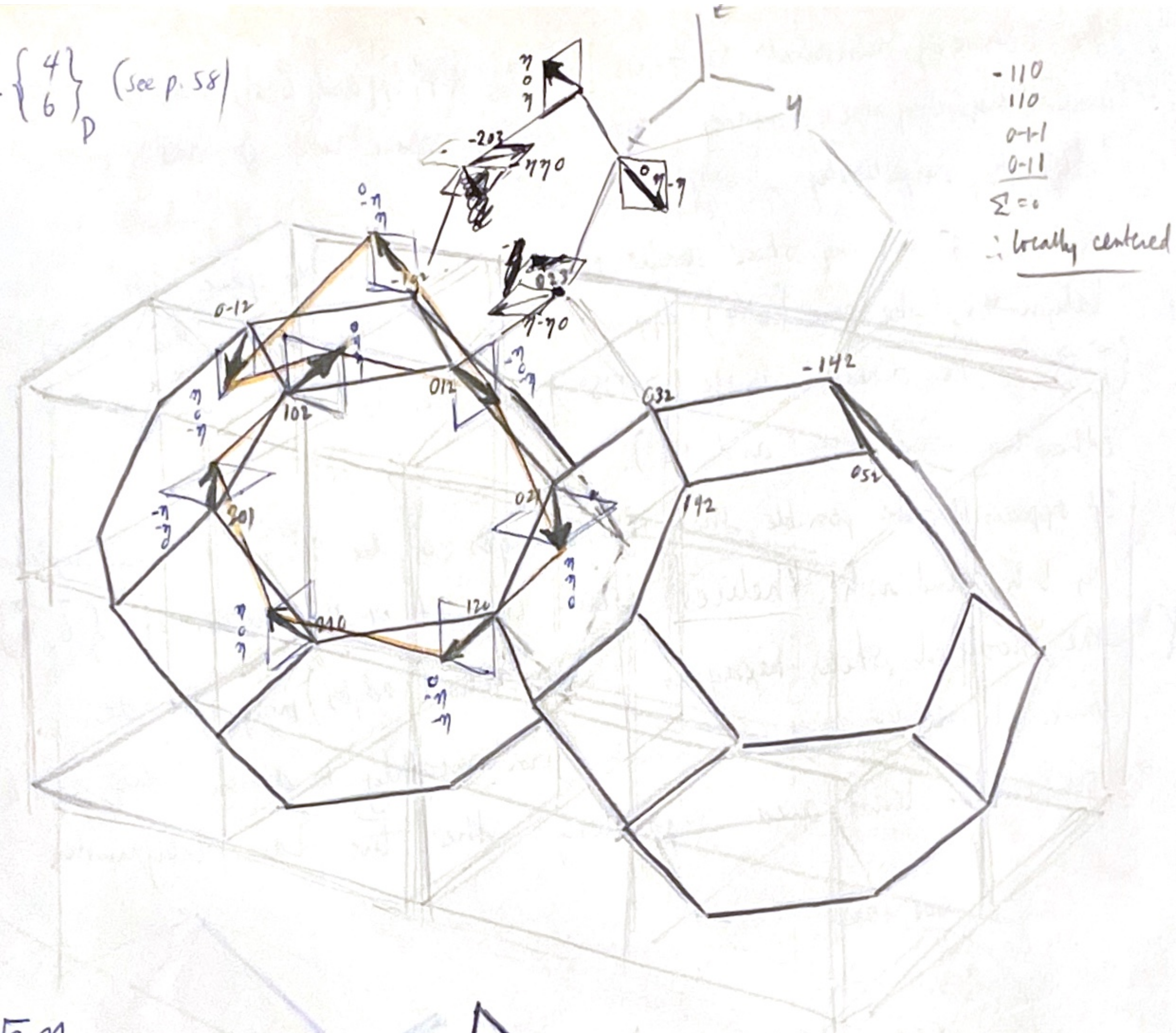


It would be useful to make some l.h. & r.h. helices of stiff copper wire, solder them together, and span them with soap — OR: make one good shillelagh for plaster forming.

"pitch" = **4**
1 revolution defines advance along helical axis of **4** diameters of corresponding cylinder

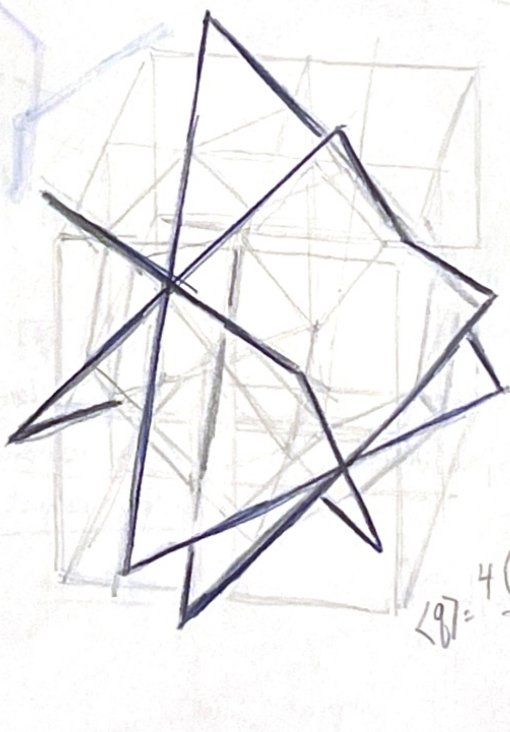
60

$\Lambda\left\{\begin{matrix}4\\6\end{matrix}\right\}_p$ (see p. 58)



$$\sigma_4 = \frac{\sqrt{2}\,\eta}{\sqrt{1+\eta^2}}$$

$$\sigma_6 = \frac{2\sqrt{2}\,\eta}{\sqrt{3+\eta^2}}$$



When $\eta = 1$,
this figure collapses
into a space-filling
assembly of these
symmetry domains
$\left(\begin{matrix}\sigma_4 = 1 \\ \sigma_6 = \sqrt{1/2}\end{matrix}\right)$.

$$\langle q\rangle = \frac{4(6)+6(2)}{10} = \frac{18}{5}$$