Friday, May

I have finally succeeded in obtaining a completely general description of all regular [infinite] 3-dim. $\{\tilde{p}, \tilde{q} | r\}$. The main point is not to ignore the hole r. The laves figures are truly intermediate between the Schwarz and Coxeter figures. The holes are helical polygons of intermediate pitch between those of S & C's figures. Whether any other $\{\tilde{p}, \tilde{q} | r\}$'s exist is a matter which cannot be settled exhaustively without examining the cubic space groups in gory detail. (I don't think I'll bother to do it!)

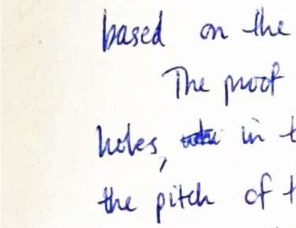
Sat., May 4 2PM

I have bothered to do it. I have succeeding in showing that there are only 3 families of warped polyhedra:

Regular skew polyhedra	Regular skew saddle polyhedra	Regular saddle polyhedra
$\{6, \tilde{4} \mid 4(0)\}_P$	$\{\tilde{6}, \tilde{4} \mid 4(4/\pi)\}_D$	$\{\tilde{6}, 4 \mid 4(\infty)\}_D$
$\{4, \tilde{6} \mid 4(0)\}_P$	$\{\tilde{4}, \tilde{6} \mid 4(2/\pi)\}_L$	$\{\tilde{4}, 6 \mid 4(2\sqrt{2}/\pi)\}_D$
$\{6, \tilde{6} \mid 3(0)\}_D$	$\{\tilde{6}, \tilde{6} \mid 3(3/2\pi)\}_L$	$\{\tilde{6}, 6 \mid 3(3\sqrt{3}/2\pi)\}_P$

based on the $\{6,4\}$, $\{4,6\}$, $\{6,6\}$ maps.

The proof depends on considering all those helical variants of the plane regular polygonal holes, ~~etc~~ in the regular skew polyhedra, which preserve hole regularity, and verifying that the pitch of the circumhelix of the regular helical polygonal hole can have no other values than those shown in parentheses (above). For any other values of the pitch, the polyhedron fails to "close back" on itself in a simple way (i.e., in such a way as to avoid multiple coverings of the ~~the~~ underlying minimal surface with a regular map; the case of $\{\tilde{6}, \tilde{4} | 4(2/\pi)\}$ (A) is an example of a kind of "self-intersecting polyhedron". It is analogous to the self-intersecting Schwarz-Schoenflies IPMS, based on the quadrilateral, which intersects itself only at edges, and only a finite # of times in any finite region of space, of course. Another kind of "self-intersecting polyhedron" is $\{\tilde{6}, \tilde{4} | 4(1/\pi)\}$ (B). This is analogous to those Schwarz-Schoenflies IPMS which intersects themselves not only at edges. (A) is a space-filling of diamond net symmetry domains. (B) is a "polyhedron" which is a "slightly skewed" $\{6, \tilde{4} | 4\}$. In (A), 6 faces come together at every edge. Thus, it intersects itself twice at every edge ( ), We could speak of density = 3.

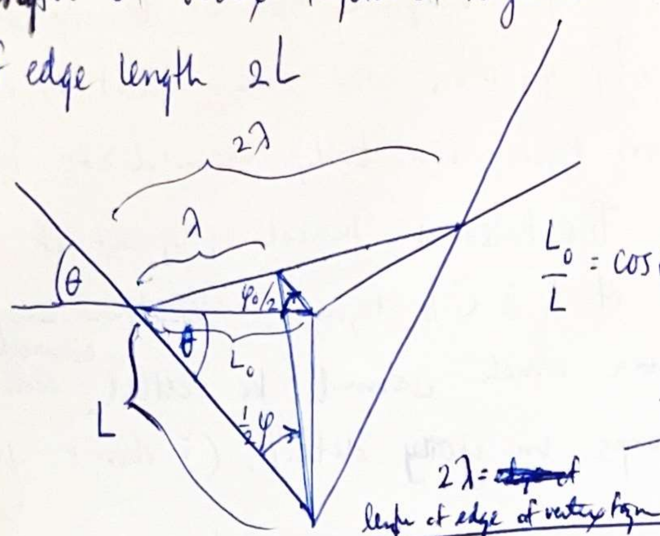


We have here analogs of Riemann surfaces — the difference being that these surfaces are not related to the sphere. ~~Here~~ The discussion in Coxeter on pp. 104-105 (ff.) is relevant here.

We will use the term "regularity transformations" to describe those [non-linear] transformations of edges and vertices which preserve the regularity

(cont. on p. 96 (opp.)

Length of vertex figure of regular skew polygon $\{\tilde{p}\}$ of edge length $2L$



$\lambda = L_0 \sin \frac{\varphi_0}{2}$ $\qquad \sin\frac{\varphi_0}{2} = \cos\frac{\pi}{p}$

$\varphi_0 = \left(1-\frac{2}{p}\right)\pi$; $\frac{\varphi_0}{2} = \frac{\pi}{2} - \frac{\pi}{p}$

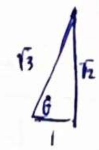
$\frac{L_0}{L} = \cos\theta$ $\qquad \therefore \lambda = L_0 \cos\frac{\pi}{p} = L\cos\theta\cos\frac{\pi}{p}$

$\left(\cos\theta = \frac{\sin}{\sin}\right.$

$\therefore$ $\boxed{2\lambda = (2L\cos\theta)\cos\frac{\pi}{p}}$

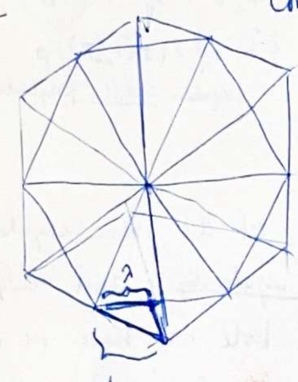
Example: Consider $60° \{\tilde{6}\}$ with $l = 2\sqrt{2}$ $(\approx 2.828^{+})$

$2\lambda = \cancel{2}(2\sqrt{2})(.57735 = \frac{1}{\sqrt{3}})(\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}) = \cancel{2}\sqrt{2}$ Correct



Example:

[5/25/71 (!)]

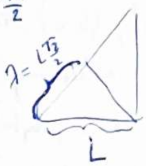


Consider $90° \{\hat{6}\}$ with edge length $2L$, $\varphi = \frac{\pi}{2}$; $\theta = \cos^{-1}\left(\sqrt{\frac{1-\cos\varphi}{1-\cos\varphi_0}} = \frac{\sqrt{6}}{3}\right)$

Then

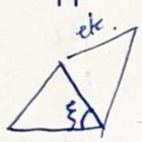
$\lambda = L\cos\theta\cos\frac{\pi}{p}$ $\qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

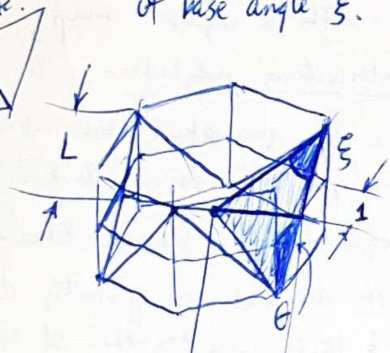
$= L\frac{\sqrt{6}}{3}\frac{\sqrt{3}}{2} = L\frac{\sqrt{2}}{2}$



✓

5/25/71 Suppose we wish to approximate a regular skew polygon by a collection of ~~2~~ p plane isosceles triangles of base angle ξ. How does ξ depend on p, θ, in general? (i.e, not necessarily a $\{\hat{6}\}$)

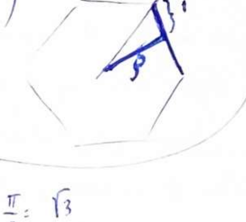




$L = \frac{1}{\cos\theta} = \frac{\sin\varphi_0/2}{\sin\varphi/2}$

$\boxed{\tan\xi = \frac{\rho}{L} = \cot\frac{\pi}{p}\cos\theta}$

$\boxed{\rho = \cot\frac{\pi}{p}}$



For $\{\hat{6}\}$, $\cot\frac{\pi}{p} = \sqrt{3}$

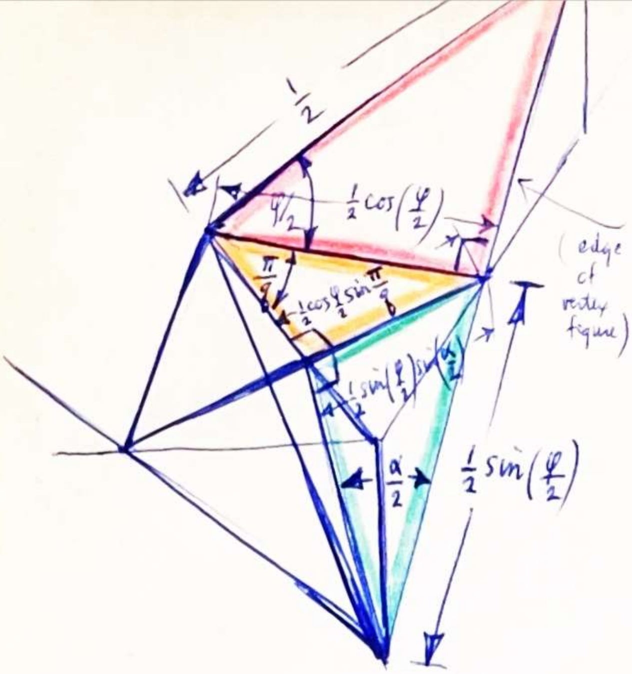
For example: $\boxed{\{\hat{6}\}_{90°}}$ $\left(\cos\theta = \frac{\sin 45°}{\sin 60°} = \sqrt{\frac{2}{3}}\right)$; $\tan\xi = \sqrt{3}\cdot\sqrt{\frac{2}{3}} = \sqrt{2}$ $(\xi = 54°45')$

$\{\hat{6}\}_{60°}$ $\quad \cos\theta = \frac{1}{\sqrt{3}}$; $\tan\xi = \frac{\sqrt{3}}{\sqrt{3}} = 1$ $(\xi = 45°)$

Relation between α and φ

(α = face angle of vertex figure $\{\tilde{q}\}$

φ = face angle of face $\{\tilde{p}\}$)



$$\frac{1}{2}\cos\frac{\varphi}{2}\sin\frac{\pi}{q} = \frac{1}{2}\sin\frac{\varphi}{2}\sin\frac{\alpha}{2}$$

$$\sqrt{\frac{1+\cos\varphi}{2}}\sqrt{\frac{1-\cos\frac{2\pi}{q}}{2}} = \sqrt{\frac{1-\cos\varphi}{2}}\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\therefore \cos\alpha = \frac{(1-C)-(1+C)\cos\varphi}{1-\cos\varphi} \quad \left(C = 1-\cos\left(\frac{2\pi}{q}\right)\right)$$

or $$\boxed{\cos\alpha = \frac{\cos\left(\frac{2\pi}{q}\right) - \left(2-\cos\frac{2\pi}{q}\right)\cos\varphi}{1-\cos\varphi}}$$

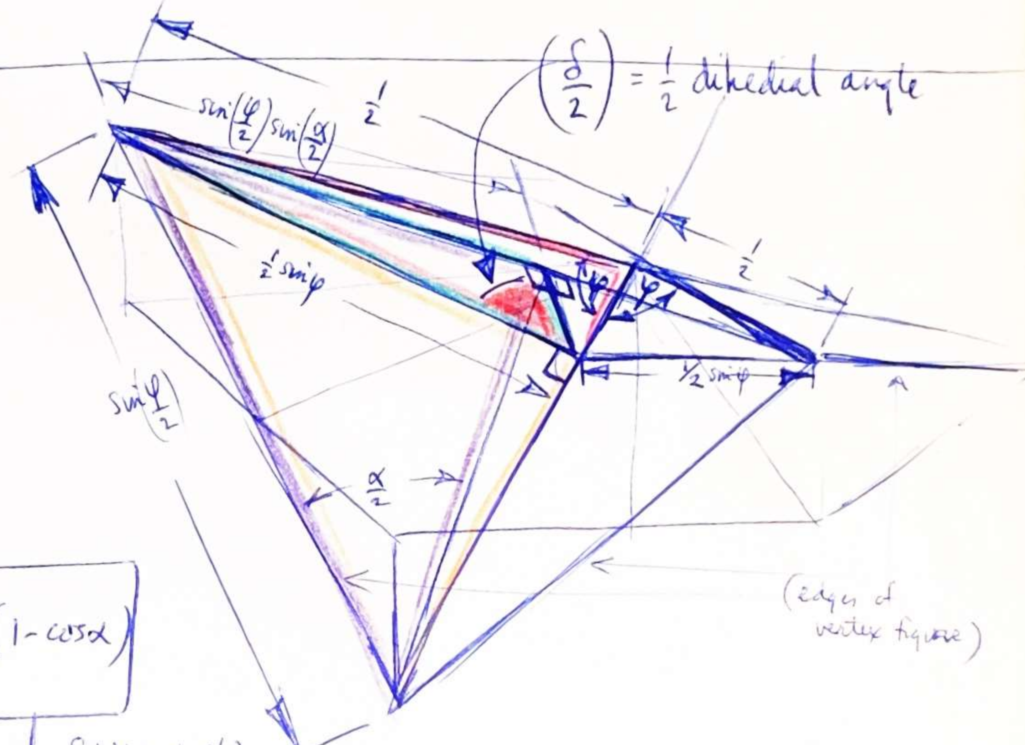
$$\cos\alpha_4 = \frac{-2\cos\varphi}{1-\cos\varphi}$$

$$\cos\alpha_6 = \frac{1}{2}\frac{(1-3\cos\varphi)}{(1-\cos\varphi)}$$

$$\cos\alpha_8 = \boxed{\frac{\frac{\sqrt{2}}{2}-\left(2-\frac{\sqrt{2}}{2}\right)\cos\varphi}{1-\cos\varphi}}$$

Dihedral angle δ

$\left(\frac{\delta}{2}\right) = \frac{1}{2}$ dihedral angle



$$\sin\left(\frac{\varphi}{2}\right)\sin\left(\frac{\alpha}{2}\right) = \left(\frac{1}{2}\sin\varphi\right)\left(\sin\frac{\delta}{2}\right)$$

$$\sqrt{\frac{1-\cos\varphi}{2}}\sqrt{\frac{1-\cos\alpha}{2}} = \frac{1}{2}\sin\varphi\sqrt{\frac{1-\cos\delta}{2}}$$

$$\left(\frac{1-\cos\varphi}{2}\right)\left(\frac{1-\cos\alpha}{2}\right) = \frac{\sin^2\varphi}{4}\left(\frac{1-\cos\delta}{2}\right)$$

$$\therefore 1-\cos\delta = \frac{2}{\sin^2\varphi}(1-\cos\varphi)(1-\cos\alpha)$$

$$\boxed{\cos\delta = 1 - \frac{2}{\sin^2\varphi}(1-\cos\varphi)(1-\cos\alpha)}$$

or $$\boxed{\cos\delta = 1 - 2\left(\frac{1-\cos\alpha}{1+\cos\varphi}\right)}$$

Substitute $\alpha = \alpha(\varphi)$ from above:

$$\boxed{\cos\delta_{[q=4]} = -\frac{(1+\cos\varphi)}{(1-\cos\varphi)}}$$

Cf. pp. 156-157 Coxeter Int. to Geom.

$$\boxed{\cos\delta_{[q=6]} = \frac{-\cos\varphi}{1-\cos\varphi}}$$

In general $\cos\delta = \dfrac{\left(-1+2\cos\frac{2\pi}{q}\right) - \cos\varphi}{\ldots}$ ☆

(1

① $\Lambda\{\tilde{4},6\}_D = \begin{Bmatrix}\tilde{4}\\ \tilde{4}\end{Bmatrix}^D_6$ $\eta=\pm\frac{1}{2}$ cubes ①

② $\Lambda\{\tilde{6},4\}_D = \begin{Bmatrix}\tilde{6}\\ \tilde{6}\end{Bmatrix}^D_4$ $\eta=\pm 2$ diamond tetrahedra ②

③ $\Lambda\{\tilde{6},6\}_P = \begin{Bmatrix}\tilde{6}\\ \tilde{6}\end{Bmatrix}^P_6$ $\eta=\pm\frac{1}{2}$ " "

④ $\Lambda\{4,\tilde{6}\}_P = \begin{Bmatrix}\tilde{4}\\ \tilde{4}\end{Bmatrix}^P_6$ $\eta=\pm\frac{1}{2}$ tetragonal tetrahed ③

⑤ $\Lambda\{6,\tilde{4}\}_P = \begin{Bmatrix}\tilde{6}\\ \tilde{6}\end{Bmatrix}^P_4$ $\eta=\pm 1$ expanded octahedr ④

~~($\Lambda\{6,\tilde{6}\}_D$ does not exist~~

⑥ $\Lambda\left[\begin{Bmatrix}4\\ 6\end{Bmatrix}_D = t\{\tilde{4},6\}_D\right] = \begin{Bmatrix}\tilde{4}\\ \tilde{6}\end{Bmatrix}^D_4$ $\eta=1$ tetrahedral ⑤ "decahedron

⑦ $\Lambda\left[\begin{Bmatrix}4\\ \tilde{6}\end{Bmatrix}_P = t\{4,\tilde{6}\}_P\right] = \begin{Bmatrix}\tilde{4}\\ \tilde{6}\end{Bmatrix}^P_4$ $\eta=1$ tetragonal tetrahedra

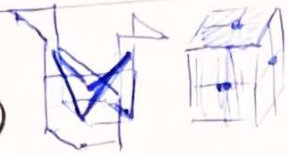
$\eta=-1$ expanded octahedra

⑧ (=②) $\Lambda\left[\begin{Bmatrix}6\\ \tilde{6}\end{Bmatrix}_D = t\{6,\tilde{6}\}_D\right] = \begin{Bmatrix}\tilde{6}\\ \tilde{6}\end{Bmatrix}^D_4$ $\eta=1$ diamond tetrahedra

)

[Verified] Summary of Space Fillings of Polyhedra reached by $\Lambda_\eta\{\tilde{p},\tilde{q}|\lambda\}$ (10)
and $\Lambda_\eta\left\{\begin{smallmatrix}\tilde{p}\\ \tilde{q}\end{smallmatrix}\right\}$

1) (p.42) $\Lambda\{\tilde{4},6\}_D = \boxed{\left\{\begin{smallmatrix}\tilde{4}\\ \tilde{4}\end{smallmatrix}\right\}_{6(D)}}$



Alternate vertices of the IBCC are centers of squares of edge length 2

I.D. & S.D. of

(SD) S.C. $(100)_6$

For $\eta = \frac{1}{2}$, $\Lambda_{\frac{1}{2}}\{\tilde{4},6\}$ = space filling of cubes $(\sigma_A \to 0,\ \sigma_B \to \infty)$

2) p. $\Lambda\{\tilde{6},4\}_{D,\ \eta=\pm 2} = \boxed{\left\{\begin{smallmatrix}\tilde{6}\\ \tilde{6}\end{smallmatrix}\right\}_{4(D)}}$ space filling of diamond saddle tetrahedra (SD or ID) $(111)_4$

3) $\Lambda\{\tilde{6},6\}_{P,\ \eta=\pm\frac{1}{2}} \boxed{\left\{\begin{smallmatrix}\tilde{6}\\ \tilde{6}\end{smallmatrix}\right\}_{6(P)}}$ " " " " " (SD or ID) $(111)_4$

4) p.53 $\Lambda\{4,\tilde{6}\}_{P,\ \eta=\pm\frac{1}{2}} = \boxed{\left\{\begin{smallmatrix}4\\ \tilde{4}\end{smallmatrix}\right\}_{6(P)}}$ space filling of tetragonal tetrahedra $\left\{\begin{matrix}\text{SD of IBCC } (100)_4\\ \text{ID of (BCC) } (111)_8\end{matrix}\right\}$

5) $\Lambda\{6,\tilde{4}\}_{P,\ \eta=\pm 1} \boxed{\left\{\begin{smallmatrix}\tilde{6}\\ \tilde{6}\end{smallmatrix}\right\}_{4(P)}}$ " " " expanded octahedra $\left\{\begin{matrix}\text{SD} \\ \text{ID}\end{matrix}\right\}$ of $\left\{\begin{matrix}\text{BCC } (111)_8\\ \text{IBCC } (100)_4\end{matrix}\right\}$

(6) $\Lambda\{6,\tilde{6}\}_D$ (Impossible to use Λ on vertices, because holes are triangles)

$\Lambda\{\tilde{p},\tilde{q}|\lambda\}_2$ does not lead to space-fillings of finite polyhedra

6) p.58 $\Lambda\left[\left\{\begin{smallmatrix}4\\ 6\end{smallmatrix}\right\}_D = t\{\tilde{4},6\}_D\right]_{\eta=1}$ = space filling of ~~tetrahedra~~ $\langle \tilde{4}[1], \tilde{6}[\sqrt{2}]\rangle$ (ID) of 6-connected FCC

7) p.83 $\Lambda\left[\left\{\begin{smallmatrix}4\\ \tilde{6}\end{smallmatrix}\right\}_P = t\{\tilde{4},6\}_P\right]_{\eta=1}$ → space filling of ~~diamond~~ tetragonal tetrahedra $\left\{\begin{matrix}\text{SD of (IBCC)}\\ \text{ID of (BCC)}\end{matrix}\right\}$ ~~SD or ID~~ (111)

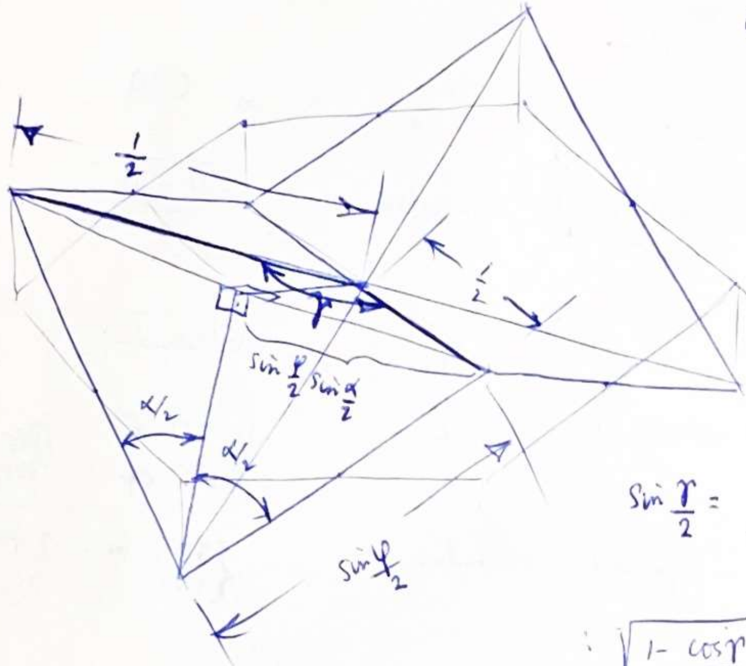
$\eta=-1$ → space filling of expanded octahedra $\left\{\begin{matrix}\text{SD of BCC}\\ \text{ID of IBCC}\end{matrix}\right\}$

8) = (2) ☆ p.85 $\Lambda\left[\left\{\begin{smallmatrix}6\\ \tilde{6}\end{smallmatrix}\right\}_D = t\{6,\tilde{6}\}\right]_{\eta=1}$ → space-filling of diamond tetrahedra

~~(4) Λ [~~

Relation between γ, face angle of helical polygon <u>hole</u>, and φ and α

γ = central ∡ subtended by 2 alternate vertices of the vertex figure.



$$\sin\frac{\gamma}{2} = \frac{\sin\frac{\varphi}{2}\sin\frac{\alpha}{2}}{\frac{1}{2}} = 2\sin\frac{\varphi}{2}\sin\frac{\alpha}{2}$$

$$\therefore \sqrt{\frac{1-\cos\gamma}{2}} = 2\sqrt{\frac{1-\cos\varphi}{2}}\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\therefore \boxed{\cos\gamma = 1 - 2(1-\cos\varphi)(1-\cos\alpha)}$$

Since $\cos\alpha = \dfrac{\cos\left(\frac{2\pi}{q}\right) - \left(2-\cos\frac{2\pi}{q}\right)\cos\varphi}{1-\cos\varphi}$,

$$\cancel{1-\cos\alpha = \frac{1-\cos\frac{2\pi}{q} + \left(1-\cos\frac{2\pi}{q}\right)\cos\varphi}{1-\cos\varphi}}$$

$$\cancel{\cos\gamma = -1 + 2\left[\cos\frac{2\pi}{q} - \left(1-\cos\frac{2\pi}{q}\right)\cos\varphi\right]}$$

$$1-\cos\alpha = \frac{(1+\cos\varphi)\left[1-\cos\frac{2\pi}{q}\right]}{1-\cos\varphi}$$

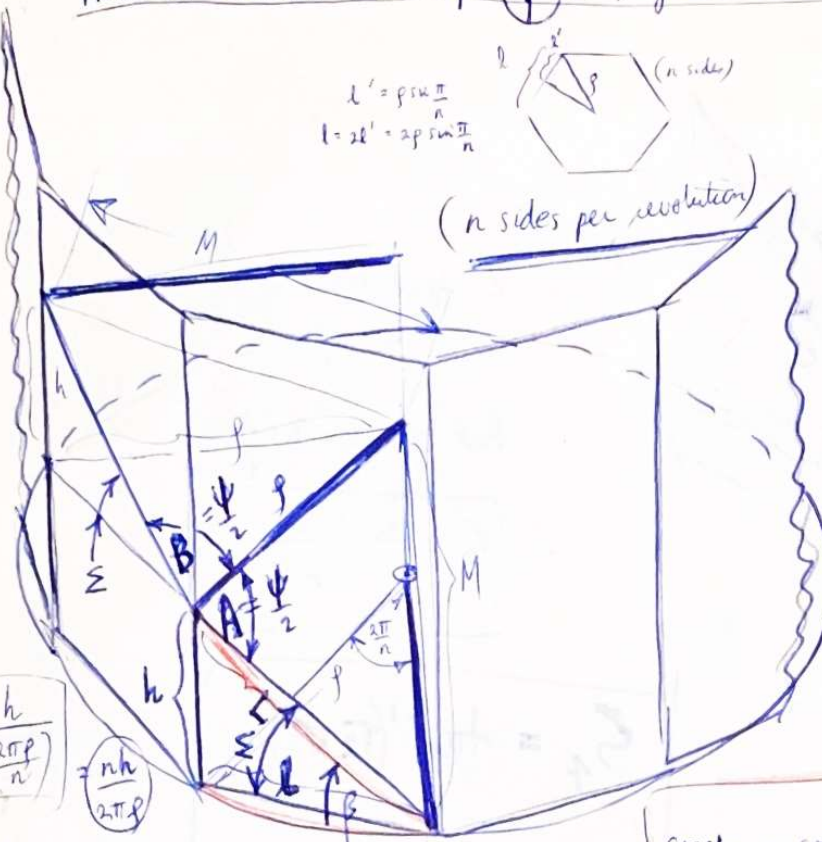
$$\boxed{\cos\gamma = 1 - 2(1+\cos\varphi)\left(1-\cos\frac{2\pi}{q}\right)}$$

γ = face angle of <u>hole</u>

φ = " " " <u>face</u>

$\{q\}$ = vertex figure

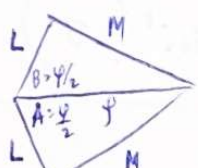
Relation between face angle (ψ) of regular helical polygon and slope $\tau = \tan\beta$ of circum-helix and $\boxed{\tan\Sigma = \omega}$



$$\boxed{\tan\beta = \frac{h}{\left(\frac{2\pi\rho}{n}\right)}} = \frac{nh}{2\pi\rho}$$

β = helix angle

τ = slope = $\tan\beta$



$$\cos\frac{\psi}{2} = \frac{L^2+\rho^2-M^2}{2L\rho}$$

$$= \frac{(h^2+l^2)+\rho^2-(\rho^2+h^2)}{2L\rho} = \frac{l^2}{2L\rho}$$

$L^2 = h^2 + l^2$

$l = 2\rho\sin\frac{\pi}{n}$

$$\therefore \cos\frac{\psi}{2} = \frac{\sin^2\frac{\pi}{n}}{\sqrt{\frac{\pi^2}{n^2}\tan^2\beta + \sin^2\frac{\pi}{n}}}$$

$$\boxed{\cos\psi = 2\cos^2\frac{\psi}{2} - 1 = \frac{2\sin^2\left(\frac{\pi}{n}\right)}{1+\left(\frac{\pi^2}{n^2}\right)\left(\frac{\tan^2\beta}{\sin^2\left(\frac{\pi}{n}\right)}\right)} - 1}$$

(n need not be integer)

p = pitch = $\frac{1}{2\pi}$ (axial advance per revolution)

Define a = "radius-normalized axial advance per revolution"

Then $a = \frac{2\pi p}{\rho} = 2\pi\tan\beta$ $\left(\tan\beta = \frac{p}{\rho}\right)$

$\therefore \tan\beta = \frac{a}{2\pi}$

$\underline{n=4}$ $$\boxed{\cos\psi = \frac{-\tan^2\beta}{\frac{8}{\pi^2} + \tan^2\beta}}$$

$$\tan\Sigma = \frac{h}{l} = \frac{\frac{2\pi\rho\tan\beta}{n}}{2\rho\sin\frac{\pi}{n}}$$

$\underline{n=3}$ $$\boxed{\cos\psi = \frac{\frac{27}{8\pi^2} - \tan^2\beta}{\frac{27}{4\pi^2} + \tan^2\beta}}$$

or $$\boxed{\tan\Sigma = \frac{\pi\tan\beta}{n\sin\frac{\pi}{n}}}$$

For $\underline{n=4}$ $$\boxed{\cos\varphi = \frac{-1}{1+\frac{32}{a^2}}}$$

For $\underline{n=3}$ $$\boxed{\cos\varphi = \frac{27-2a^2}{27+2a^2}}$$

(I'll use ψ for the face angle of helical polygon holes, in order to avoid confusion.) ψ_0 = face angle for limiting (plane) polygon $\{n\}$

$$\boxed{\tau = \tan\beta = \left(\frac{n}{\pi}\sin\frac{\pi}{n}\right)\left[\frac{\cos\psi_0 - \cos\psi}{\cos\psi + 1}\right]^{1/2}}$$

$$\left(\text{Cf. } \sigma = \tan\theta = (1)\left[\frac{\cos\varphi_0 - \cos\varphi}{\cos\varphi - 1}\right]^{1/2}\right) \quad c = \cos\psi_0$$

$$\boxed{\omega = \tan\Sigma = (1)\left[\frac{\cos\psi_0 - \cos\psi}{\cos\psi + 1}\right]^{1/2}}$$

0	$\psi\{\tilde{6},\tilde{4}\}_L = \cos^{-1}(-2/3)$
0	$\psi\{\tilde{\infty},\tilde{6}\}_L = \cos^{-1}(-1/3)$
1/2	$\psi\{\tilde{\infty},\tilde{\infty}\}_L = \cos^{-1}(0)$

(Better not to use τ for slope of tangent to helix, in order to avoid confusion with $\underline{\text{torsion}}$)

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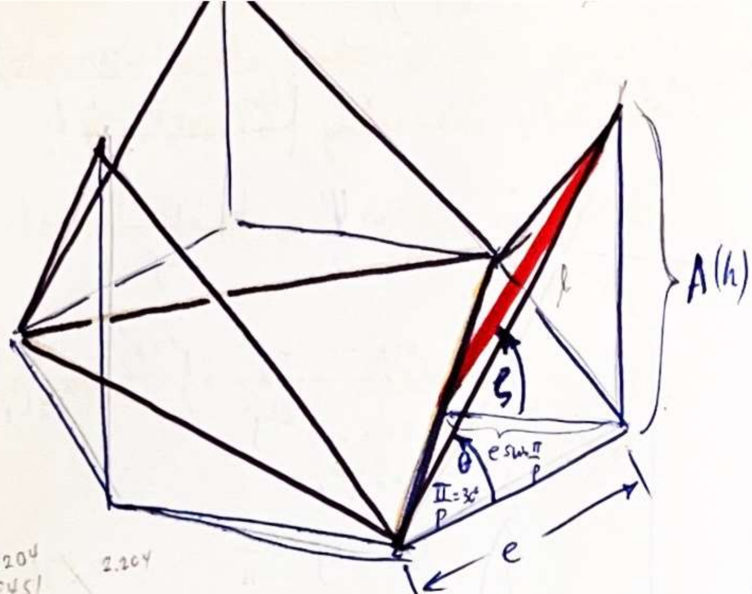
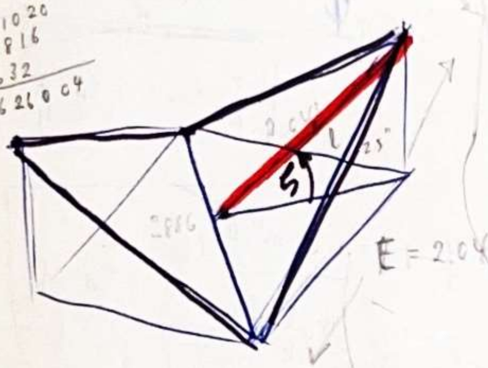


$$\zeta = \tan^{-1}\frac{A}{e\sin\frac{\pi}{p}}$$

But $\sigma = \tan\theta = \frac{A}{e}$

$$\therefore \boxed{\zeta = \tan^{-1}\frac{\sigma}{\sin\frac{\pi}{p}}}$$



$$\boxed{\begin{aligned}\zeta_4 &= \tan^{-1}(\sqrt{2}\,\sigma)\\ \zeta_6 &= \tan^{-1}(2\sigma)\end{aligned}}$$

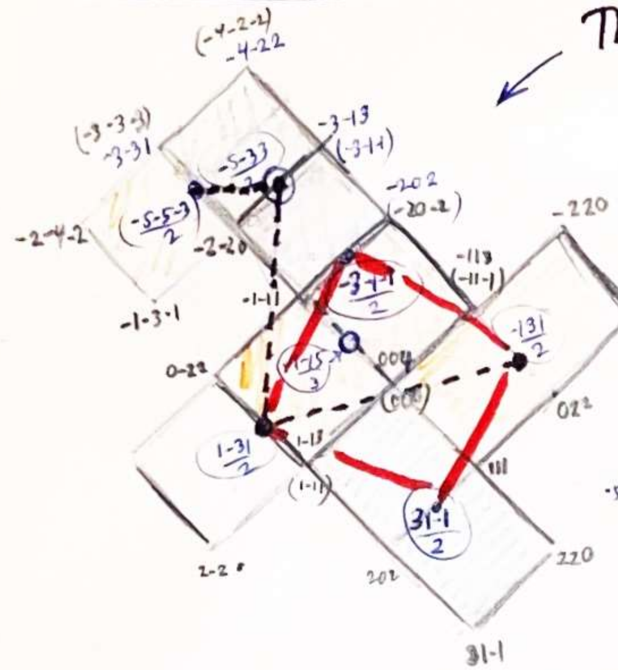
These equations are useful for the construction of shillelaghs.

$\{\tilde{4}\}$

e	φ	σ	√2σ	ζ
63.435°	36.87°	2	2.828	70.526°
54.735°	48.19° (112)	√2 = 1.414	2	63.435°
45°	60° (110)	1	1.414	54.733°
35.264°	70.526° (111)	√2/2 = .707	1	45° ~~35.264°~~
32.315° cos = .84314	73.39° (123)	$\sqrt{\frac{2}{5}}$ = .6325	[illegible]	41.806° ~~34.315°~~
e = 1.85	80.407° (112)	$\sqrt{\frac{1}{5}}$ = .4472	$\sqrt{\frac{2}{5}}$	32.313°
21.415°	82.338° 82.338° (125)	$\sqrt{\frac{2}{13}}$ = .3922		29.013° ~~33.683°~~

$\{\tilde{6}\}$

φ	σ	2σ	ζ	θ	sin θ	l	l sin θ = A	l cos θ
33.559° (112)	√8 = 2.828	5.656	79.973°	70.526°				
60° (110)	√2 = 1.4142	2.828	70.526°	54.733°				
72.542° (310)	$\sqrt{\frac{8}{7}}$ = 1.0691	2.1382	64.935°	46.912°				
90° (100)	$\frac{\sqrt{2}}{2}$ = .7071	1.414	54.733°	35.264°				
99.593° (112)	$\sqrt{\frac{2}{7}}$ = .534	1.0691	46.912°	28.125°	.47139	2.886	1.360	2.545
109.47° (111)	$\sqrt{\frac{1}{8}}$ = .3535	.7071	35.264°	19.47°				
115.688° (125)	$\sqrt{\frac{2}{43}}$ = .2157		23.335°	12.172°	.21054			



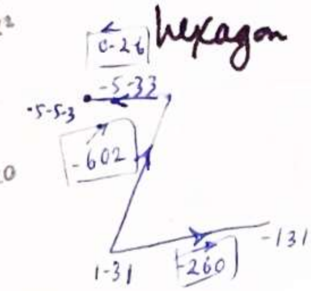
This is a top view (looking along −z axis) of the self-intersecting $\{\tilde{6}[109\tfrac{1}{2}°], \tilde{4}\}$.

Shown in red is the quadrilateral of $\{\tilde{4}, \tilde{6}\}$;

shown in black [- - - -] is the hexagon of $\{\tilde{6}, \tilde{6}\}$.

$\alpha_{\text{vertex fig.}} = 60°$ for $\{\tilde{6}, \bar{4} | \bar{4}\}$

$\psi_{\text{hole}} = 109.5°$ for $\{\tilde{6}, \bar{4} | \bar{4}\}_{dh}$



$\boxed{\{\tilde{6}\} \text{ of } \{\tilde{6}, \tilde{6} | 3\}_{dh}}$

-301
-136
$\frac{300}{10} = \frac{3}{10} = \cos\varphi$ $\therefore \boxed{\varphi = 72.542°}$ $\left(\cos\varphi = \frac{3}{10}\right)$

$$(\cos\alpha)_{\tilde{6},\tilde{6}} = \frac{1}{2}\frac{(1-3\cos\varphi)}{(1-\cos\varphi)} = \frac{1}{2}\frac{\left[1-3\left(\frac{3}{10}\right)\right]}{\left[1-\frac{3}{10}\right]} = \frac{1}{2}\frac{1/10}{7/10} = \frac{1}{14} = .0714285714285\ldots \quad \therefore \alpha = 44.425°$$

$\boxed{\alpha = 44.425°}$ $\left(\cos\alpha = \frac{1}{14}\right)$

$$(\cos\delta)_{\tilde{6},\tilde{6}} = \frac{-\cos\varphi}{1-\cos\varphi} = \frac{-3/10}{1-\frac{3}{10}} = -\frac{3}{7} \quad \therefore \delta = \cancel{.42857}\ \cancel{64.623}\ 115.377$$

$= .42857$

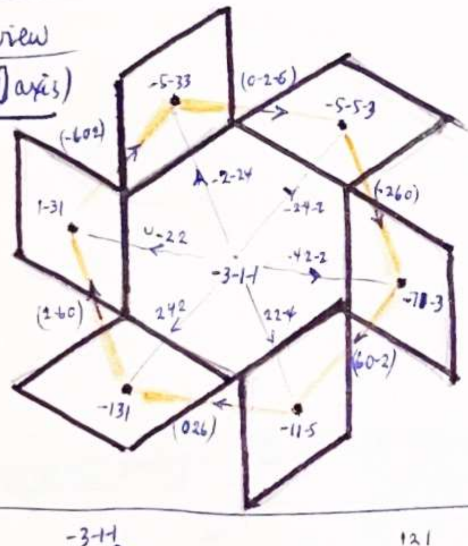
$\boxed{\delta = 115.377°}$ $\left(\cos\delta = -3/7\right)$

$$\sigma_{\text{face}} = \sqrt{\frac{\cos\varphi - \cos\varphi_c}{1-\cos\varphi}} = \sqrt{\frac{3/10 + 1/2}{1-3/10}} = \frac{8/10}{7/10} = \sqrt{\frac{8}{7}} = 1.0691$$

$\omega = \tan\Sigma = \sqrt{\frac{8}{7}}$

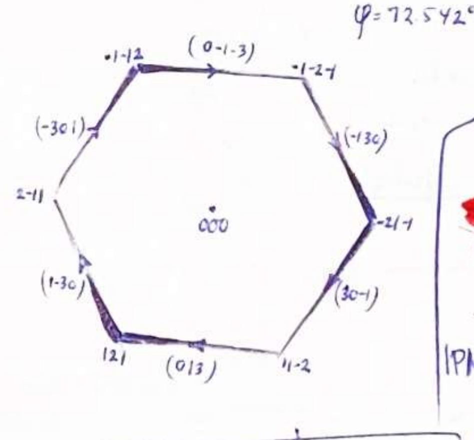
$\boxed{\sigma = \sqrt{8/7} = 1.0691}$

Top view
(along [1-1-1] axes)



Hexagonal face of $\{\tilde{6}, \tilde{6}\}_{S.I.}$

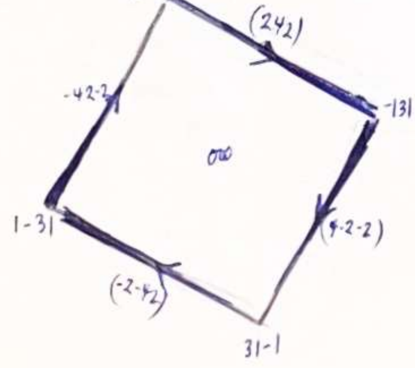
$\varphi = 72.542°$



NOTE: Because $\{\tilde{6}, \tilde{6}\} = h\{\tilde{4}, \tilde{6}\}$ [in all cases]

$\varphi\{\tilde{6}, \tilde{6}\} = \alpha\{\tilde{4}, \tilde{6}\}$

BUT: This applies only to polyhedra which correspond to regular maps on the same IPMS. Hence: $\{6, \tilde{6}\}_D = h\{\tilde{4}, 6\}_D$ and $\{\tilde{6}, 6\}_P = h\{4, \tilde{6}\}_P$ while $\{\tilde{6}, \tilde{6}\}_L = h\{\tilde{4}, \tilde{6}\}_L$



121
2-11
2-2-1
6

$\varphi = \cos^{-1}\left(\frac{1}{6}\right) = \boxed{80.407° = \varphi}$

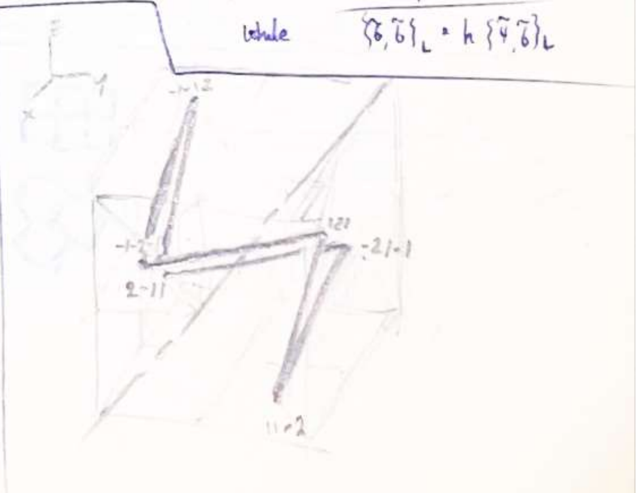
$\alpha = \cos\left(\frac{3}{10}\right) = \boxed{72.542° = \alpha}$

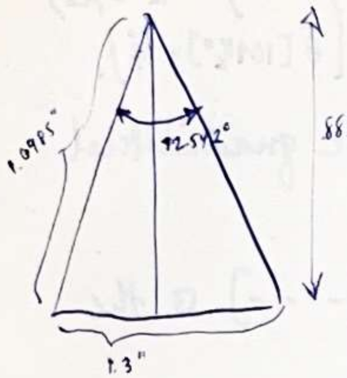
$$\delta = \cos^{-1}\left[\frac{-\cos\varphi}{1-\cos\varphi}\right] = \frac{-1/6}{5/6} = -\frac{1}{5}$$

$= \cos^{-1}(-1/5)$ $\boxed{101.537° = \delta}$

$\psi = \cos^{-1}\left(-\frac{1}{6}\right) = 99.593°$

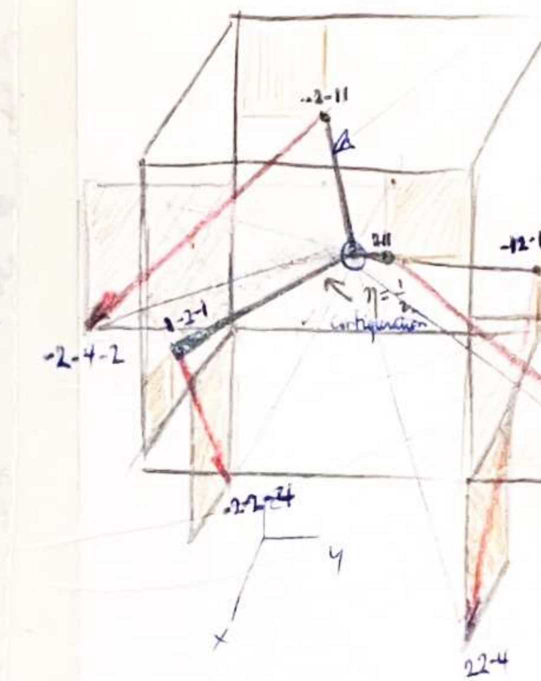
$\omega = \tan\Sigma = \frac{\sqrt{5}}{5}$





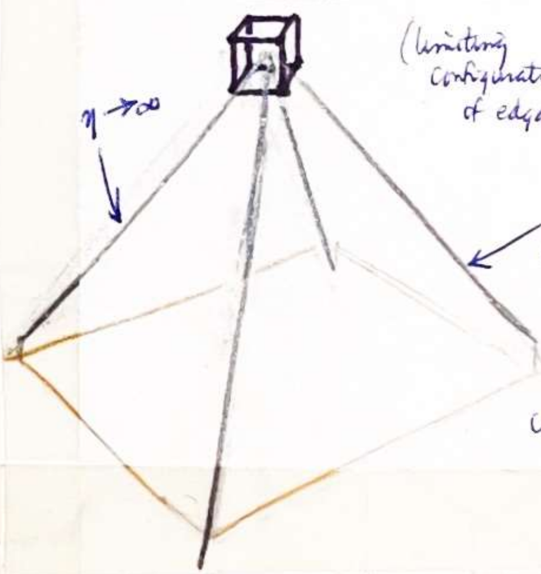
The point of view taken here is that the transformation $\Lambda\{p,q|r\}$ or $\Lambda t\{p,q|r\}$ is most conveniently described by letting the underlying space lattice remain fixed and displacing each vertex according to the fundamental rule ("golden rule" for Λ). This means that the edges change in length. Finally, for those cases where $\eta \to \infty$, the edges are all infinitely long. While this approach is mathematically convenient, it is advantageous to <u>normalize</u> the transformation to constant edge length. (See p. 88, for example.) Then the unit cell of the lattice shrinks monotonically, starting with the fully expanded state (the <u>regular</u> $\{p,q|r\}$).

<u>locally-centered</u> net



This is the fully deployed configuration $= \{\tilde{6},\tilde{4}\}_L$. ($\eta=\frac{1}{2}$, starting with $(6\,1[\sqrt{2}]\cdot 6\,[0])$)

<u>not locally-centered</u> net

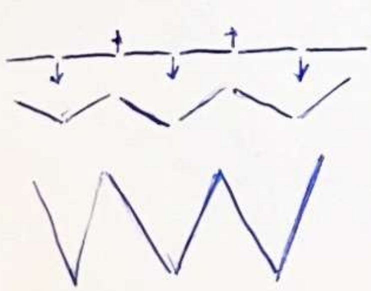


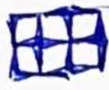
(limiting configuration of edges)

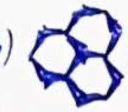
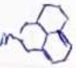
each edge has rotated 90°, when $\eta \to \infty$, starting with the $\{\tilde{6},\tilde{4}\}_L$ configuration.

<u>The golden rule says:</u>
Λ is a transformation in which every vertex is displaced along the local symmetry axis by an amount = to that of every other. Adjacent vertices are displaced into alternate latitudes.

I

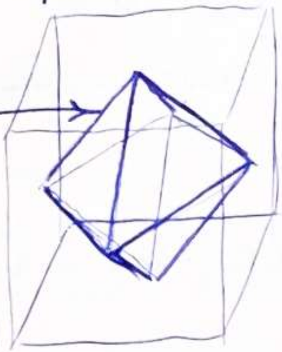


II a)  from 

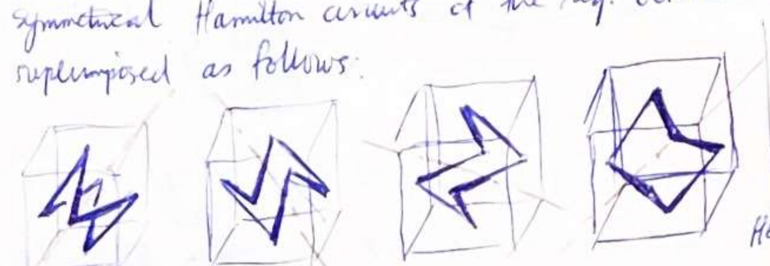
b)  from 

c)

Now let us start to consider in some detail the expandable space frame applications of the $\{p,q|n\}$, especially the Laves figures (because they have unlimited range of "de-deployment", i.e., collapse, as compared to the small amount of collapse of the primitive & diamond labyrinth regular figures. In fact, it appears that the Laves figures have the property that they undergo complete collapse — i.e., that a strictly infinite 3-dimensional $\{p,q|n\}_L$ collapses to a finite symmetrical polyhedron when $\eta \to \infty$. The example on p. 91 $[N(\{\tilde{6},\tilde{4}\})]$ will be studied to show this. In this case, all of the polygons of the net develop $\sigma = \sqrt{2}$ when $\eta \to \infty$, and they all become superimposed to form one finite regular octahedron! Thus, the 4 distinct orientations of the $\sigma = \sqrt{2}$ hexagons, which are the most symmetrical Hamilton circuits of the reg. octahedron, become superimposed as follows:



Each edge is a superposition of an infinite number of edges of the original regular warped polyhedron)



Each hexagon shares two common [parallel] edges with every other

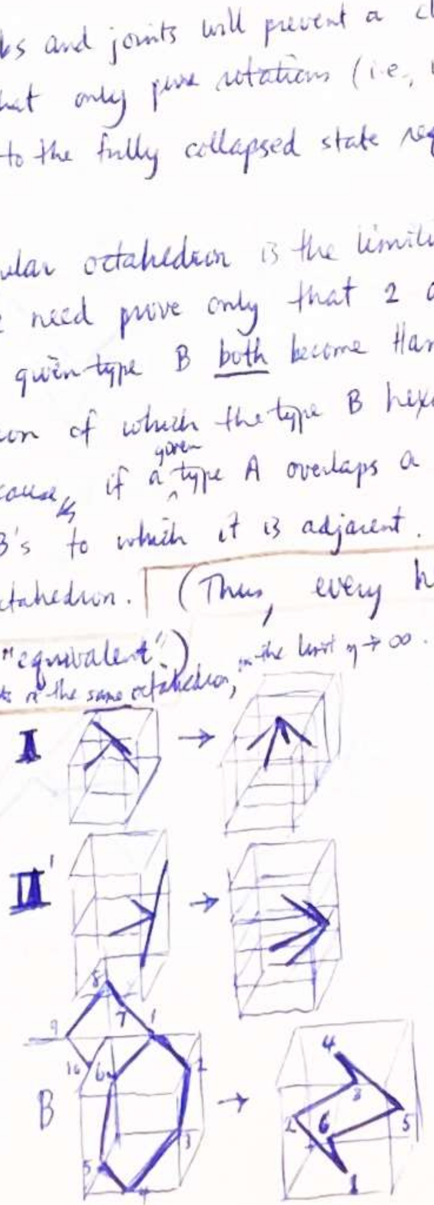
Hence this is not a regular polyhedron

In practice, only the finite thickness of the webs and joints will prevent a close approach to this behavior. Furthermore, it turns out that only pure rotations (i.e., in a single plane) are required of each web. From $\{\tilde{6},\tilde{4}\}$ to the fully collapsed state requires only a 90° rotation for each edge.

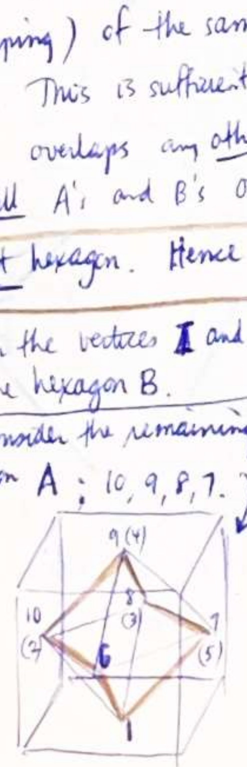
Let us now prove that the finite regular octahedron is the limiting state of $\{\tilde{6},\tilde{4}\}$. Refer to the illustration on p. 91. We need prove only that 2 distinct polygons of type A which are adjacent to ~~the~~ a given type B both become Hamilton circuits ~~&~~ (overlapping) of the same regular octahedron of which the type B hexagon becomes a Hamilton circuit. This is sufficient for the proof, because, if a given type A overlaps a given type B, ~~then~~ then it also overlaps any other of the 6 type B's to which it is adjacent. From this it follows that all A's and B's overlap the same octahedron. (Thus, every hexagon "≡" every adjacent hexagon. Hence all hexagons are "equivalent", i.e., are Hamilton circuits of the same octahedron, in the limit $\eta \to \infty$.)

Thus, every plane hexagon (on p. 95) undergoes rotation through an angle just greater than 180° $\left(\cos^{-1}\left(\frac{-1}{\sqrt{12}}\right)\right)$ Each 60° skew hexagon which is adjacent to it becomes superimposed without net change in orientation. This provides a convenient way of describing the transformation.

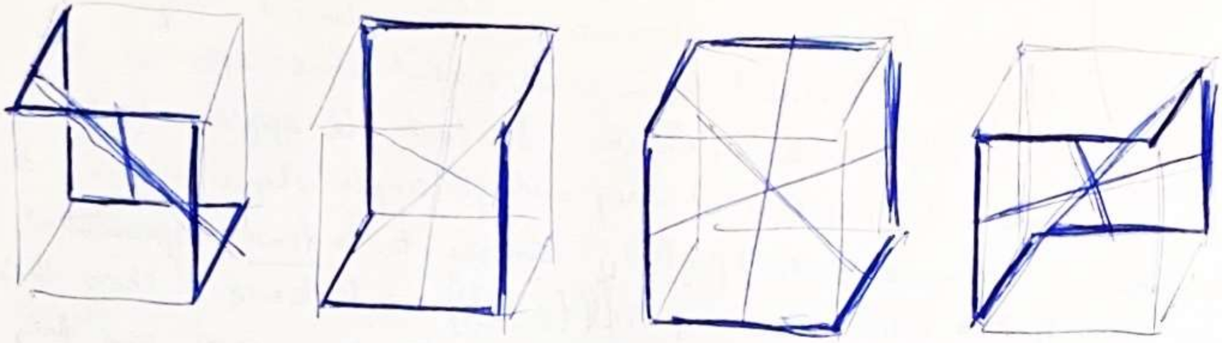
Proof: Consider the vertices I and II When $\eta \to \infty$, I and the hexagon B.



Now consider the remaining vertices of hexagon A: 10, 9, 8, 7.



This completes the proof.



Each polygonal face shares two ~~ap~~ [parallel] edges with every other

This strange ~~p~~ "pseudo-compound ~~p~~ regular ~~p~~ saddle polyhedron" can be assembled from 24 quadrilaterals

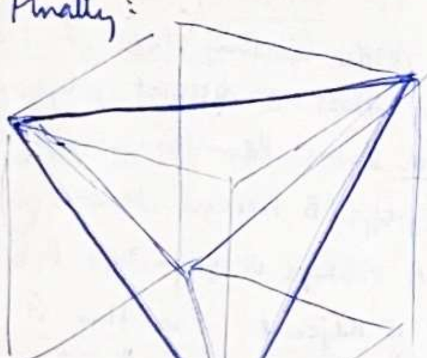
A

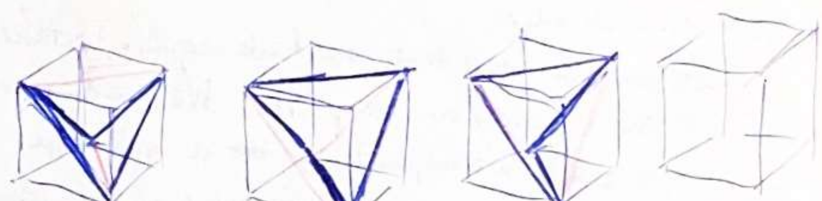
[pseudo-compound regular (90°) {hexagonal} saddle tetrahedron]

Similarly, the pseudo-compound regular (60°) {hexagonal} saddle-tetrahedron, shown on the previous page, can be assembled from 24 quadrilaterals:

B

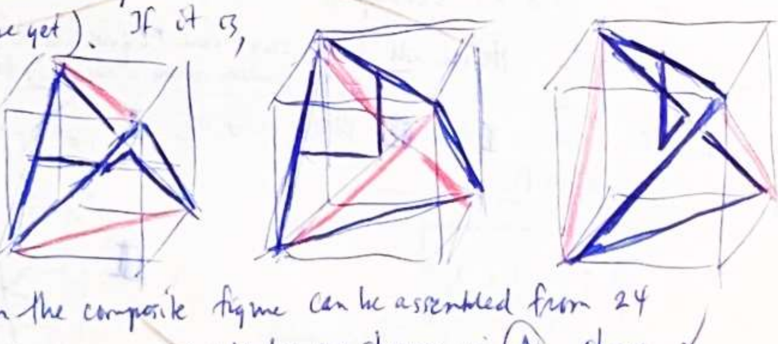
Finally:





the tetrahedron can be assembled from 3 full 60° quadrilaterals

But I suspect the actual composite figure must be the stella octangula (I haven't had time to investigate this case yet). If it is,



then the composite figure can be assembled from 24 of the same quadrilaterals as shown in A above, ✓ but with the "tips" on the outside of the figure, instead of at the center.

(After all, there are six distinct orientations of the quadrilateral faces in the original $\{\tilde{4}, \tilde{6}\}_L$

No!

I checked it. It's just the tetrahedron, with three quadrilateral faces.

It is interesting that $\lim_{\eta\to\infty}\left[\sigma\{\tilde{6},\tilde{4}\}\right] = \sqrt{2}$ ($\varphi=60°$) Regular octahedron

$\lim_{\eta\to\infty}\left[\sigma\{\tilde{4},\tilde{6}\}\right] = 1$ ($\varphi=60°$) " tetrahedron

$\lim_{\eta\to\infty}\left[\sigma\{\tilde{6},\tilde{6}\}\right] = \frac{1}{\sqrt{2}}$ ($\varphi=90°$) cube

---

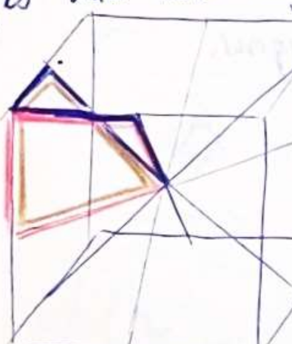
Note that in the case of the regular skew polyhedra, the "complementary polyhedra" correspondences are not the same

$\{6,\tilde{4}\} \Rightarrow$ reg. octahedron
$\{4,\tilde{6}\} \Rightarrow$ cube
$\{6,\tilde{6}\} \Rightarrow$ reg. tetrahedron

same dihedral ∡'s

Thus, all 3 regular skew saddle polyhedra "collapse" into finite regular polyhedra. At least, we may take that point of view if we disregard what happens to the faces of the reg. skew sad. poly. But a ~~f~~ more fundamental point of view is to preserve the identity of the faces of the $\{\tilde{p},\tilde{q}\}_L$, and describe the collapsed figures as quasi-compound polyhedra in which each (skew) face shares two edges with each adjoining faces, just as with the ~"helicoid" "polyhedron".

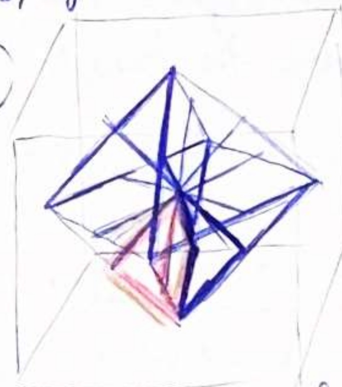
I



Use four colors

(Each vertex has three quadrilaterals meeting there.)

II



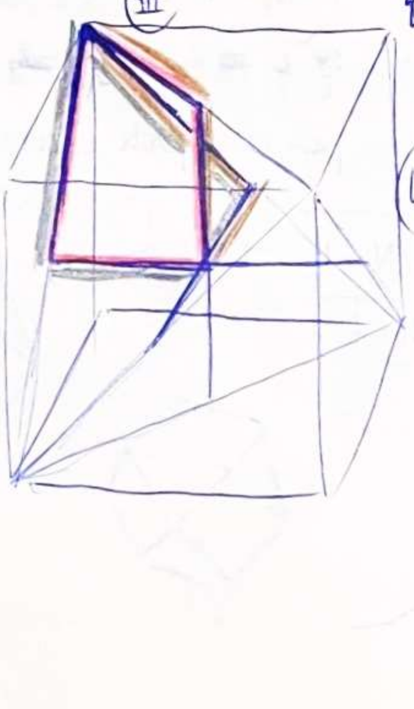
(Use four colors)

$6-12+4=-2=2-2p$
$2p=4$
$p=2$

(Each vertex has four quadrilaterals meeting there)

III

(Each vertex has three quadrilaterals meeting there.)



(Use three colors)

(110) The *finite* quasi-regular saddle polyhedron $(4[\sqrt{8}]\cdot 6[\sqrt{2}])$ is the *only* example of such a finite figure. Now there are three finite quasi-regular figures (the two others being the cuboctahedron and the

The 3 figures on the preceding page are a novel kind of compound.

Some properties of $(4[\sqrt{8}]\cdot 6[\sqrt{2}])$:

$N_0 - N_1 + N_2 = 2-2p$

$12 - 24 + 10 = 2 - 2p = -2$ $\quad \therefore \boxed{p = 2}$ genus

What is *density* here?

Core of edges is cuboctahedron
Case is cuboctahedron (larger)

Actually, the core of the polyhedron, including its *faces*, is merely the central point, i.e., no convex polyhedron can be wholly contained "inside".

Coxeter [RP], p. 96: "Just as the definition of a polygon can be generalized by allowing non-adjacent sides to intersect, so the definition of a polyhedron can be generalized by allowing non-adjacent faces to intersect; and it is natural at the same time to allow the faces to be star polygons."

Equatorial polygon $\{h\}$ of $(4[\sqrt{8}]\cdot 6[\sqrt{2}])$ is regular compound polygon ✡

(Cf. Coxeter p. 19) $\boxed{\cos^2\frac{\pi}{h} = \cos^2\frac{\pi}{p}\cos^2\theta_p + \cos^2\frac{\pi}{q}\cos^2\theta_q}$

Every edge of $(p\cdot q)$ belongs to just one equatorial $\{h\}$. $\therefore \exists\ \frac{2N_1}{h}$ such $\{h\}$'s.

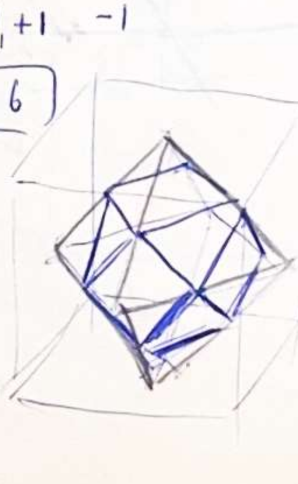
Since $h=3$ (in this context), $p=4$, $q=6$, and $\theta_p = 54°45'$, $\theta_q = 70.5°$, then equation is verified ✓

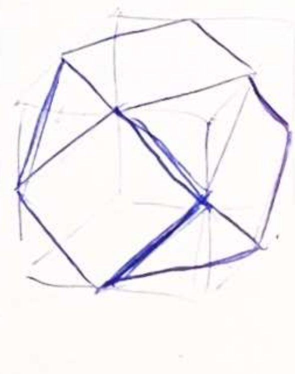
Coxeter: $(2N_1 = \text{no. of vertices of } (p\cdot q))$

Coxeter, p. 19: Each of the $\frac{2N_1}{h}$ (here $\frac{2N_1}{h}$ would be $\frac{24}{6} = 4$; there are 4 ✡'s.) equatorial h-gons meets each of the others at a pair of opposite vertices.

Hence $\left(\frac{2N_1}{h} - 1\right) = \frac{h}{2}$ or $h = \sqrt{4N_1 + 1} - 1$

Here $\boxed{h = 6}$





Note: This treatment is awkward, because $\{h\}$ is compound.

" Skew truncation (a "skew faceting") of the cube and regular octahedron.

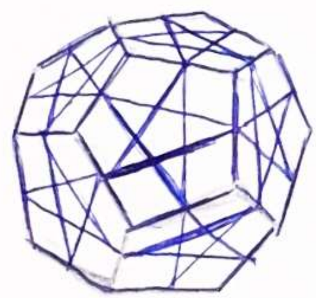
The octahedron $\begin{Bmatrix}3\\3\end{Bmatrix}$, cuboctahedron $\begin{Bmatrix}3\\4\end{Bmatrix}$, and icosidodecahedron $\begin{Bmatrix}3\\5\end{Bmatrix}$ are the 3 quasiregular polyhedra which are derivable from the Platonic figures by truncation, i.e. by relating each Platonic figure to its reciprocal (with respect to their common midsphere) and deriving the solid region interior to both polyhedra (Coxeter, p. 17). The faces of $\begin{Bmatrix}p\\q\end{Bmatrix}$ are the vertex figures of $\{p,q\}$ and $\{q,p\}$ respectively.

Another way this could have been described is to say that the common midpoints M of edges of $\{p,q\}$ & $\{q,p\}$ are simply joined to adjacent M's and the resulting regular polygons regarded as faces of $\begin{Bmatrix}p\\q\end{Bmatrix}$.

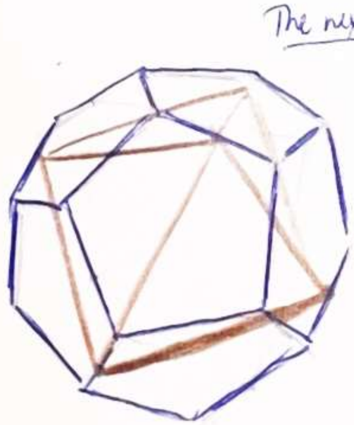
Let us define as skew truncation of reciprocal $\{p,q\}$ & $\{q,p\}$ the joining of all non-contiguous edge midpoints which are equivalently related. For the cube & octahedron, only the (112) joins define a quasi-regular figure $\left(\begin{Bmatrix}\tilde{4}\\\tilde{6}\end{Bmatrix}\right)$.

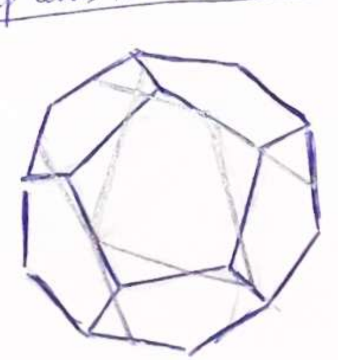
For the tetrahedron, nothing results.

Consider the dodecahedron: $\begin{Bmatrix}\frac{5}{2}\\\tilde{6}\end{Bmatrix}$ is a quasi-regular polyhedron!



The next step leads to the octahedron!





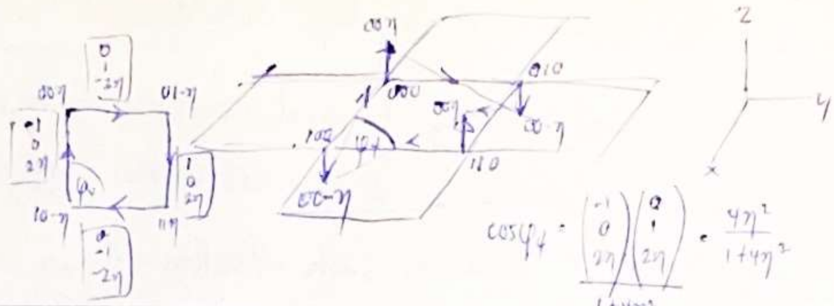
(112)

$\boxed{\Lambda\{4,4\}}$

$\boxed{\sigma = 2\eta}$

(obviously!)

(Using definition $\sigma = \tan\theta$)

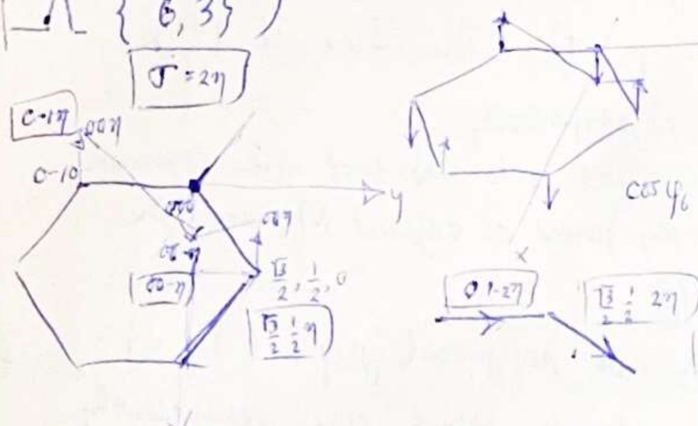


$$\cos\varphi_4 = \frac{\begin{pmatrix}-1\\0\\2\eta\end{pmatrix}\begin{pmatrix}0\\1\\2\eta\end{pmatrix}}{1+4\eta^2} = \frac{4\eta^2}{1+4\eta^2}$$

$$\sigma_4 = \sqrt{\frac{1-\frac{4\eta^2}{1+4\eta^2}}{1-\frac{4\eta^2}{1+4\eta^2}}} = \sqrt{\frac{4\eta^2}{1}} = 2\eta$$

$\Lambda\{6,3\}$

$\boxed{\sigma = 2\eta}$



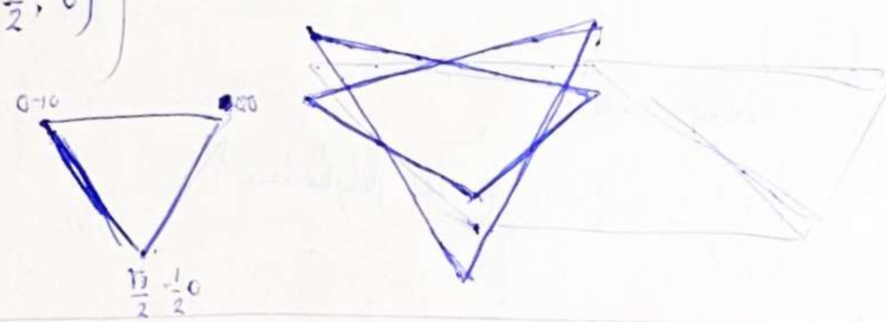
$$\cos\varphi_6 = \frac{\begin{pmatrix}0\\-1\\+2\eta\end{pmatrix}\begin{pmatrix}\frac{\sqrt3}{2}\\ \frac12\\ 2\eta\end{pmatrix}}{1+4\eta^2} = \frac{-\frac12+4\eta^2}{1+4\eta^2}$$

$$\sigma_6 = \sqrt{\frac{\frac{-\frac12+4\eta^2}{1+4\eta^2}+\frac12}{1-\frac{-\frac12+4\eta^2}{1+4\eta^2}}}$$

$$= \sqrt{\frac{-1+8\eta^2+1+4\eta^2}{2+8\eta^2+1-8\eta^2}} = \sqrt{\frac{12\eta^2}{3}} = 2\eta$$

$\Lambda\left\{\frac{6}{2}, 6\right\}$

$\boxed{\sigma = 2\eta}$



$$\frac{l_0}{l} = \frac{1}{\sqrt{1+4\eta^2}} = \cos\alpha \qquad \boxed{\tan\alpha = 2\eta} = \sigma$$

$(\alpha = \theta)$

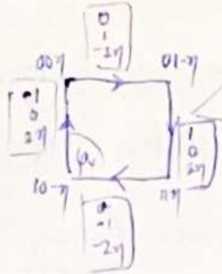
$1+4\eta^2 = \sec^2\alpha - 1 = \tan^2\alpha$

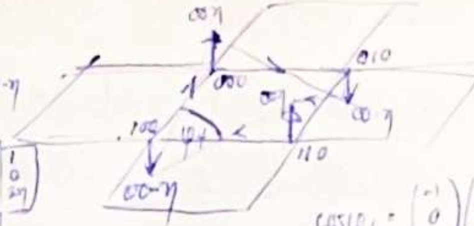
$\boxed{\Lambda\ \{4,4\}}$

$\boxed{\sigma = 2\eta}$

(obviously!)

(Known definition $\sigma = \tan\theta$)



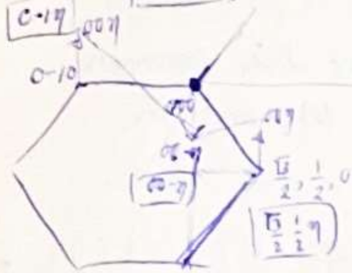


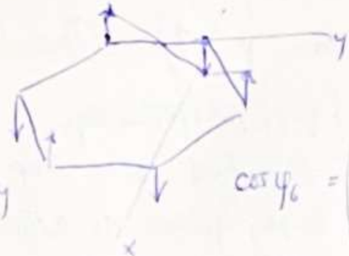
$$\cos\varphi_4 = \frac{\begin{pmatrix}-1\\0\\2\eta\end{pmatrix}\begin{pmatrix}0\\1\\2\eta\end{pmatrix}}{1+4\eta^2} = \frac{4\eta^2}{1+4\eta^2}$$

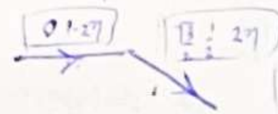
$$\sigma_4 = \sqrt{\frac{\cancel{1}\ \frac{4\eta^2}{1+4\eta^2}}{1-\frac{4\eta^2}{1+4\eta^2}}} = \sqrt{\frac{4\eta^2}{1}} = 2\eta$$

$\boxed{\Lambda\ \{6,3\}}$

$\boxed{\sigma = 2\eta}$





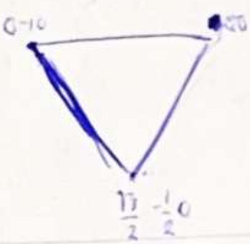


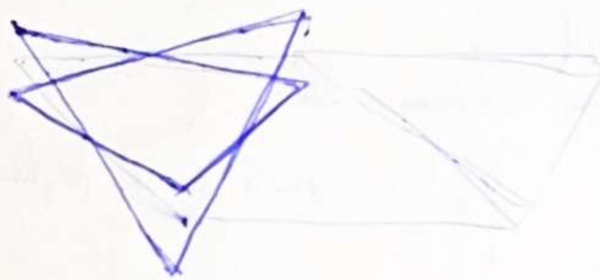
$$\cos\varphi_6 = \frac{\begin{pmatrix}0\\-1\\+2\eta\end{pmatrix}\begin{pmatrix}\frac{\sqrt3}{2}\\ \frac12\\ 2\eta\end{pmatrix}}{1+4\eta^2} = \frac{-\frac12+4\eta^2}{1+4\eta^2}$$

$$\sigma_6 = \sqrt{\frac{\frac{-\frac12+4\eta^2}{1+4\eta^2}+\frac12}{1-\frac{-\frac12+4\eta^2}{1+4\eta^2}}} = \sqrt{\frac{-1+8\eta^2+1+4\eta^2}{2+8\eta^2+1-8\eta^2}} = \sqrt{\frac{12\eta^2}{3}} = 2\eta$$

$\boxed{\Lambda\ \{\frac{6}{2},6\}}$

$\boxed{\sigma = 2\eta}$





$$\frac{l_0}{l} = \frac{1}{\sqrt{1+4\eta^2}} = \cos\alpha$$

$\boxed{\tan\alpha = 2\eta} = \sigma$

$(\alpha = \theta)$

$1+4\eta^2 = \sec^2\alpha - 1 = \tan^2\alpha$

1) $\dfrac{1-2\eta}{1+2\eta}$

1a) $\dfrac{\sqrt{2}\,\eta}{\sqrt{1+\eta^2}}$, $\dfrac{2\sqrt{2}\,\eta}{\sqrt{3+\eta^2}}$

2) $\dfrac{\sqrt{2}}{2}\,\dfrac{(2\eta-1)}{\eta+1}$

2b) η " "

3)

3b) $\sqrt{2}\left(\dfrac{2+\eta}{1-\eta}\right)$, $\sqrt{2}\left(\dfrac{\eta}{3+\eta}\right)$

---

4) $\dfrac{2\eta}{\sqrt{1+4\eta^2}}$

4b) $\dfrac{\sqrt{2}\,\eta}{1+\eta}$, $\sqrt{2}\,\dfrac{(1+2\eta)}{(1-\eta)}$

5) $\dfrac{\sqrt{2}\,\eta}{\sqrt{3+\eta^2}}$

5b) $\sqrt{2}\,\dfrac{(1+\eta)}{(1-\eta)}$, $\dfrac{2\sqrt{2}\,\eta}{3+\eta}$

6) $\sqrt{2}\,\dfrac{(1+\eta)}{(1-2\eta)}$

$\left(\sqrt{2}\,\dfrac{1-\eta}{1+2\eta}\right)$

6b) $\dfrac{\sqrt{2}\,\eta}{\sqrt{3+\eta^2}}$

---

7) $\dfrac{1-2\eta}{\sqrt{2+4\eta+4\eta^2}}$

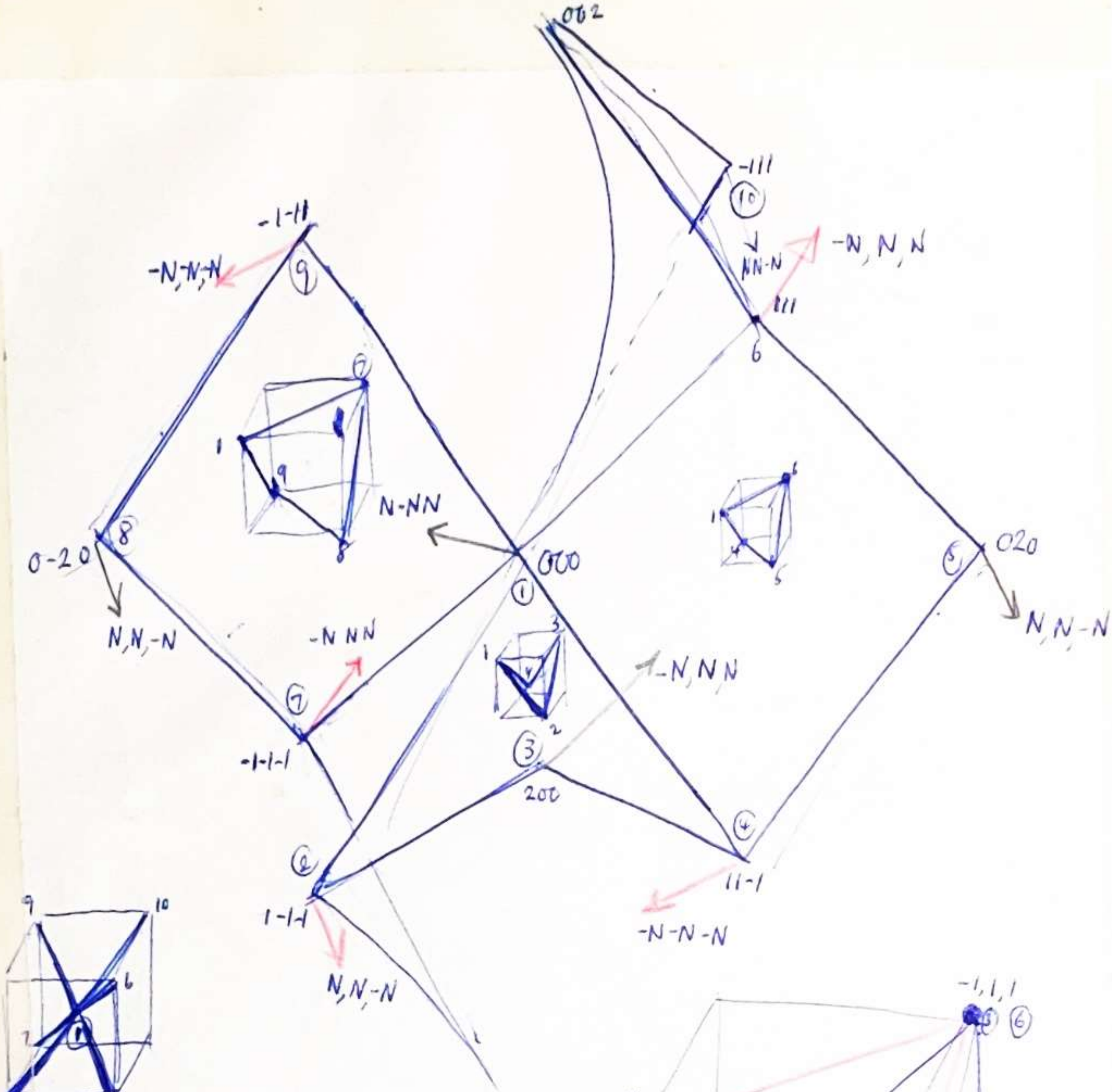
7b) $\dfrac{\sqrt{2}\,\eta}{\sqrt{\eta^2-2\eta+2}}$, $\sqrt{2}\,\dfrac{(2\eta-1)}{(2+\eta)}$

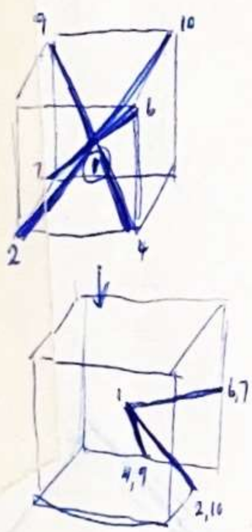
8) $\dfrac{\sqrt{2}\,(1-\eta)}{\sqrt{7+4\eta+\eta^2}}$

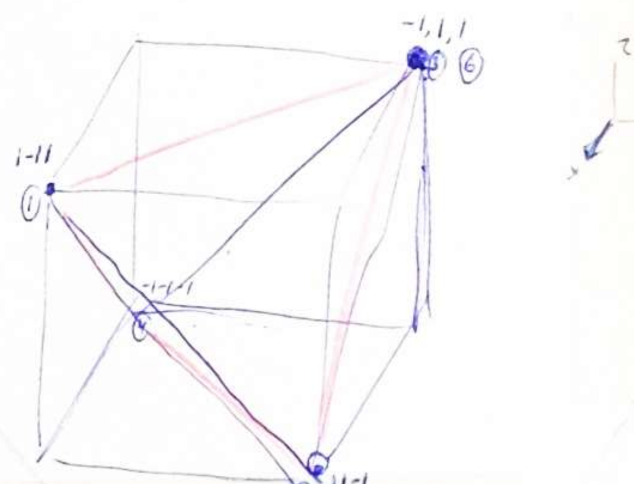
8b) $\dfrac{\sqrt{2}\,(1-\eta)}{\sqrt{5+2\eta+\eta^2}}$, $\dfrac{2\sqrt{2}\,\eta}{\sqrt{21-6\eta+x^2}}$

9) $\dfrac{1-2\eta}{\sqrt{2(1+2\eta+4\eta^2)}}$

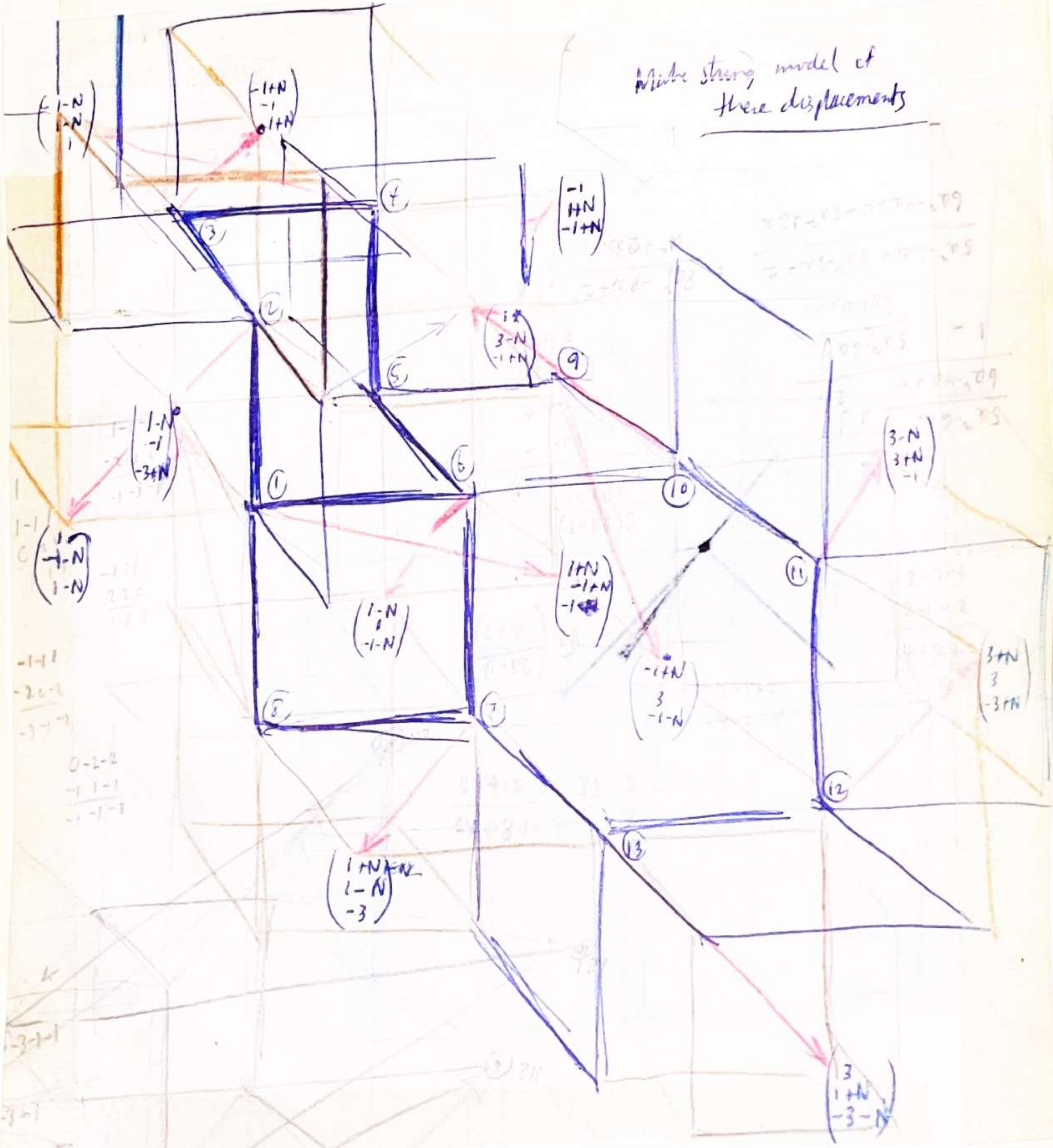
9b) $\dfrac{\sqrt{2}\,(1-\eta)}{\sqrt{1+\eta+\eta^2}}$, $\dfrac{\sqrt{2}\,\eta}{\sqrt{3-3\eta+\eta^2}}$


002
-111
10
-N, N, N
NN-N
111
6
-1-11
-N,-N,-N
9
N-NN
0-20
8
000
1
020
5
N, N, -N
N, N, -N
-N NN
-N, NN
7
-1-1-1
3
200
4
11-1
2
1-1-1
-N -N -N
N, N, -N


9
10
6
2
4
6,7
4,9
2,10


-1, 1, 1
6
1-11
1
-1-1-1
11-1
2
5

Make string model of these displacements



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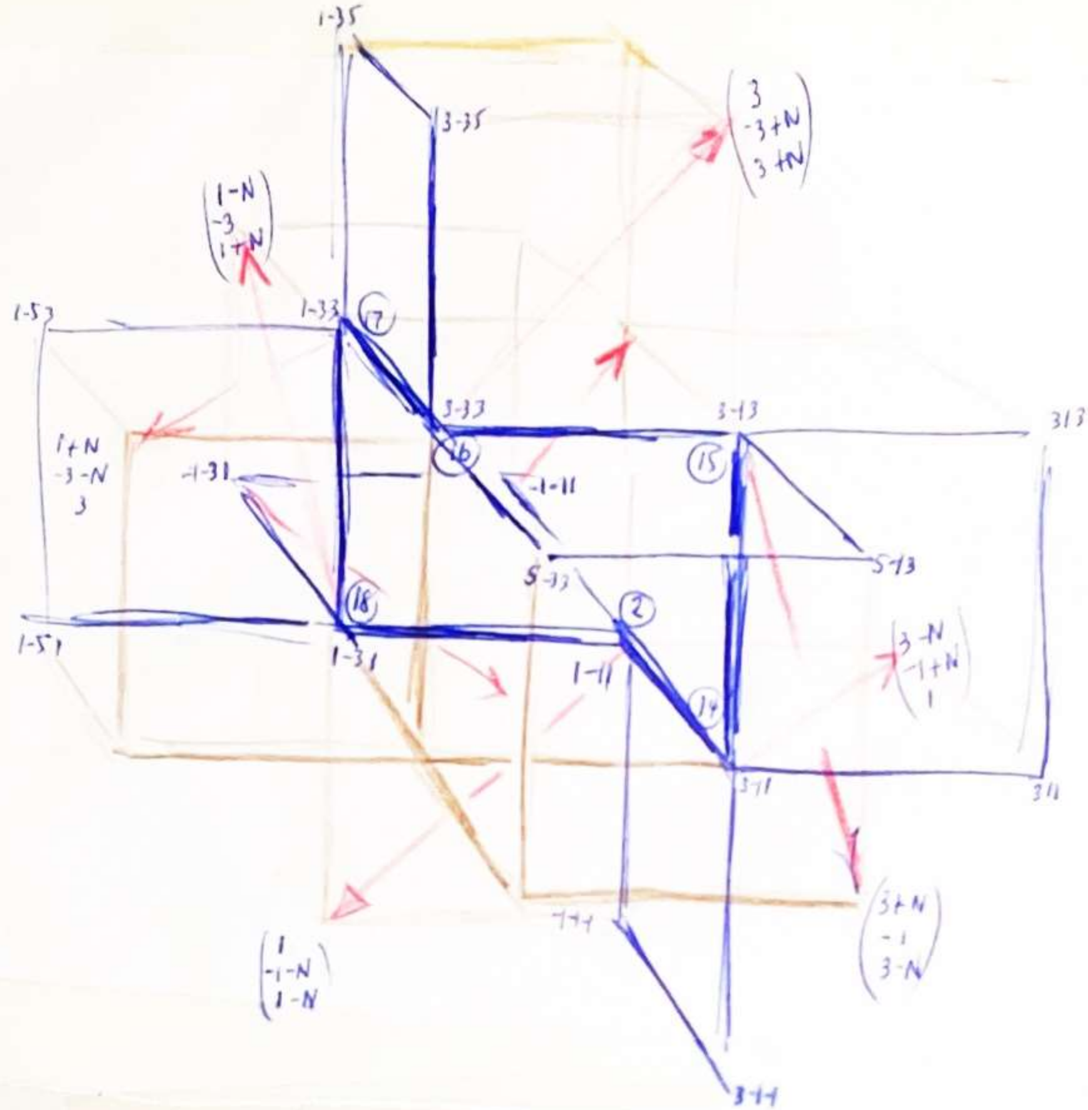
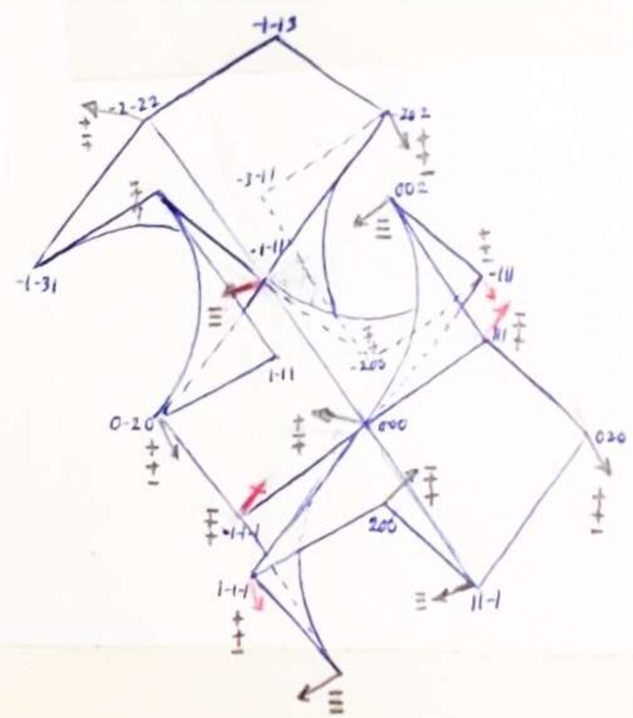


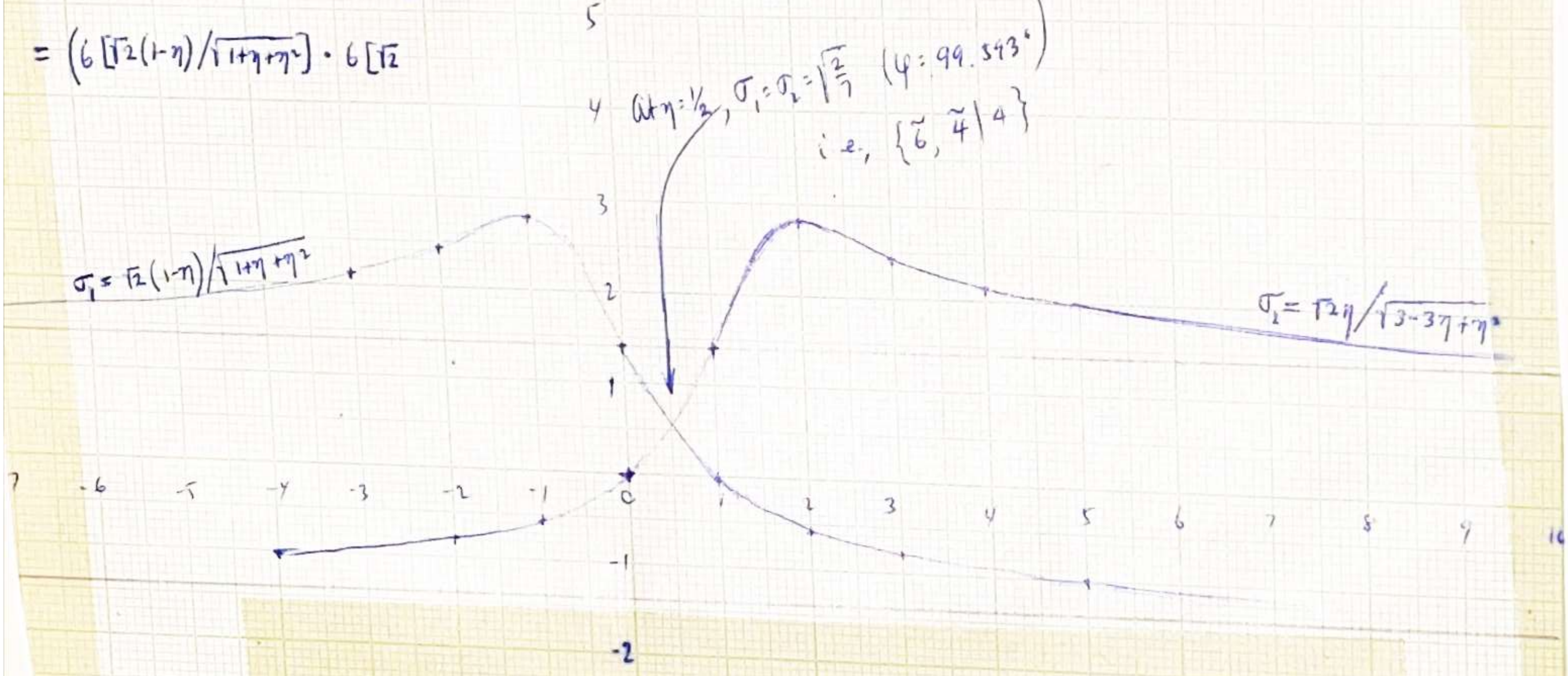
1-35
3-35
3 -3+N 3+N
1-N -3 1+N
1-53
1-33
17
3-33
3-13
313
1+N -3-N 3
-1-31
16
-1-11
15
5-33
5-13
18
2
1-51
1-31
1-11
3-N -1+N 1
14
3-11
311
1 -1-N 1-N
3+N -1 3-N
3-11



-1-13
-2-22
202
-3-11
002
-1-31
-1-11
-111
-200
1-11
0-20
000
020
200
1-1-1
11-1

$$\Lambda\left(6[0]\cdot 6[(2)^{1/2}]\right)_L^2$$

$$=\left(6\left[\sqrt{2}(1-\eta)/\sqrt{1+\eta+\eta^2}\right]\cdot 6[\sqrt{2}\right.$$



Name	Schläfli symbol	N_0	N_1	N_2	h	d (genus)	Schläfli symbol	N_2	χ $=2-2p$	$\left\{\begin{matrix}p\\q\end{matrix}\right\}$
regular tetrahedron	$\{3,3\}$	4	6	4	4	1 (0)	$\{\tilde{4},3\}_3$	3	1	$4\{3\}+3\{4\}$
octahedron	$\{3,4\}$	6	12	8	6	1 (0)	$\{\tilde{6},4\}_3$	4	-2	$6\{4\}+4\{6\}$
cube	$\{4,3\}$	8		6			$\{\tilde{6},3\}_4$		0	$8\{3\}+4\{6\}$
icosahedron	$\{3,5\}$	12	30	20	10	1 (0)	$\{\tilde{10},5\}_3$	6	-12	$12\{5\}+6\{10\}$
dodecahedron	$\{5,3\}$	20		12			$\{\tilde{10},3\}_5$		-4	$20\{3\}+6\{10\}$
small stellated dodecahedron	$\{\frac{5}{2},5\}$	12	30	12	6	3 (4)	$\{\tilde{6},5\}_{5/2}$	10	-8 (not in Coxeter & Moser)	$12\{5\}+10\{6\}$
great dodecahedron	$\{5,\frac{5}{2}\}$						$\{\tilde{6},5/2\}_5$		-8	$12\{\frac{5}{2}\}+10\{6\}$
great stellated dodecahedron	$\{\frac{5}{2},3\}$	20	30	12	$\frac{10}{3}$	7 (0)	$\{\tilde{\frac{10}{3}},3\}_{5/2}$	6	-4	$20\{3\}+6\{\frac{10}{3}\}$
great icosahedron	$\{3,\frac{5}{2}\}$	12		20			$\{\tilde{\frac{10}{3}},\frac{5}{2}\}_3$		-12	$12\{\frac{5}{2}\}+6\{\frac{10}{3}\}$

$1 + \beta(01-1)$

$(\ldots N, -N)$ $(2, N, -N)$

$2x + Ny - Nz = 2\varphi$

$2x$

$\cos\alpha = \frac{(211)\cdot(01-1)}{\sqrt{6}\cdot\sqrt{2}}$

$= \frac{1-1}{\sqrt{12}} = 0$

$\therefore \alpha = 90^\circ$

$x = \cos\theta\cos\varphi$

$y = \cos\theta \sin\varphi$

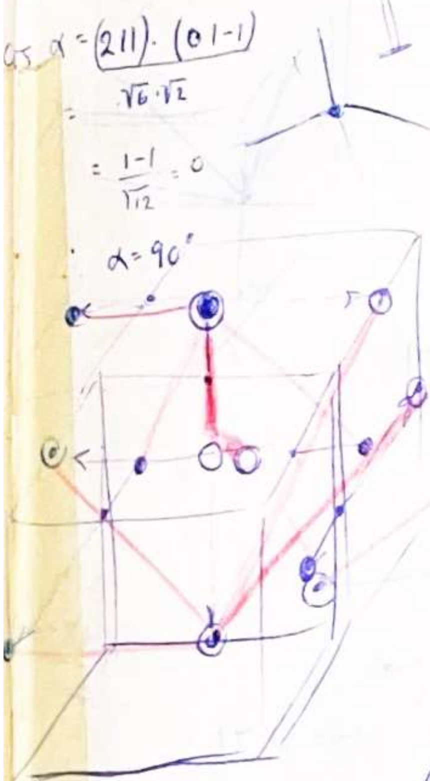
$z = \sin\theta$

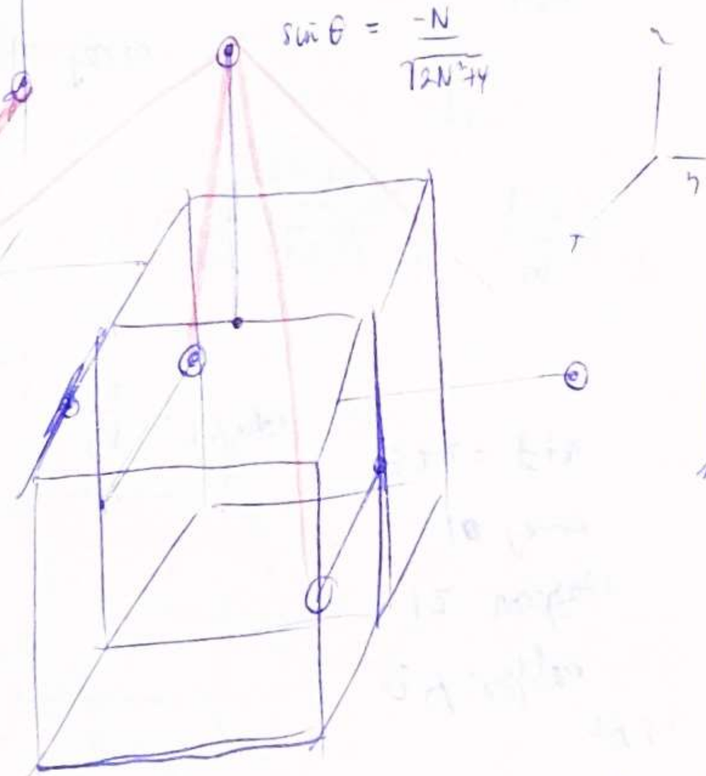
$\vec{n}_3 = \left(\frac{2, N, -N}{\sqrt{2N^2+4}}\right)$

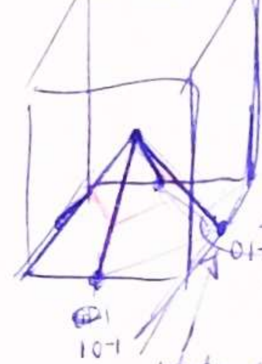
$\cos\theta\cos\varphi = \frac{2}{\sqrt{2N^2+4}}$ $\tan\varphi = \frac{N}{2} \to \infty \quad \therefore \varphi = \frac{\pi}{2}$

$\cos\theta\sin\varphi = \frac{N}{\sqrt{2N^2+4}}$ $\sin\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\theta = -45^\circ$

$\sin\theta = \frac{-N}{\sqrt{2N^2+4}}$







plane regular vertex figure: Square !

!?!

what must take (211)

$10-1$
$01-1$
$\frac{1}{2}$

60°

What is limiting form?

$(211) + \beta(01-1) = [2, 1+\beta, 1-\beta]$ $\lim \to (2, N, -N) \to (0 1, -1)$

$(-12-1) + \beta(10-1) = [-1+\beta, 2, -(1+\beta)]$ $\to (N, 2, -N)$ $(1 0 -1)$

$(-2-11) + \beta(0-1-1)$ $[-2, -(1+\beta), (1-\beta)]$ $\to [-2, -N, -N]$ $(0 -1 -1)$

$(1-2-1) + \beta(-10-1)$ $[1-\beta, -2, -(1+\beta)]$ $[-N, -2, -N]$ $(-1 0 -1)$

This looks extremely interesting!

$\alpha_4 = \frac{-2\cos\varphi}{1-\cos\varphi} = -\frac{2(\frac{1}{2})}{1-\frac{1}{2}} = -\frac{2}{\frac{1}{2}} = -2.$

(118)

Nam

regular tetrahed

octahedro

cube

icosahedro

dodecahe

small stellated dodecahe

great dodecahe

great stellated dodecahe

great icosahed

(120)

When the translations $\vec{\eta}$ are normalized to the length of the edge of the collapsing graph, i.e., when the length of the unit vector along the local $\vec{\eta}$ axis is equal to the edge length, then the relation between η and α depends only on $\{\tilde{p}, \tilde{q}\}$ (or $(\tilde{p}\cdot\tilde{q})$), and not on the labyrinth!

$\{p,q\}$ or $(\tilde{p}\cdot\tilde{q})$	$\cos\alpha = \frac{l_0}{l}$	$\eta / \tan\alpha = C$
$(\tilde{6}\cdot\tilde{4})$	$(1+ \frac{9}{3}\eta^2)^{-1/2}$	$\sqrt{3}/3 = .577$
$\{\tilde{4}, \tilde{6}\}$	$(1+ \frac{8}{3}\eta^2)^{-1/2}$	$\sqrt{6}/4 = .612$
$\{\tilde{6}, \tilde{4}\}$	$(1+ \frac{6}{3}\eta^2)^{-1/2}$	$\frac{\sqrt{2}}{2} = .707$
$\{\tilde{6}, \tilde{6}\}$	$(1+ \frac{4}{3}\eta^2)^{-1/2}$	$\frac{\sqrt{3}}{2} = .866$

$\{\tilde{6},\tilde{6}\}_L = \frac{\sqrt{3}}{2}\tan\alpha = \frac{1}{2}\left[\frac{\sqrt{3}}{1}\right] \therefore \alpha = 45°$

laves $\frac{\sqrt{6}}{4}\tan\alpha = \frac{1}{2}\left[\frac{\sqrt{3}}{\sqrt{2}}\right] = \frac{\sqrt{6}}{4} \therefore \alpha = 45°$

$\{4,6\}$ $\frac{\sqrt{6}}{4}\tan\alpha = \frac{1}{2}[$

1
2 $\frac{1}{2}$
3
4

5
6 $\frac{\sqrt{3}}{3}$ .577 $(6\cdot4)$
7

8
9 $\frac{\sqrt{6}}{4} = .612$ $\{4,6\}$
10
11 laves

12
13 $\frac{\sqrt{2}}{2}$ $\{6,4\}$
14

15
16 $\frac{\sqrt{3}}{2}$ .866 $\{6,6\}$
17

$\{p,q\}$	①	of graph ②	$\tan\alpha$ ③	④	$\lvert\lambda\rvert^2$ ⑤	⑥ $\left(\frac{\eta}{\tan\alpha}\right)\left(\frac{\lambda}{\eta}\right)\left(\frac{l}{l_0}\right)$	In col. ③ $(\eta/\tan\alpha)$ = ratio obtained if the components of λ are — e.g., in $\{\bar4,6\}_D$ — $\eta\eta\eta$.
$\{\bar4,6\}_D$	$(111)(\eta)$ $\lambda=\sqrt3\eta$	(110)	$\frac12$	$4\eta^2$ $\frac{l_0}{l}=\frac{1}{\sqrt{1+4\eta^2}}$	$\frac83\eta^2$	$\frac{\sqrt6}{4}$ ✓ .8165	
$\{\bar6,4\}_D$	$(100)(\eta)$ $\lambda=\eta$	~~(100)~~ (100)	$\sqrt2$, $\frac{l_0}{l}=\frac{1}{\sqrt{1+\frac12\eta^2}}$	$\frac12\eta^2$	$2\eta^2$	$\frac{\sqrt3}{3}$ .577	
$\{6,\bar6\}_D$	—	(110)	—		—	—	
$(\bar6\cdot\bar4)_D$	$\lambda=\sqrt2\eta$ $(110)(\eta)$	(110)	$\frac{\sqrt3}{3}$	$\frac{l}{l_0}=\frac{1}{\sqrt{1+3\eta^2}}$ $3\eta^2$	$3\eta^2$	$\frac{\sqrt3}{3}$	
$\{4,\bar6\}_P$	$(111)(\eta)$ $\lambda=\sqrt3\eta$	(100)	$\frac{\sqrt2}{4}$ ✓	$8\eta^2$ $\frac{l_0}{l}=\frac{1}{\sqrt{1+8\eta^2}}$	$\frac83\eta^2$	$\frac{\sqrt6}{4}$ ✓	
$\{6,\bar4\}_P$	$(100)(\eta)$	(110)	1 ✓	η^2 ✓ $\frac{l_0}{l}=\frac{1}{\sqrt{1+\eta^2}}$	$2\eta^2$	$\frac{\sqrt3}{3}$	
Laves graph	$\lambda=\eta$					$\sqrt6/4$	$\lambda=\Lambda\tan\alpha$ $\lambda'=\frac12\frac{\frac{\sqrt3}{\frac{\sqrt2}{2}}}{}=\frac{\sqrt6}{2}=\frac{\sqrt6}{4}\tan\alpha'$ $\therefore\alpha'=\tan^{-1}2$ $\boxed{\alpha'=63°26'}$
$\{\bar6,6\}_P$	$\lambda=\sqrt3\eta$ $(111)(\eta)$	(110)	$\frac{\sqrt2}{2}$	$2\eta^2$ $\frac{l_0}{l}=\frac{1}{\sqrt{1+2\eta^2}}$	$\frac43\eta^2$	$\frac{\sqrt3}{2}$ ✓	
$(\bar6\cdot\bar4)_P$	$\lambda=\sqrt2\eta$ $(110)(\eta)$	(112)	$\frac{\sqrt2}{2}$	$2\eta^2$	$3\eta^2$	$\frac{\sqrt3}{3}$	
$\{\bar4,\bar6\}_L$ .81650 3)2.44949	$\lambda=\sqrt3\eta$ $(111)(\eta)$	(111)	$\frac{\sqrt6}{4}$ ✓	$\frac83\eta^2$ $\frac{l_0}{l}=\frac{1}{\sqrt{1+\frac83\eta^2}}$	$\frac83\eta^2$	$\frac{\sqrt6}{4}$ ✓ $\alpha=39.13.9'$	$\lambda=\Lambda\tan\alpha$ $\lambda'=\frac{\sqrt6}{4}\tan\alpha'$ $=\frac{\lvert(111)\rvert}{2}\cdot\frac{1}{\sqrt3}=\frac12\therefore\tan\alpha'=\frac{\sqrt6}{3}=.81651$ $\boxed{\alpha'=39.°13.9'}$
$\{\bar6,\bar4\}_L$	$\lambda=\eta$ $(100)(\eta)$	(112)	$\sqrt3$	$\frac13\eta^2$ $\frac{l_0}{l}=\frac{1}{\sqrt{1+\frac13\eta^2}}$	$2\eta^2$	$\frac{\sqrt3}{3}$	
$\{\bar6,\bar6\}_L$	$\lambda=\sqrt3\eta$ $(111)(\eta)$	(100)	$\frac12$	$4\eta^2$ $\frac{l_0}{l}=\frac{1}{\sqrt{1+4\eta^2}}$	$\frac43\eta^2$	$\frac{\sqrt3}{2}$	$\Lambda=\sqrt2/2$ $\alpha'=45°$ $\lambda=\Lambda\tan\alpha$ $\lambda'=\frac{\sqrt3}{2}\tan\alpha'$ $=\frac{\lvert(111)\rvert}{2}=\frac{\sqrt3}{2}\therefore\boxed{\alpha'=45°}$
$(\bar6\cdot\bar4)$	$\lambda=\sqrt2\eta$		$\sqrt{}$		$3\eta^2$	$\sqrt3$	

Book Inventory June 9, 1967 PVP, just before move to Mass! AMS & CFS

#	name	Face angles	$\sigma = \tan\theta$ (if {p̃})	θ (if {p̃})	ς (if {p̃})	$\sin\epsilon$ (if {p̃})	$\cos\epsilon$ (if {p̃})	l	e $= l\cos\epsilon$	h $= l\sin\epsilon$		Date received	Dihedral angle $= 180° - 2\varsigma$
X 1)	90° hex	90°	$\frac{\sqrt{2}}{2}$ = .7071	35.264°	54.733°	.57735	.81650	2.000"	1.633"	1.155"			
X 2)	60° hex F	60°	$\sqrt{2}$ = 1.414	54.733	70.526	.81650	.57735	2.828	1.633	2.309			
X 3)	90-90-90-60 quad	90°-90°-90°-60°	—	—			—	3.535"	2.5"	2.5"		9-22-67	
X 4)	60-90-60-90 quad	60°-90°-60°-90°						3.535	3.062	1.768			
X 5)	60° {4̃}	60°	1	45°	54.733	.7071	.7071	2.828	2"	2"			
X 6)	70.5° {4̃}	70.526°	$\frac{\sqrt{2}}{2}$ = .707	35.264°	45°			2.500"	2.041	1.443			90°
7)	109.5° {6̃}	109.474°	$\frac{\sqrt{2}}{4}$ = .3535	19.47°	35.264°	.33338	.94280	~~2.250~~ 1.155	~~2.121~~ 1.089	~~.750"~~ .385	★	redone, is good	
8)													
9)	73.39° {4̃}	Hold											
✓ 10)	99.593 {6̃}	99.593° P.C	$\sqrt{\frac{2}{7}}$ = .534	28.125°	46.912°	.47139	.88192	2.449	2.160"	1.154			
✓ 11)	open book	Delivered			P.O								
✓ 12)	48.19° {4}	90-90-90-48.19			P.O							dihedral 109.47°	3.53 … hypot = 7.07
✓ 13)	33.58° (6)	90-66-90-33.58			P.O							dihedral 70.526	$\frac{10\sqrt{2}}{4}$ … $\frac{10\sqrt{2}}{2}$ hyp
14)		round edges	P.O										
15)		LH "	P.O										
16)	60° {4̃}			45°		.7071	.7071	1.155 $\frac{2\sqrt{3}}{3}$	.8163 $=\frac{\sqrt{6}}{3}$	.8163	★		
17)	70.5° {4̃}	70.526°	$\sqrt{\frac{2}{43}}$ = .2152		P.O			1.732"	1.414"	1.000"		done	
18)	82.338° {4̃}	82.338°	$\sqrt{\frac{2}{13}}$ = .3922	21.415°	29.013°	.36512 Hold	.93096	1.826"	1.700"	.666"			
19)	115.680 {6̃}	115.680°	$\sqrt{\frac{2}{43}}$ = .2157	12.172°	23.333°	.21084	.97752	1.826"	1.785"	.385"			
20)	LH part (F)												
21)	RH part (F)												
22)	Starfish I LH ½-face	B_l											
23)	Starfish I RH ½-face	B_r											
24)	Starfish II LH ½-face	D_l											
25)	Starfish II RH ½-face	D_r											
26)	Starfish III LH face	F_l											
27)	Starfish III RH face	F_r											
28)	pseudo-circle hex												
29)	12-gon												
30)	Necessary												
31)													
32)	fcc												
33)													

Hal Robinson Tim

19 Orlando Ave

Arlington M18-4074

Rudy Langer

4 Court St. Place

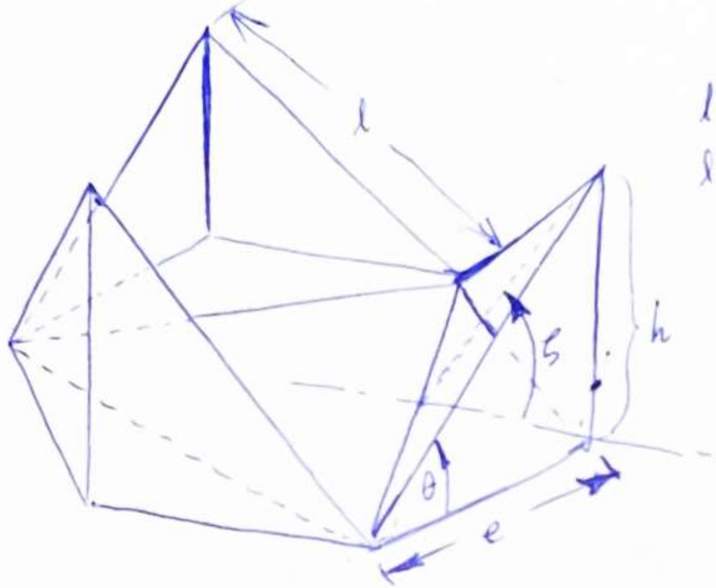
Arlington 02174

M18-2525

Michael Goldberg

Seidler – ~~Elemente~~ Elementare Mathematik

(Basel 1942)



$l \sin \theta = h$

$l \cos \theta = e$