Summary 3/06/2001

Schm 9/1980 US 4223890 Set of tiles for covering a surface

www delphini.com

3/6/2001

Summary:

The ratio of volumes of the two rhombohedra $\boxed{\rho = \tau}$:

$$\frac{V_{obtuse}}{V_{acute}}$$

"Attitude angle" φ for acute rh $= \cos^{-1}\left(\frac{1}{\sqrt{3-\tau}}\right) \cong 31.71747^\circ$

" " ξ for obtuse rh $= \sin^{-1}\left(\frac{1}{\sqrt{3-\tau}}\right)$ 58.28252558 (complementary angles)

Obtuse rh $V = 2(\tau - 1)$

Acute " " $= 2(2-\tau)$

$$\boxed{\text{ratio } \rho = \frac{V_{obt}}{V_{acut}} = \frac{2(\tau-1)}{2(2-\tau)} = \tau}$$

46	20	16	15	15	12	8	8	4
47	20	16	15	15	12	8	6	6
48	20	16	15	15	10	10	8	4
49	20	16	15	15	10	10	6	6
50	20	16	15	15	10	8	8	6
51	20	16	15	12	12	10	9	4
52	20	16	15	12	12	9	8	6
53	20	16	15	12	10	10	9	6
54	20	16	15	12	10	9	8	8
55	20	15	15	12	12	10	10	4
56	20	15	15	12	12	10	8	6
57	20	15	15	12	10	10	8	8
58	20	16	15	15	12	10	6	4
59	16	15	15	12	12	10	10	8

kyokoh@seagreen.

Check vol. of obtuse rhombohedra:

Volume of acute rhombohedron = base area × altitude

$= [2(\tau-1)] \times [\tau-1] = 2(2-\tau) = .763932$

Volume of obtuse rhombohedron = base area × altitude

$= [2(\tau-1)] \times [1] = 2(\tau-1) = 1.23606$

The rhombohedral volume ratio τ = Ratio $\frac{\cancel{2}(\tau-1)}{\cancel{2}(2-\tau)} = \tau$!

Verify $2\tau - \tau^2 = \tau - 1$

$2\tau - (\tau+1) = \tau - 1$

$\tau - 1 = \tau - 1$ ✓

Hence vol. $= 10[2(2-\tau)] + 10[2(\tau-1)]$

$= 20(2-\tau) + 20(\tau-1) = 20[2-\cancel{\tau}+\cancel{\tau}-1] = 20$!

V sphere $= \frac{4\pi}{3}\tau^3 = 17.7440000510$

area ~~triacontahedron~~ $= 2(\tau-1)\cdot 30 = 60(\tau-1) = 37.0820393250$

~~$\frac{A^3}{V^2} = 60(\tau-1)^2$~~ Normalized to unit inscribed sphere,

Area $= \frac{60(\tau-1)}{\tau^2}$

$= 14.16407865$

(~ n = 27 best)

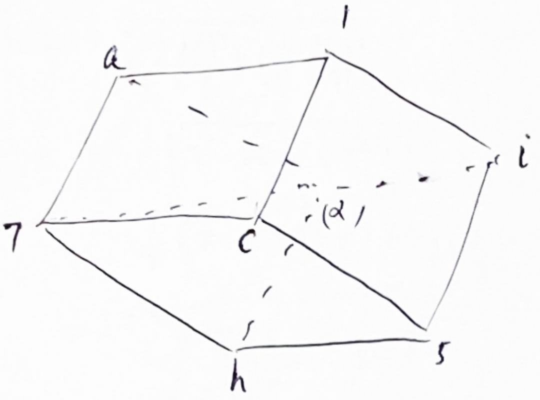
(plausible.)

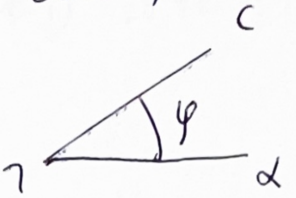
3/5/2001

Check vol. of obtuse rhombohedra:

Calc. φ (because (edge)(sin φ) = "height" of rhombohedron)

redo ACUTE rhombo vol.





$\vec{7c} = (t, c, 1) \to (1,1,1)$

$\vec{7\alpha} = (t, 0, 1) \to$

$\vec{\alpha} = \vec{a} + \vec{7h} = (\tau-1, 0, \tau) + (t, 0, 1) \to (\tau, \tau-1, 0)$

$= \quad " \quad + (0, \tau-1, -1) = \boxed{(\tau-1, \tau-1, \tau-1)}\ \vec{\alpha}$

NB length of "axis" $|\vec{c\alpha}|$ is $\sqrt{\ }|(\tau-1, \tau-1, \tau-1) - (1,1,1)| = \cancel{1.1151151272}$ $\quad 1-(\tau-1) = 2-\tau$

$= \sqrt{3}(\tau-1) \cong 1.0704662093$ $= \left|(\tau-1)[1,1,1] - [1,1,1]\right| = \left|(2-\tau)(1,1,1)\right| = \sqrt{2-\tau}\sqrt{3}$

$= \sqrt{3}(\tau-1)$

WRONG

Check vol. of obtuse rhombohedra :

$\text{vol.}_{\text{triacontahedron}} = 20\left[2(\tau-1)\right] = 40(\tau-1) = 24.7213595500$

$\vec{r}_5 = (1, \tau, 0)$

$\vec{r}_7 = (\tau, 0, 1)$

$57_{\text{mdpt}} = \left(\frac{\tau+1,\ \tau,\ 1}{2}\right) = \frac{1}{2}(\tau+1, \tau, 1)$

$\text{inradius} = \sqrt{\left(\frac{\tau+1}{2}\right)^2 + \left(\frac{\tau}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

$= \frac{1}{2}\sqrt{\tau^2+2\tau+1+\tau^2+1}$

$= \frac{1}{2}\sqrt{\tau+1+2\tau+1+\tau+1+1}$

$= \frac{1}{2}\sqrt{4\tau+4} = \frac{1}{2}\cdot 2\sqrt{\tau+1}$

$= \boxed{\sqrt{\tau+1}} = \tau$

$\frac{d}{dr}\left(\frac{4\pi}{3}r^3\right) = 4\pi r^2$

vol. of inscribed sphere $= \frac{4\pi}{3}(\tau+1)^{3/2}$ 17.7440000516 impossible! (?)

$\frac{4\pi}{3} = 4.1887902048$

$(\tau+1)^{3/2}$

sphere vol $= \frac{4\pi}{3}\tau^3 = 17.7440000516$

Volume of acute rhombohedra

= area of base × altitude

$\boxed{\text{altitude}} = \text{edge length } (= |\vec{7c}|) \times \sin\varphi \ (= \frac{\tau}{2})$

$= (\sqrt{3-\tau})\frac{\tau}{2}$ ($= .9510565162$ altitude)

area of base $= 2\sqrt{2-\tau}$

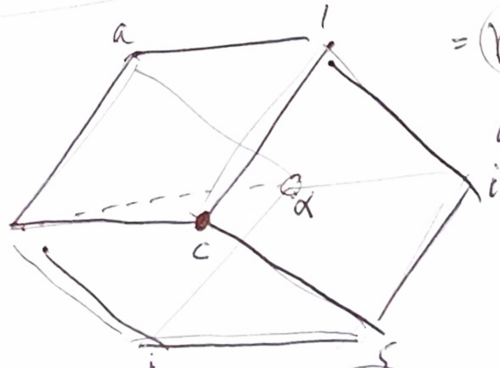
Vol. $= 1.1755705$

Total vol $= 23.5114100916$

$|\vec{7c}| = \sqrt{3-\tau}$

$|\vec{7\alpha}| = 2\tau-2$

Height of acute rhombohedra =



$\varphi = \measuredangle(\vec{7c}, \vec{7\alpha})$

$\vec{7\alpha} = (\tau, 0, 1) \rightarrow (\tau-1, \tau-1, \tau-1)$

$\vec{7c} = (\tau, 0, 1) \rightarrow (1, 1, 1)$

$\therefore \vec{7\alpha} = (-1, \tau-1, \tau-2)$

$\vec{7c} = (-(\tau-1), 1, 0)$

$\vec{7\alpha}\cdot\vec{7c} = |\vec{7\alpha}||\vec{7c}|\cos\varphi$

$\therefore \cos\varphi = \frac{3\tau-4}{(\sqrt{3-\tau})(2\tau-2)}$

$.58778525$

$\boxed{\varphi = 54^\circ}$ (!)

$\sin 54^\circ = .8090169943 = \frac{\tau}{2}$

$\vec{7\alpha}\cdot\vec{7c} = [\tau-1+\tau-1+\tau-2] = 3\tau-4$

$|\vec{7\alpha}| = \sqrt{1+(\tau-1)^2+(\tau-2)^2} = \sqrt{1+\tau^2-2\tau+1+\tau^2-4\tau+4}$

$= \sqrt{1+\cancel{\tau}+1-2\cancel{\tau}+1+\cancel{\tau}+1-4\tau+4}$

$= \sqrt{8-4\tau} = \sqrt{4(2-\tau)} = 2\sqrt{2-\tau} = 2(\tau-1)$

$= 2\tau-2$

$|\vec{7c}| = \sqrt{(\tau-1)^2+1}$

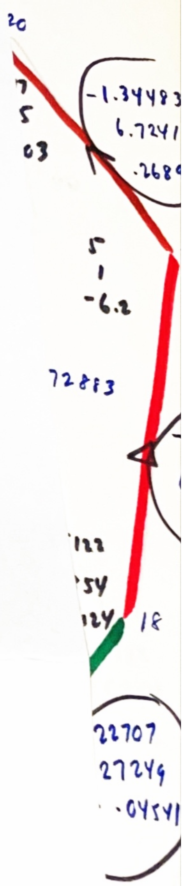
$= \sqrt{\tau^2-2\tau+1+1} = \sqrt{\tau+1-2\tau+2} = \sqrt{3-\tau} = \cancel{\sqrt{\tau}}$

$(2-\tau)(3-\tau) = 6-5\tau+\tau^2 = 6-5\tau+\tau+1$

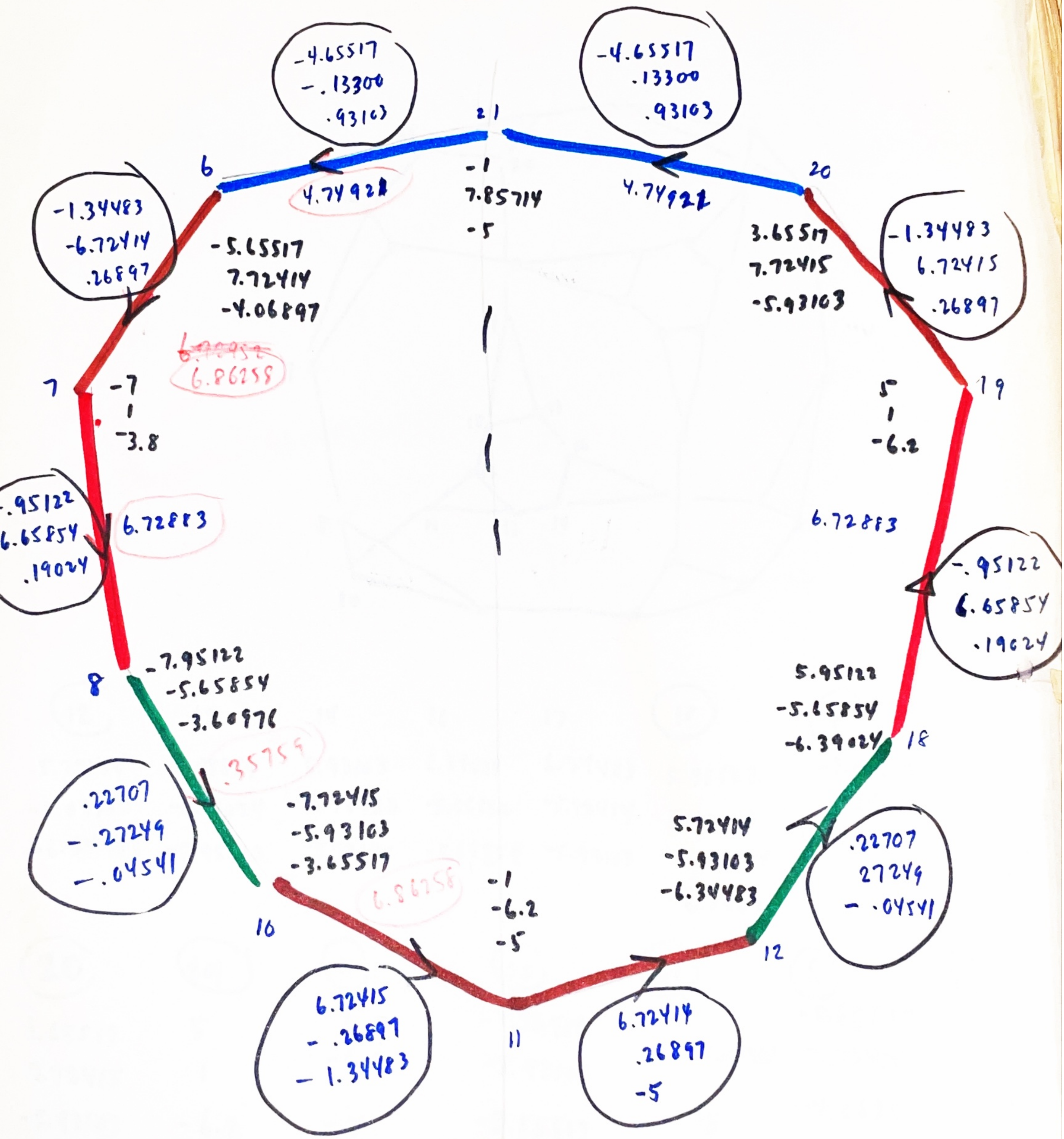
$= 7-4\tau$

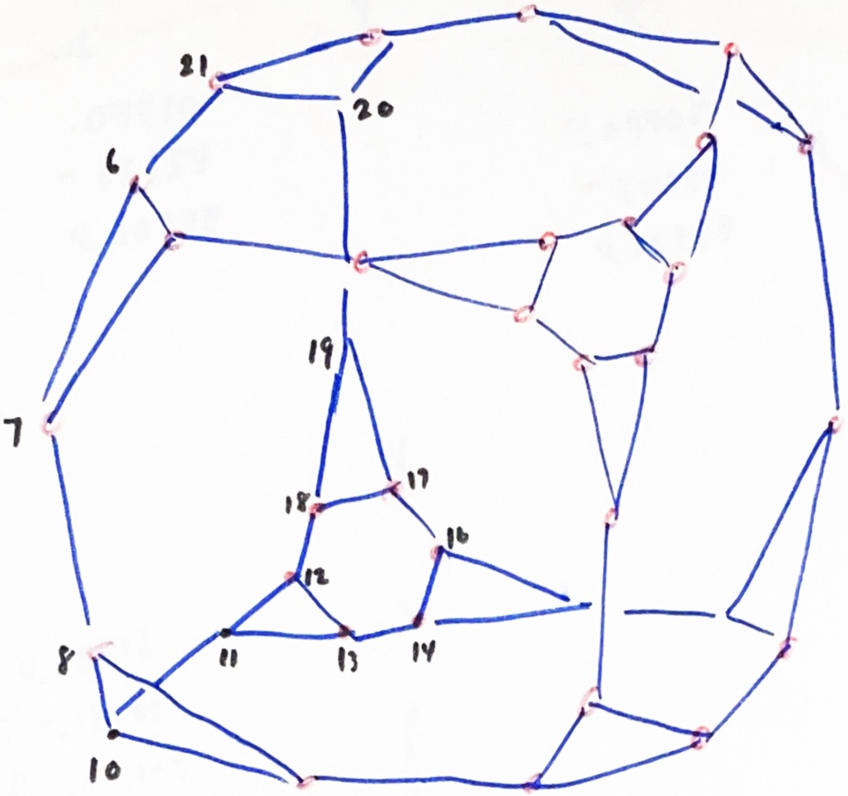
$\frac{3-\tau}{6}$

Check vol. of other tetrahedra :



Face H

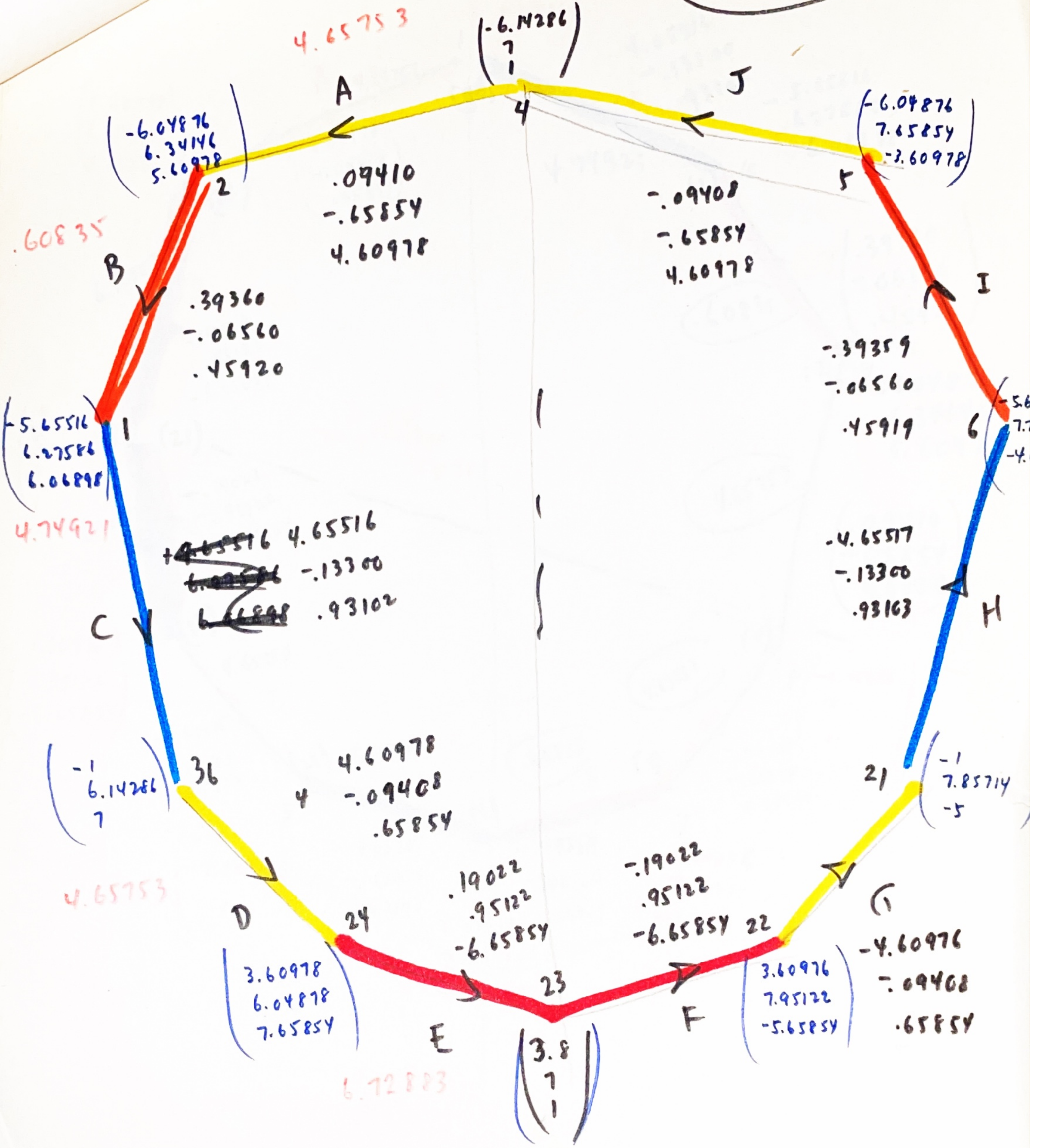

21
20
6
7
8
10
11
12
18
19
-4.65517
-.13300
.93163
-4.65517
.13300
.93163
-1
7.85714
-5
4.74928
4.74928
-1.34483
-6.72414
.26897
-5.65517
7.72414
-4.06897
3.65517
7.72415
-5.93103
-1.34483
6.72415
.26897
6.86258
-7
1
-3.8
5
1
-6.2
.95122
6.65854
.19024
6.72883
6.72883
-.95122
6.65854
.19024
-7.95122
-5.65854
-3.60976
5.95122
-5.65854
-6.39024
.35759
.22707
-.27249
-.04541
-7.72415
-5.93103
-3.65517
5.72414
-5.93103
-6.34483
.22707
.27249
-.04541
6.86258
-1
-6.2
-5
6.72415
-.26897
-1.34483
6.72414
.26897
-5



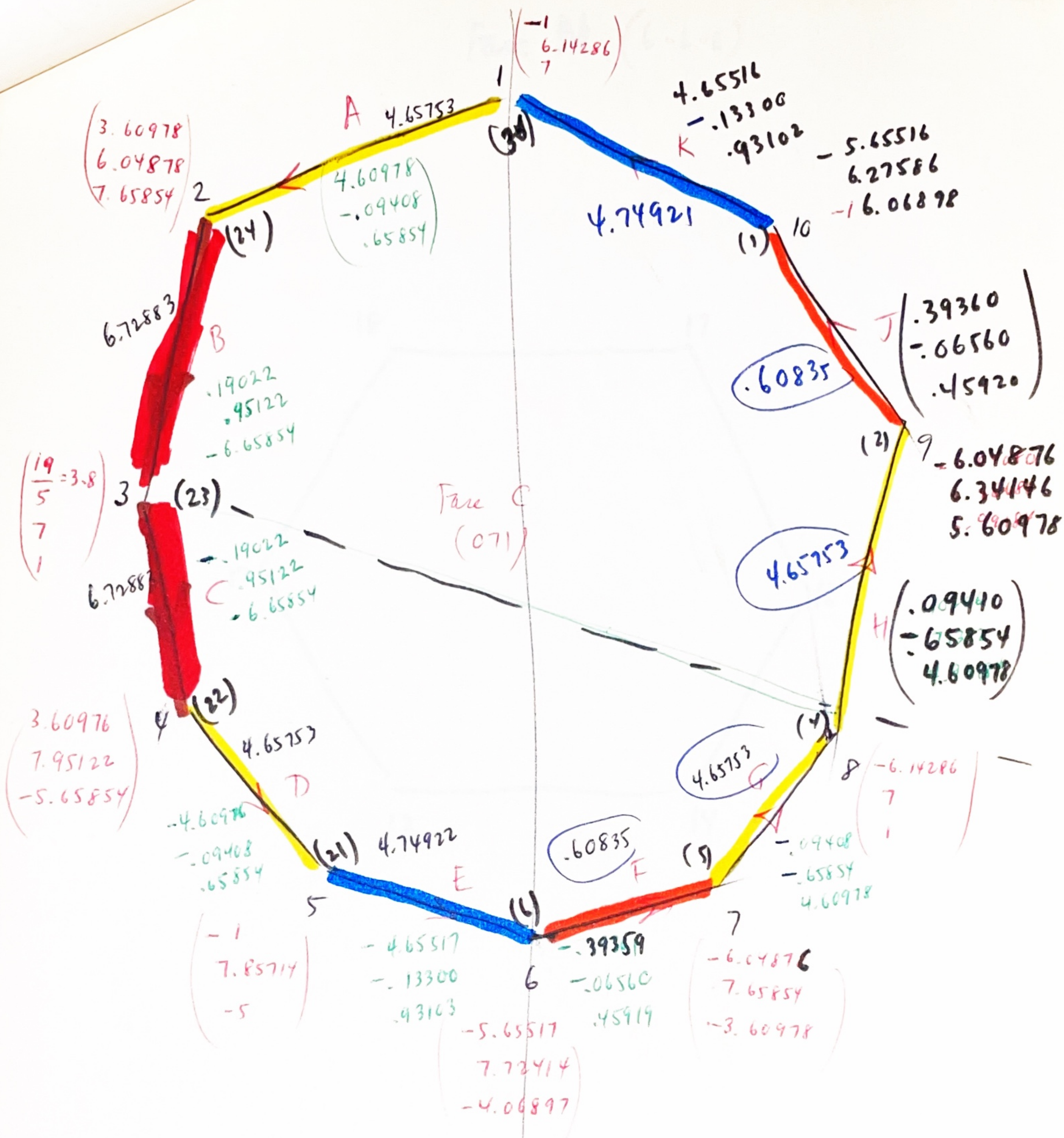
12	13	14	16	17	18	8
5.72414	5.65854	5.93163	6.39024	6.34483	5.95122	-7.95122
-5.93103	-6.39024	-6.34483	-5.95122	-5.72414	~~5.72414~~	-5.65854
-6.34483	-5.95122	-5.72414	-5.65854	-5.93163	-5.65854	-3.60976
					-6.39024	

20	19	11	10	21	6	7
3.65517	5	-1	-7.72415	-1	-5.65517	-7
7.72415	1	-6.2	-5.93103	~~5~~ 7.85714	7.72414	1
-5.93103	-6.2	-5	-3.65517	-5	-4.06897	-3.8

Face C

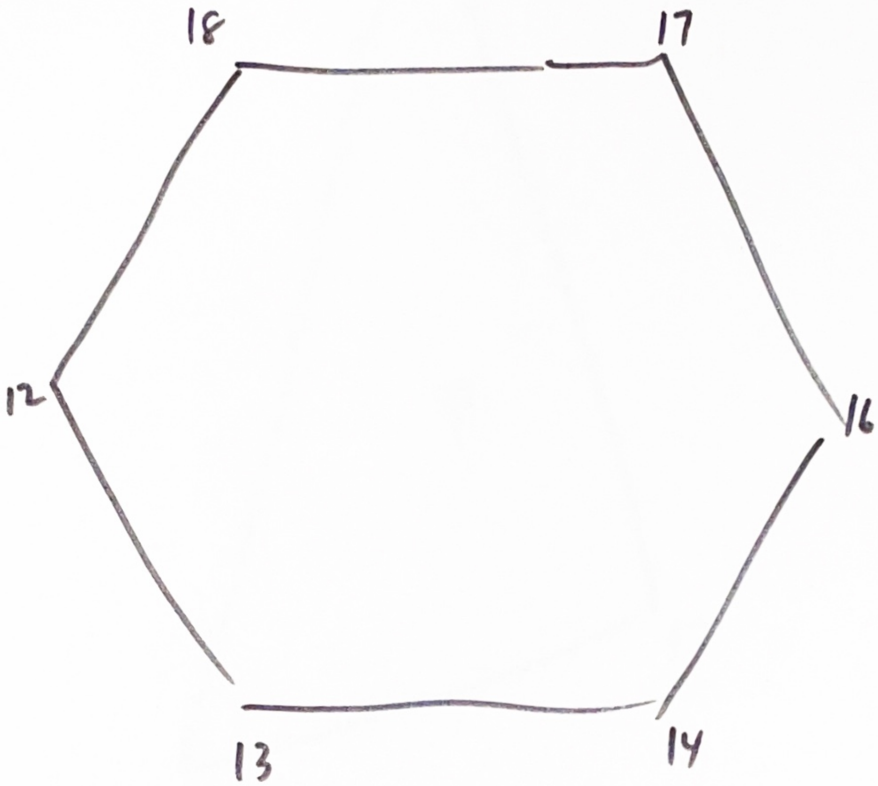


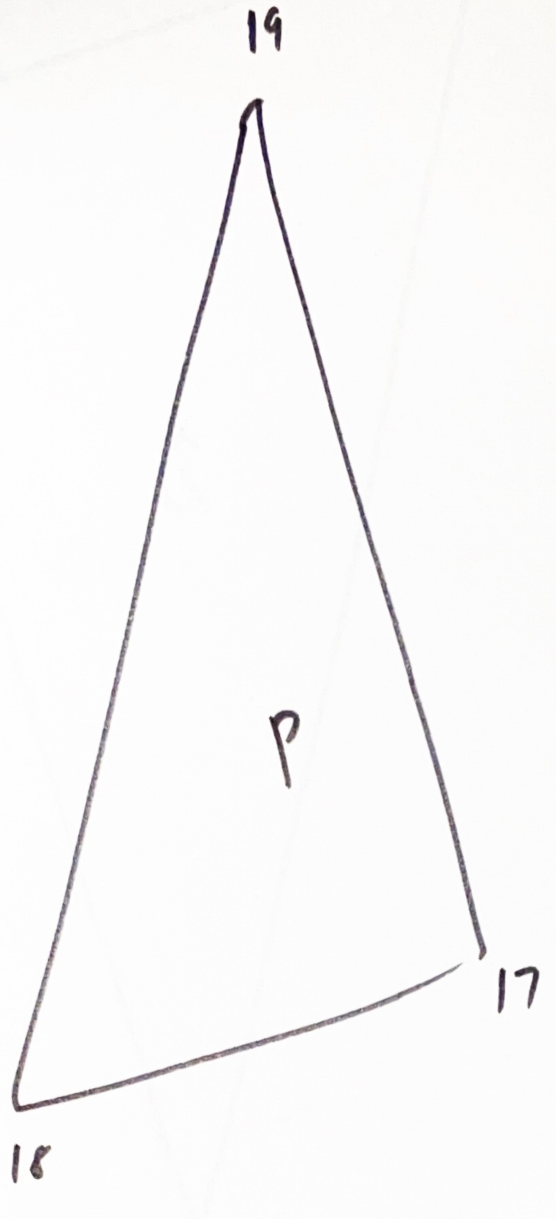
4.65753
-6.14286
7
1
A
4
J
-6.04876
6.34146
5.60978
2
.09410
-.65854
4.60978
-.09408
-.65854
4.60978
-6.04876
7.65854
-3.60978
5
.60835
B
.39360
-.06560
.45920
I
-.39359
-.06560
.45919
-5.65516
6.27586
6.06898
1
6
4.74921
4.65516
-.13300
.93102
-4.65517
-.13300
.93103
C
H
-1
6.14286
7
36
4.60978
-.09408
.65854
21
-1
7.85714
-5
4.65753
D
.19022
.95122
-6.65854
-.19022
.95122
-6.65854
22
G
24
-4.60976
-.09408
.65854
3.60978
6.04878
7.65854
23
3.60976
7.95122
-5.65854
E
F
3.8
7
1

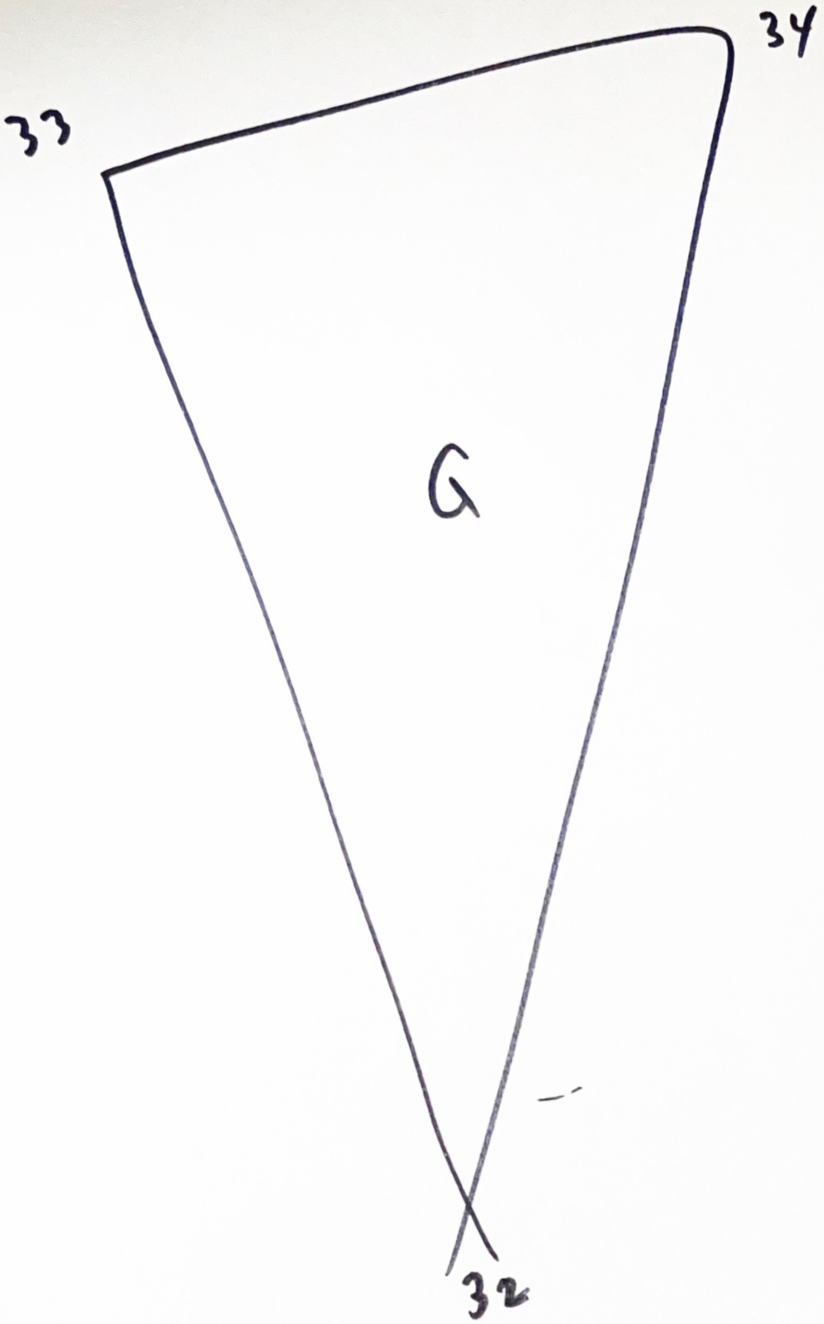


1
-1
6.14286
7
A
4.65753
3.60978
6.04878
7.65854
2
4.60978
-.09408
.65854
(30)
4.65516
-.13300
.93102
K
-5.65516
6.27586
-16.06898
4.74921
(1)
10
(24)
6.72883
B
.19022
.95122
-6.65854
.39360
-.06560
.45920
J
.60835
(2)
9
-6.04876
6.34146
5.60978
19/5 = 3.8
7
1
3
(23)
Face C
(071)
-.19022
.95122
-6.65854
C
6.72883
4.65753
.09410
-.65854
4.60978
H
(22)
4
3.60976
7.95122
-5.65854
4.65753
D
(4)
8
4.65753
G
-6.14286
7
1
-4.60976
-.09408
.65854
(21)
4.74922
E
.60835
F
(5)
-.09408
-.65854
4.60978
5
-1
7.85714
-5
(6)
-4.65517
-.13300
.93103
6
-.39359
-.06560
.45919
7
-6.04876
7.65854
-3.60978
-5.65517
7.72414
-4.06897

Face **N** (6-6-6)




19
p
17
18


33
34
G
32

→ 32 → 36

1 -5.65516, 6.27586, 6.06898

-6.04876 6.34146 5.60978

2 ~~.39360 -.06560 .15920~~

3 -6.14286 7 1

4 ~~3.60976 7.95122 -5.65854~~

5 -6.04876 7.65854 -3.60978

6 -5.65517 7.72414 -4.06897

7 -7 1 -3.8

8 -7.95122 -5.65854 -3.60976

9 -6.04080 6.28688 5.99184

10 -7.72415 -5.93103 -3.65517

11 -1 -6.2 -5

12 5.72414 -5.93103 -6.34483

13

14

15

16

17

18 5.95122 -5.65854 -6.39024

19 5 1 -6.2

20 3.65517 7.72415 -5.93103

21 4.~~[illegible]~~ -1 7.85714 -5

22 3.60976 7.95122 -5.65854

23 3.8 7 1

24 3.60978 6.04878 7.65854

25

26

27

28

29

30

31

34

35

~~35 -1 6.14286 7~~

~~36 -5.65516, 6.27586, 6.06898~~

36 -1, 6.14286, 7

edge length of triacontahedron

$= \sqrt{3-\tau} = 1.175\ 570\ 504\ 58$

indices $= \tau$

single pyramid volume $= 2/3$ $\exists$ 20 pyramids. $\therefore V_{\text{triacontahedron}} = \frac{40}{3} = 13\tfrac{1}{2}$

area of rhombus face

$= 2\sqrt{2-\tau}$

$= 2(\tau-1) \cong 1.236067$

$\frac{\tau}{1+\tau} = \tau - 1$

$\frac{2\tau+1}{\tau+1} = \tau$

~~Vol = $13\tfrac{1}{3}$?~~

$1-(\tau-1)$

$1-\tau+1$

a	$\tau-1$	0	τ	1	0	1	τ
b	1	-1	1	2	0	-1	τ
c	1	1	1	3	0	1	$-\tau$
d	$-(\tau-1)$	0	τ	4	0	-1	$-\tau$
e	-1	-1	1	5	1	τ	0
f	0	$-\tau$,	$\tau-1$	6	1	$-\tau$	0
g	τ	$-(\tau-1)$	0	7	τ	0	1
h	τ	$\tau-1$	0	8	τ	0	-1
i	0	τ	$\tau-1$	9	-1	τ	0
j	-1	1	1	10	-1	$-\tau$	0
k	1	-1	-1	11	$-\tau$	0	1
l	1	1	-1	12	$-\tau$	0	-1
m	0	$+\tau$	$-(\tau-1)$				
n	$-\tau$	$\tau-1$	0				
o	$-\tau$	$-(\tau-1)$	0				
p	-1	-1	-1				
q	$\tau-1$	0	$-\tau$				
r	-1	1	-1				
s	$-(\tau-1)$	0	$-\tau$				
t	0	$-\tau$	$-(\tau-1)$				

volume of one acute rhombohedron =

area of base × altitude

$= (2\sqrt{2-\tau}) \times (\sqrt{3-\tau}\ \frac{\tau}{2})$

$= \sqrt{4-4\tau} = \tau\sqrt{7-4\tau}$

$= 1.175\ 570\ 504\ 58$

$\therefore \text{Vol}_{\text{triacontahedron}} = 20\tau\sqrt{7-4\tau} =$

$23.511410\ 0916\ldots$

Cf. $\frac{4\pi}{3}\tau^3 = 17.744\ 000\ 0510$ (!) ?

vol is about 32% bigger than insphere vol.

CRAZY! ?

diametrical pairs

a s

b r

c p

d q

e l

f m

g n

h o

i t

j k

$\frac{A^3}{V^2} = \frac{(60)^3(\tau-1)^3}{(40)^2(2-\tau)^2}$

$\frac{A^3}{V^2}$

$\text{Vol}_{\text{triacont}} = 20(2)(2-\tau)$

$= \boxed{40(2-\tau)}$

$\cong 15.27864045$

Cf sphere vol $= 17.7440000510$

$\text{Area}_{\text{triacont}} = 30 \cdot 2 \cdot (\tau-1) = \boxed{60(\tau-1)} = 37.082039325$

$A^2 =$

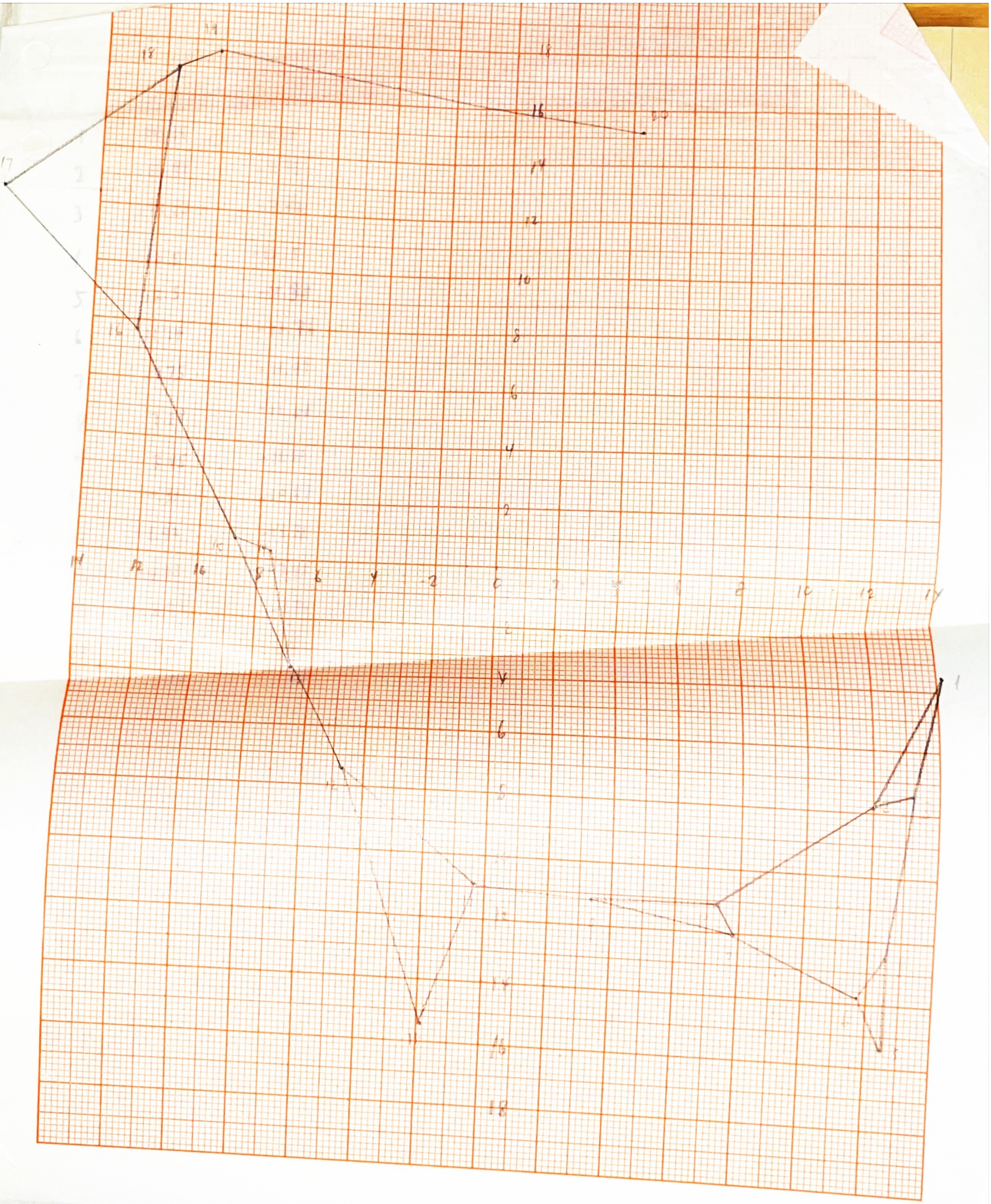
sphere $\frac{(4\pi r^2)^3}{(\frac{4\pi}{3}r^3)^2} = \frac{(4\pi)^3}{(\frac{4\pi}{3})^2} = \frac{9 \cdot 64\pi^3}{16\pi^2} = 36\pi$

sphere IQ: $\boxed{36\pi = 113.0973}$

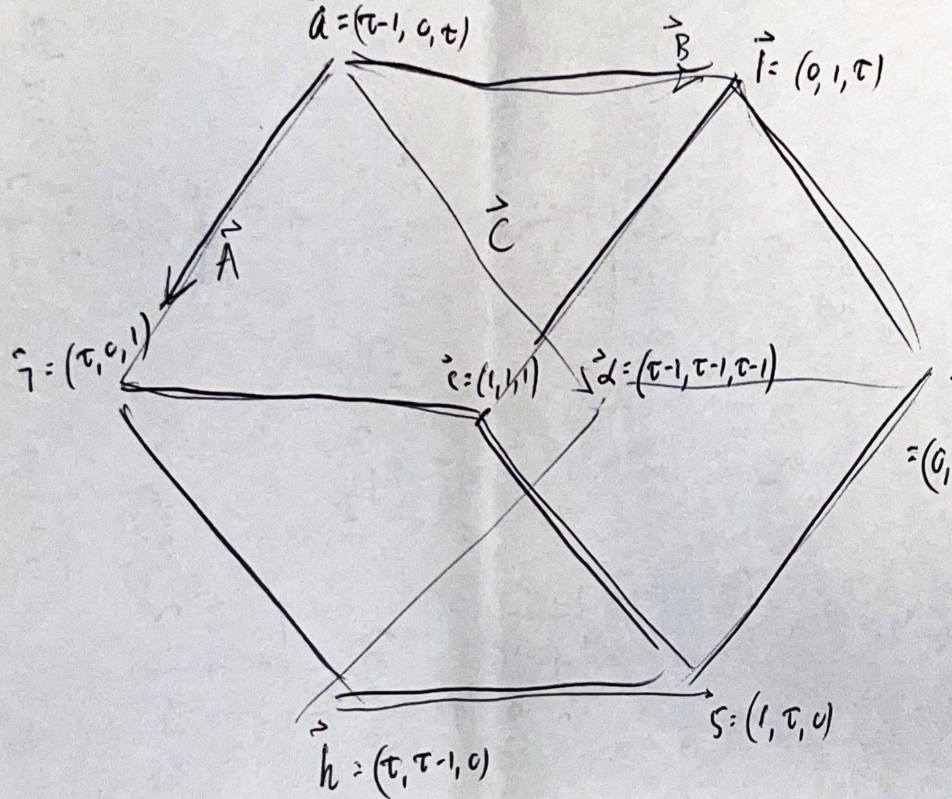
```
LIST
1   DIM X[20]
2   DIM Y[20]
3   DIM Z[20]
19   PRINT ,"INPUT"
20   FOR N=1 TO 20
25      PRINT "X(N)=";
30      INPUT X[N]
35      PRINT ,"Y(N)=";
40      INPUT Y[N]
45      PRINT ,"Z(N)=";
50      INPUT Z[N]
55      PRINT " "
60   NEXT N
65   PRINT "X","Y","Z"
70   FOR N=1 TO 20
75      PRINT X[N]/(Y[N]+25),
80      PRINT "1",
85      PRINT Z[N]/(Y[N]+25)
90   NEXT N
RUN
                    INPUT
X(N)=? 4            Y(N)=? 3.25         Z(N)=? -1
X(N)=? 3.4          Y(N)=? 3.4          Z(N)=? -2.2
X(N)=? 3.727272Y(N)=? 3.181818Z(N)=? -2.090909
X(N)=? 3.4          Y(N)=? 2.2          Z(N)=? -3.4
X(N)=? 3.25         Y(N)=? 1            Z(N)=? -4
X(N)=? 3.181818Y(N)=? 2.090909Z(N)=? -3.727272
X(N)=? 2.2          Y(N)=? 3.4          Z(N)=? -3.4
X(N)=? 2.090909Y(N)=? 3.727272Z(N)=? -3.181818
X(N)=? 1            Y(N)=? 4            Z(N)=? -3.25
X(N)=? -.090909Y(N)=? 4.272727Z(N)=? -3.181818
X(N)=? -.5          Y(N)=? 1            Z(N)=? -4
X(N)=? -1.4         Y(N)=? 4.6          Z(N)=? -2.2
X(N)=? -2           Y(N)=? 4.75         Z(N)=? -1
X(N)=? -2.272727                        Y(N)=? 4.818181Z(N)=? .090909
X(N)=? -2.6         Y(N)=? 4.6          Z(N)=? .2
X(N)=? -3.5         Y(N)=? 1            Z(N)=? 2
X(N)=? -3.9090909                       Y(N)=? -2.272727              Z(N)=? 2.818
X(N)=? -2.6         Y(N)=? -2.6         Z(N)=? 3.8
X(N)=? -2.272727                        Y(N)=? -2.818181              Z(N)=? 3.909
X(N)=? 1            Y(N)=? -2           Z(N)=? 3.5
X                   Y                   Z
 .141593            1 → 14.16           -3.53982E-2   →  - 3.54
 .119718            1   11.97           -7.74648E-2   →  - 7.75
 .132258            1                   -7.41935E-2   →  - 7.42
 .125               1                   -.125             -12.5
 .125               1                   -.153846          -15.4
 .11745             1                   -.137584          -13.8
 7.74648E-2         1                   -.119718
 7.27848E-2         1                   -.118759
 3.44828E-2         1                   -.112069
-3.10559E-3         1                   -.108696
-1.92308E-2         1                   -.153846
-4.72973E-2         1                   -7.43243E-2
-6.72269E-2         1                   -3.36134E-2
-7.62195E-2         1                    3.04878E-3
-8.78378E-2         1                    6.75676E-3
-.134615            1                    7.69231E-2
-.172               1                    .123992
 .116071            1                    .169643
-.102459            1                    .176225
 4.34783E-2         1                    .152174
```

```
RUN
                          INPUT
X(N)=? -2.6          Y(N)=? -3.8          Z(N)=? 2.6
X(N)=? -2.818181
                                          Y(N)=? 3.909090Z(N)=? 2.272727
X(N)=? -3.8          Y(N)=? -2.6          Z(N)=? 2.6
X(N)=? -2            Y(N)=? -3.5          Z(N)=? -1
X(N)=? -.2           Y(N)=? -2.6          Z(N)=? 4.6
X(N)=? -.090909Y(N)=? -2.272727                      Z(N)=? -4.818181
X(N)=? 1             Y(N)=? -2            Z(N)=? -4.75
X(N)=? 2.2           Y(N)=? -1.4          Z(N)=? -4.5
X(N)=? 3.181818Y(N)=? -.090909Z(N)=? -4.272727
X(N)=? 4             Y(N)=? -.5           Z(N)=? -1
X(N)=? 4.818181Y(N)=? -.090909Z(N)=? 2.272727
X(N)=? 4.6           Y(N)=? -.2           Z(N)=? 2.6
X(N)=? 4.75          Y(N)=? 1             Z(N)=? 2
X(N)=? 4.6           Y(N)=? 2.2           Z(N)=? 1.4
X(N)=? 1             Y(N)=? 4             Z(N)=? .5
X(N)=? 4.272727Y(N)=? 3.181818Z(N)=? .090909
X(N)=? 0             Y(N)=? 1             Z(N)=? 0
X(N)=? 0             Y(N)=? 1             Z(N)=? 0
X(N)=? 0             Y(N)=? 1             Z(N)=? 0
X(N)=? 0             Y(N)=? 1             Z(N)=? 0
X                    Y                    Z
-.122642             1                     .122642
-9.74842E-2          1                     7.86163E-2
-.169643             1                     .116071
-9.30233E-2          1                    -4.65116E-2
-8.92857E-3          1                     .205357
-.004                1                    -.212
 4.34783E-2          1                    -.206522
 9.32203E-2          1                    -.190678
 .127737             1                    -.171533
 .163265             1                    -4.08163E-2
 .193431             1                     9.12409E-2
 .185484             1                     .104839
 .182692             1                     7.69231E-2
 .169118             1                     5.14706E-2
3.44828E-2           1                     1.72414E-2
.151613              1                     3.2258E-3
0                    1                     0
0                    1                     0
0                    1                     0
0                    1                     0


*READY
```


18
16
14
12
10
8
6
4
2
14
12
10
8
6
4
2
0
8
10
12
14
6
8
12
14
16
18

1	14.16	-3.54
2	11.97	-7.75
3	13.23	-7.42
4	12.5	-12.5
5	12.5	-15.38
6	11.74	-13.76
7	7.75	-11.97
8	7.28	-11.08
9	3.45	-11.21
10	-.31	-10.87
11	-1.92	-15.38
12	-4.73	-7.43
13	-6.72	-3.36
14	-7.62	.30
15	-8.78	.68
16	-13.46	7.69
17	-17.2	12.40
18	-11.61	16.96
19	-10.25	17.62
20	4.35	15.22



Volume $= \vec{C} \cdot \vec{A} \times \vec{B}$

$\vec{A} = \vec{a7} = (\tau-(\tau-1), 0-0, 1-\tau)$

$\boxed{\vec{A} = (1, 0, 1-\tau)}$

$\vec{B} = \vec{a1} = (0-(\tau-1), 1-0, \tau-\tau)$

$\boxed{\vec{B} = (1-\tau, 1, 0)}$

$\boxed{\vec{C} = [0, \tau-1, -1]}$

edge length $= \sqrt{1+\tau^2-2\tau+1}$

$= \sqrt{\tau+1-2\tau+2} = \sqrt{3-\tau}$

$\boxed{= 1.1755705}$

$\vec{\alpha} = \vec{a} + \vec{c5} = (\tau-1, 0, \tau) + [(1-1, \tau-1, 0-1) = (0, \tau-1, -1)]$

$= (\tau-1, \tau-1, \tau-1)$

ACUTE

$\vec{b}$: ~~cfg~~ 267

$$\begin{matrix} 0 & -1 & \tau \\ 1 & -\tau & 0 \\ \tau & 0 & 1 \end{matrix}$$

$\tau+1 \quad -(\tau+1) \quad \tau+1$

$\tau^2 - 2\tau + 1 + \tau^2$

$\cancel{\tau^2}+1 - \cancel{2\tau} + 1 + \cancel{\tau^2}+1 \quad 3$

k 4,6,8

$$\begin{matrix} 0 & -1 & -\tau \\ 1 & -\tau & 0 \\ \tau & 0 & -1 \\ \hline 1 & -1 & -1 \end{matrix}$$

$\vec{d}$ 1, 2, 11 ~~$0\ 1\ \tau$~~

$$\begin{matrix} 0 & 1 & \tau \\ 0 & -1 & \tau \\ -\tau & 0 & 1 \\ \hline -\tau & 0 & 2\tau+1 \end{matrix}$$

$\frac{-\tau}{\tau+1} \quad 0 \quad \frac{2\tau+1}{\tau+1} = -(\tau-1), 0, \tau$

m 359

$$\begin{matrix} 0 & 1 & -\tau \\ 1 & \tau & 0 \\ -1 & \tau & 0 \\ \hline 0 & 2\tau+1 & -\tau \end{matrix}$$

$0 \quad \tau \quad -(\tau-1)$

e 2, 10, 11

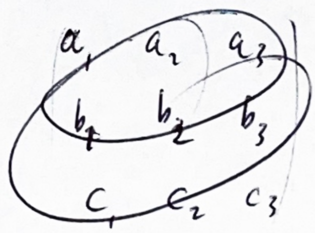
$$\begin{matrix} 0 & -1 & \tau \\ -1 & -\tau & 0 \\ -\tau & 0 & 1 \\ \hline -(\tau+1) & -(\tau+1) & \tau+1 \end{matrix}$$

f 2, 6, 10

$$\begin{matrix} 0 & -1 & \tau \\ 1 & -\tau & 0 \\ -1 & -\tau & 0 \\ \hline 0 & -(2\tau+1) & \tau \end{matrix}$$

$0, -\tau, \tau-1$

n



g 6, 7, 8

$$\begin{matrix} 1 & -\tau & 0 \\ \tau & 0 & 1 \\ \tau & 0 & -1 \\ \hline 2\tau+1, & -\tau, & 0 \end{matrix}$$

$\tau, -(\tau-1), 0$

$i \quad j \quad k$

$a_1 \quad a_2 \quad a_3$

$b_1 \quad b_2 \quad b_3$

$(a_2 b_3 - b_2 a_3)$

$a_3 b_1 - b_3 a_1$

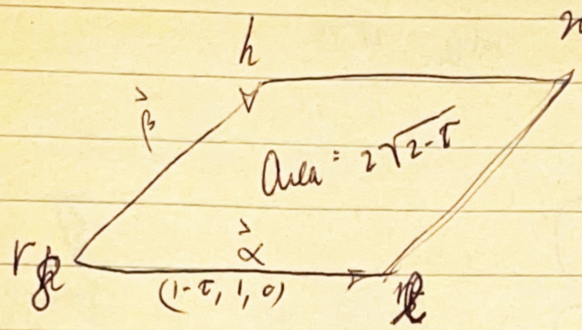
j 1, 9, 11

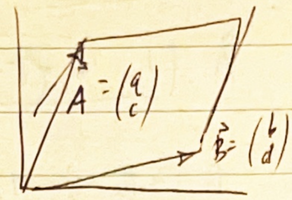
$$\begin{matrix} 0 & 1 & \tau \\ -1 & \tau & 0 \\ -\tau & 0 & 1 \\ \hline -(\tau+1) & \tau+1 & (\tau+1) \end{matrix}$$

$-1 \quad 1 \quad 1$

style pyramid vol = $2/3$

$\frac{20 \cdot 2}{3} = 13\frac{1}{3}$





$|ad - bc| = $ area

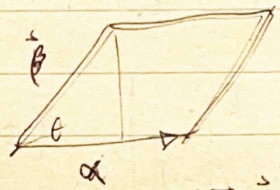
$\vec{r_g} = (\tau, 0, -1)$

$\vec{l} = (1, 1, -1)$

$\vec{h} = (\tau, \tau-1, 0)$ a

$\vec{\alpha} = \overrightarrow{r_g \to l} = (1-\tau, 1, 0)$

$\vec{\beta} = \overrightarrow{r_g \to h} = (0, \tau-1, 1)$



$|\vec{\alpha} \times \vec{\beta}| = ab \sin\theta$

$$\vec{\alpha} \times \vec{\beta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1-\tau & 1 & 0 \\ 0 & -(1-\tau) & 1 \end{vmatrix} = \hat{i}(1) + \hat{j}(1-\tau) + \hat{k}\left(-(1-\tau)^2\right)$$

$$= \left[1, 1-\tau, -(1-\tau)^2\right]$$

$= \cancel{\hat{i}\left[-(1-\tau)^2\right]}$

Area $= |\vec{\alpha} \times \vec{\beta}| \; \sqrt{1 + (\tau-1)^2 + (\tau-1)^4}$

$= \sqrt{1 + \tau^2 - 2\tau + 1 + 5 - 3\tau}$

$= \sqrt{1 + 2 - \tau + 5 - 3\tau}$

$= \sqrt{1 + 7 - 4t}$

$= \sqrt{8 - 4t}$

$= \sqrt{4(2-\tau)}$

$= \boxed{2\sqrt{2-\tau}}$

.1458

$(\tau-1)^2$

$= \tau^2 - 2\tau + 1$

$= \tau + 1 - 2\tau + 1$

$= 2 - \tau$

$(\tau-1)^4 = (\tau-2)^2$

$= \tau^2 - 4\tau + 4$

$= \tau + 1 - 4\tau + 4$

$= 5 - 3\tau$

Pyramid vol $= \frac{1}{3} Bh$

$= \frac{1}{3}\left(2\sqrt{2-\tau}\right)\tau = \frac{2}{3}$

$= \frac{2}{3}(\tau-1)\tau = \frac{2}{3}(\tau^2 - \tau)$

$= \frac{2}{3}(\tau + 1 - \tau)$

$= \frac{2}{3}$

But

$\sqrt{2-\tau} = \tau - 1$

$\tau^2 - 2\tau + 1 = 2 - \tau$

$= \tau + 1 - 2\tau + 1$

Now recalculate the volume of an obtuse rhombohedron.

$\overrightarrow{|h\delta|} = (0, -(\tau-1), -1)$

$\gamma = \vec{i} + \overrightarrow{h\delta} = (0, \tau, \tau-1) + (0, -(\tau-1), -1)$ ✓

$0 \quad -(\tau-1) \quad -1)$

$\boxed{\gamma = (0, 1, \tau-2)}$

$\vec{\delta\gamma} = (\tau\ 0\ -1) - (0, 1, \tau-2)$

$0 \quad 1 \quad \tau-2$

$\boxed{\overrightarrow{\delta\gamma} = -\tau, +1, +(\tau-1)}$

$\xi = \angle(\overrightarrow{\delta h}, \overrightarrow{\delta\gamma})$ $\boxed{\begin{array}{l}\vec{\delta h} = (0, \tau-1, 1) \\ \vec{\delta\gamma} = (-\tau, +1, +(\tau-1))\end{array}}$

$\overrightarrow{h\delta} = \tau\ 0\ -1$

$-(\tau\ \ \tau-1\ \ 0)$

$= 0\ -(\tau-1)\ -1$

$i = 0 \quad \tau \quad \tau-1$

$h\delta \quad 0 \quad 1-\tau \quad -1$

$\boxed{\gamma = (0 \quad 1 \quad \tau-2)}$ ✓

$\cancel{\delta}\gamma \quad 0 \quad 1 \quad \tau-2$

$\delta \quad \tau \quad 0 \quad -1$

$\boxed{\vec{\delta\gamma} = -\tau, \ 1, \ \tau-1}$

~~$\overrightarrow{\delta h} \cdot \vec{\delta\gamma} = (0, \tau-1, 1)\ (\tau, +1, +(\tau-1)) = (0 + (1-\tau) + 1-\tau) = 2-2\tau = 2(1$~~

Check again:

$h = (\tau, \ \tau-1, 0)$

$\vec{\delta} = (\tau, \ 0, -1)$

$\overrightarrow{\delta h} = (0, \tau-1, 1)$

$\gamma =$

$\cancel{\delta\gamma} = \vec{\delta h} \cdot \vec{\delta\gamma} = (0, \tau-1, 1) \cdot (-\tau, 1, \tau-1) = 0 + \tau-1+\tau-1 = 2\tau-2$

$= \boxed{2(\tau-1)}$

$|\vec{\delta h}| = \sqrt{0^2 + \tau^2 - 2\tau + 1 + 1} = \sqrt{\tau + 1 - 2\tau + 1 + 1} = \sqrt{3-\tau}$ ✓

$|\vec{\delta\gamma}| = \sqrt{\tau^2 + 1 + \tau^2 - 2\tau + 1} = \sqrt{\cancel{\tau} + 1 + 1 + \cancel{\tau} + 1 - \cancel{2\tau} + 1} = 2$

$\therefore \vec{\delta h} \cdot \vec{\delta\gamma} = \boxed{2(\tau-1) = 2\sqrt{3-\tau}\cos\xi}$

$\therefore \cos\xi = \dfrac{\cancel{2}(\tau-1)}{\cancel{2}\sqrt{3-\tau}}$

$\sin\varphi = \sqrt{1 - \dfrac{(\tau-1)^2}{3-\tau}}$

$= \sqrt{\dfrac{3-\tau-[\tau^2-2\tau+1]}{3-\tau}}$

$= \sqrt{\dfrac{3-\tau-(\tau+1-2\tau+1)}{3-\tau}}$

$= \sqrt{\dfrac{3-\cancel{\tau}-\cancel{\tau}-1+2\tau-1}{3-\tau}} = \sqrt{\dfrac{1}{3-\tau}}$

$\therefore$ Altitude $=$ edge length $\times \sin\varphi = \sqrt{3-\tau} \cdot \dfrac{1}{\sqrt{3-\tau}} = 1$

volume = area of base × altitude

~~vol~~ area$_{\text{base}} = 2(\tau-1)$

Altitude $= 1$

$\therefore$ volume $= 2(\tau-1)$

Now recalculate the volume of an obtuse rhombohedron.

$\overrightarrow{h\delta} = (0, -(\tau-1), -1)$

$\gamma = \vec{i} + \overrightarrow{h\delta} = (0, \tau, \tau-1) + (0, -(\tau-1), -1)$ ✓

$0 \; -(\tau-1) \; -1)$

$\boxed{\gamma = (0, 1, \tau-2)}$

$\overrightarrow{\delta\gamma} = (\tau \; 0 \; -1) - (0, 1, \tau-2)$

$0 \; 1 \; \tau-2$

$\boxed{\overrightarrow{\delta\gamma} = -\tau, +1, +(\tau-1)}$

$\xi = \angle(\overrightarrow{\delta h}, \overrightarrow{\delta\gamma})$ $\boxed{\begin{array}{l}\overrightarrow{\delta h} = (0, \tau-1, 1) \\ \overrightarrow{\delta\gamma} = (-\tau, +1, +(\tau-1))\end{array}}$

$\overrightarrow{h\delta} = \tau \; 0 \; -1$

$-(\tau \; \tau-1 \; 0)$

$= 0 \; -(\tau-1) \; -1$

$i = 0 \; \tau \; \tau-1$

$h\delta \; 0 \; 1-\tau \; -1$

$\boxed{\gamma = (0 \; 1 \; \tau-2)}$ ✓

$\boxed{\gamma \; 0 \; 1 \; \tau-2}$

$\delta \; \tau \; 0 \; -1$

$\boxed{\overrightarrow{\delta\gamma} = -\tau, 1, \tau-1}$

~~$\overrightarrow{\delta h} \cdot \overrightarrow{\delta\gamma} = (0, \tau-1, 1) \cdot (\tau, +1, +(\tau-1)) = (0 + (1-\tau) + 1-\tau) = 2-2\tau = 2(1$~~

Check again:

$\left.\begin{array}{l} h = (\tau, \tau-1, 0) \\ \vec{\delta} = (\tau, 0, -1) \end{array}\right\}$ $\overrightarrow{\delta h} = (0, \tau-1, 1)$ $\gamma =$

$\cancel{\delta\gamma} = \overrightarrow{\delta h} \cdot \overrightarrow{\delta\gamma} = (0, \tau-1, 1) \cdot (-\tau, 1, \tau-1) = 0 + \tau-1+\tau-1 = 2\tau-2$

$= \boxed{2(\tau-1)}$

$|\overrightarrow{\delta h}| = \sqrt{0^2 + \tau^2 - 2\tau + 1 + 1} = \sqrt{\tau+1-2\tau+1+1} = \sqrt{3-\tau}$ ✓

$|\overrightarrow{\delta\gamma}| = \sqrt{\tau^2 + 1 + \tau^2 - 2\tau + 1} = \sqrt{\cancel{\tau} + 1 + 1 + \cancel{\tau} + 1 - \cancel{2\tau} + 1} = 2$

$\therefore \overrightarrow{\delta h} \cdot \overrightarrow{\delta\gamma} = \boxed{2(\tau-1) = 2\sqrt{3-\tau}\cos\xi}$ $\therefore \cos\xi = \dfrac{\cancel{2}(\tau-1)}{\cancel{2}\sqrt{3-\tau}}$

$\sin\varphi = \sqrt{1 - \dfrac{(\tau-1)^2}{3-\tau}}$

$= \sqrt{\dfrac{3-\tau-[\tau^2-2\tau+1]}{3-\tau}}$

$= \sqrt{\dfrac{3-\tau-(\tau+1-2\tau+1)}{3-\tau}}$

$= \sqrt{\dfrac{3-\cancel{\tau}-\cancel{\tau}-1+\cancel{2\tau}-1}{3-\tau}} = \sqrt{\dfrac{1}{3-\tau}}$

volume = area of base × altitude

~~volume~~ $area_{base} = 2(\tau-1)$

altitude = 1

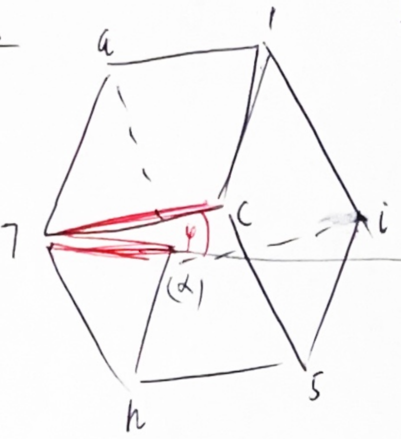
$\therefore$ volume $= 2(\tau-1)$

$\therefore$ altitude = edge length × $\sin\varphi = \sqrt{3-\tau} \cdot \dfrac{1}{\sqrt{3-\tau}} = 1$

Acute Rh.

$\vec{7a}$: $\tau-1, 0, \tau$
$\quad -\tau \quad 0 \quad 1$
$\boxed{\vec{7a} = (-1, 0, \tau-1)}$

$\vec{7c}$: $1, 1, 1$
$\quad -(\tau \quad 0 \quad 1)$
$\boxed{\vec{7c} = (1-\tau, 1, 0)}$



Find angle $\varphi = \measuredangle(\vec{7c}, \vec{7\alpha})$

This page is probably all correct!

$(\vec{7\alpha}) : (\tau-1, \tau-1, \tau-1) - (\tau, 0, 1)$
$\quad \tau \quad 0 \quad 1$
$= (-1, \tau-1, \tau-2)$

$\vec{7c} = (1,1,1) - (\tau, 0, 1)$
$\quad \tau \quad 0 \quad 1$
$\quad 1-\tau, 1, 0$

$$\boxed{\begin{aligned}\vec{7\alpha} &= (-1, \tau-1, \tau-2)\\ \vec{7c} &= (-(\tau-1), 1, 0)\end{aligned}}$$

$\vec{\alpha} = \vec{a} + (\vec{1i}) = (\tau-1, 0, \tau) + [\overset{(i)}{(0, \tau, \tau-1)} - \overset{(1)}{(0, 1, \tau)}]$

$\quad 0 \quad 1 \quad \tau$; $\quad 0, \tau-1, -1$

$\quad 0 \quad \tau-1 \quad -1$; $\quad \tau-1 \quad \tau-1 \quad \tau-1$

$\vec{\alpha} = (\tau-1, \tau-1, \tau-1)$

$$\boxed{\begin{aligned}\vec{\alpha} &= (\tau-1)(1,1,1)\\ \vec{c} &= (1,1,1)\end{aligned}}$$

$|\vec{7\alpha}| = \sqrt{1+\tau^2-2\tau+1+\tau^2-4\tau+4}$ $\quad 1 \; -2 \; 1$
$= \sqrt{1+\tau+1-2\tau+1+\tau+1-4\tau+4}$
$= \sqrt{8-4\tau} = \sqrt{4(2-\tau)} = 2\sqrt{2-\tau}$
$\boxed{7\alpha = 2(\tau-1)}$

$|\vec{7c}| = \sqrt{\tau^2-2\tau+1+1} = \sqrt{\tau+1-2\tau+2}$
$\boxed{|\vec{7c}| = \sqrt{3-\tau} = \text{edgelength} \checkmark}$

$\vec{7\alpha} \cdot \vec{7c} = (-1, \tau-1, \tau-2) \cdot (-(\tau-1), 1, 0)$
$= \tau-1+\tau-1 = 2(\tau-1) = 2(\tau-1)\sqrt{3-\tau}\cos\varphi$
$\therefore \cos\varphi = \dfrac{1}{\sqrt{3-\tau}}$
$\varphi \approx 31.7174744^\circ$

Altitude = edgelength · $\sin\varphi$
$= \sqrt{3-\tau}\sqrt{1-\left(\dfrac{1}{\sqrt{3-\tau}}\right)^2} = \sqrt{3-\tau}\sqrt{1-\dfrac{1}{3-\tau}}$
$= \sqrt{3-\tau}\sqrt{\dfrac{3-\tau-1}{3-\tau}} = \sqrt{2-\tau} = (\tau-1)$

$\boxed{\text{Altitude} = \tau-1}$

What is "short axis" length?

$|\vec{c\alpha}| = |(1,1,1) - (\tau-1)(1,1,1)|$
$= |(2-\tau)(1,1,1)|$
$= \sqrt{2-\tau}\,|(1,1,1)|$
$= \sqrt{3}\sqrt{2-\tau} = \boxed{\sqrt{3}(\tau-1) \text{ short axis}}$
≈ 1.070466

$\therefore$ volume = area of base × altitude
$= 2(\tau-1)(\tau-1)$
$= 2(\tau^2-2\tau+1) = 2(\tau+1-2\tau+1)$
$= 2(2-\tau)$

~~Correct~~

vol. $= 2(2-\tau)$

~~Total triacontahedron vol~~
~~$= V_{tria} = 20 \cdot \text{vol.} = 40(2-\tau) \approx \boxed{15.27864045}$~~
$V_{sphere} = \dfrac{4\pi}{3}\tau^3 = \boxed{17.7440000510}$ Good

Area of base = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & \tau-1 \\ 1-\tau & 1 & 0 \end{vmatrix}$ ✓

$\tau^2-2\tau+1$
$= \tau+1-2\tau+1$
$= 2-\tau$
$(2-\tau)^2 = 4-4\tau+\tau^2$
$= \tau+1+4-4\tau$
$= 5-3\tau$

$= |(1-\tau, -(\tau-1)^2, -1)| = \sqrt{(\tau-1)^2+(\tau-1)^4+1}$
$= \sqrt{2-\tau+5-3\tau+1}$
$= \sqrt{8-4\tau} = \sqrt{4(2-\tau)} = 2\sqrt{2-\tau} = 2(\tau-1)$

rhomb Area ≈ 1.236067
$= 2(\tau-1)$

$i = \frac{1}{3} r_{1,5,9} = \frac{1}{3}\big((0,1,\tau) + (1,\tau,0) + (-1,\tau,0)\big)$

$= \frac{1}{3}(0, 2\tau+1, \tau)\left(\frac{3}{1+\tau}\right) =$

$\left(0, \frac{2\tau+1}{1+\tau}, \frac{\tau}{1+\tau}\right)$

$= (0, \tau, \tau-1)$

$\boxed{\frac{\tau}{1+\tau} = \tau-1}$

$\boxed{\frac{2\tau+1}{\tau+1} = \tau}$

$\cancel{\tau^2+\tau = 2\tau+1}$

$\vec{h} = \frac{578}{3} = \left(\frac{1}{3}\right)\left(\frac{3}{\tau+1}\right)\big[(1,\tau,0) + (\tau,0,1) + (\tau,0,-1)\big]$

$= \left(\frac{1}{\tau+1}\right)\big[2\tau+1, \tau, 0\big] = \left[\frac{2\tau+1}{\tau+1}, \frac{\tau}{\tau+1}, 0\right] = (\tau, \tau-1, 0)$

~~vol. pancake:~~

vol. flatR: $\vec{C}\cdot\vec{A}\times\vec{B}$ $A\times B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1-\tau \\ 1-\tau & 1 & 0 \end{vmatrix}$

$\vec{C} = (0, \tau-1, -1)$

$\vec{C}\cdot\vec{A}\times\vec{B} = (0, \tau-1, -1)\cdot(\tau-1, -\tau+2, 1)$

$= 0 + (\tau-1)(-\tau+2) - 1$

$= -\tau^2+\tau+2\tau-2-1$

$= -(\tau-1)+3\tau-3$

$= -\tau+1+3\tau-3$

$= 2\tau-2 =$

$\boxed{\text{volume} = 2\tau-2}$

$= \vec{i}\big(0-(1)(1-\tau)\big) - \vec{j}\big(0-(\tau-1)^2\big) + \vec{k}(1)$

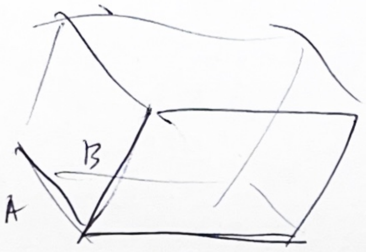
$= \vec{i}(\tau-1) + \vec{j}(\tau^2-2\tau+1) + \vec{k}$

$= (\tau-1, \tau+1-2\tau+1, 1)$

$= (\tau-1, -\tau+2, 1)$

~~vol. of each ...~~

vol. of both rhombohedra $= 2\tau - 2 = 2(\tau-1) = 1.236067977 58$



~~Scale 2~~ $= \frac{3}{1+\tau}(\tau, 0, 1+2\tau)$

vol $= |C \cdot A \times B|$

$a = \frac{3}{1+\tau}\left(\frac{\vec{r}_1+\vec{r}_2+\vec{r}_7}{3}\right)$

$= \frac{3}{1+\tau}\left(\frac{\tau}{3}, 0, \frac{1+2\tau}{3}\right)$

$\frac{L_2}{L_1} = \tau = \left(\frac{\tau}{1+\tau}, 0, \frac{1+2\tau}{1+\tau}\right)$

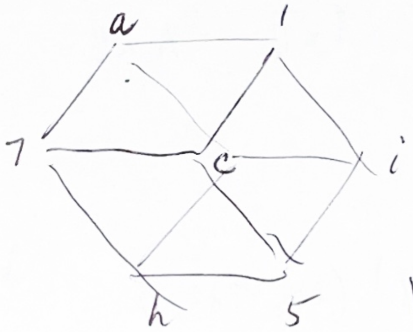
1	0	1	τ
2	0	-1	τ
7	τ	0	1

$\frac{3}{1+\tau} \quad \frac{1}{3}(\tau \quad 0 \quad 1+2\tau)$

$\left(\frac{\tau}{\tau+1}, 0, \frac{2\tau+1}{\tau+1}\right)$

$\boxed{a = (\tau-1, 0, \tau)}$

1	0	1	τ
5	1	τ	0
7	τ	0	1
2	0	-1	τ



~~$a = (\tau-1, 0, \tau+1)$~~

$c - a = [1-(\tau-1), 1, [1-(\tau+1)]$

$= \cancel{2-\tau, 1, -\tau}$

$\boxed{\vec{c} - \vec{a} = (2-\tau, 1, -\tau)}$

$\vec{7} - \vec{1} = (\tau, -1, 1-\tau)$

$C = \frac{3}{1+\tau}\left(\frac{\vec{r}_1+\vec{r}_5+\vec{r}_7}{3}\right)$

$|\vec{7}-\vec{1}| = \sqrt{\tau^2+1+\tau^2-2\tau+1}$

$= \sqrt{\cancel{\tau}+1+1+\cancel{\tau}+1-2\cancel{\tau}+1}$

$= \frac{3}{1+\tau}\left(\frac{1+\tau}{3}, \frac{1+\tau}{3}, \frac{1+\tau}{3}\right)$ $|\vec{71}| = 2$

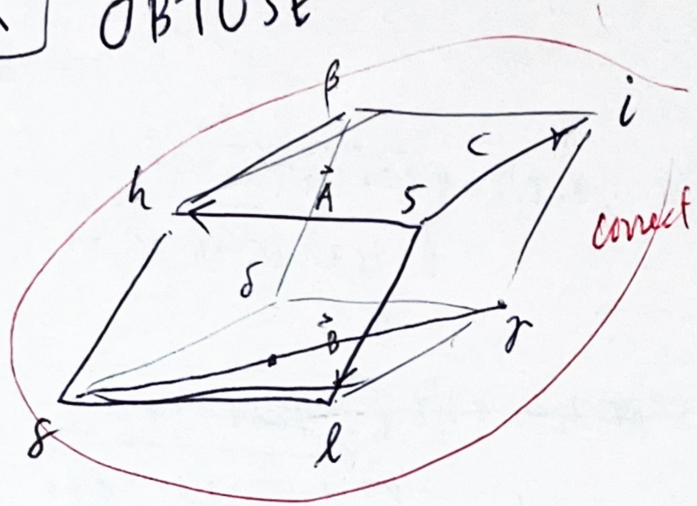
$= (1, 1, 1)$

$\vec{ac} = \sqrt{\tau^2-4\tau+4+1+\tau^2}$

$= \sqrt{\tau+1-4\tau+4+1+\tau+1}$

$= \sqrt{7-2\tau}$

longR OBTUSE



$\vec{h} = (\tau, \tau-1, 0)$ $\qquad \vec{i} = 0, \tau, \tau-1$

$\vec{5} = (1, \tau, 0)$

$8 = (\tau, 0, -1)$

$l = \left(\frac{3}{1+\tau}\right)\left(\frac{1}{3}\left(r_{3,5,8} : \begin{matrix} 0, & 1, & -\tau \\ 1 & \tau & 0 \\ \tau & 0 & -1 \\ \hline \tau+1 & \tau+1 & -(\tau+1) \end{matrix}\right)\right) = (1, 1, -1)$

$= (1, 1, -1)$

$\therefore \vec{C} = \vec{5i} = \cancel{(0, \tau, \tau-1)} (0, \tau, \tau-1) - (1, \tau, 0)$
$= (-1, 0, \tau-1)$

$\vec{C} \cdot \vec{A} \times \vec{B} = (-1, 0, \tau-1) \cdot$
$(\tau-1, -1, 0) \times (0, 1-\tau, -1)$

$\vec{A} = \vec{5h} = (\tau, \tau-1, 0) - (1, \tau, 0)$
$= (\tau-1, -1, 0)$

$\vec{B} = \vec{5l} = (1, 1, -1) - (1, \tau, 0) = (0, 1-\tau, -1)$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \tau-1 & -1 & 0 \\ 0 & -(\tau-1) & -1 \end{vmatrix} = \cancel{\hat{i}(1) - \hat{j}(-1)(\tau-1)^2 \hat{k}}$

$= \hat{i}(1) - \hat{j}(-1)(\tau-1) + \hat{k}(-1)(\tau-1)^2$

$= (1, \tau-1, -[\tau^2 - 2\tau + 1])$

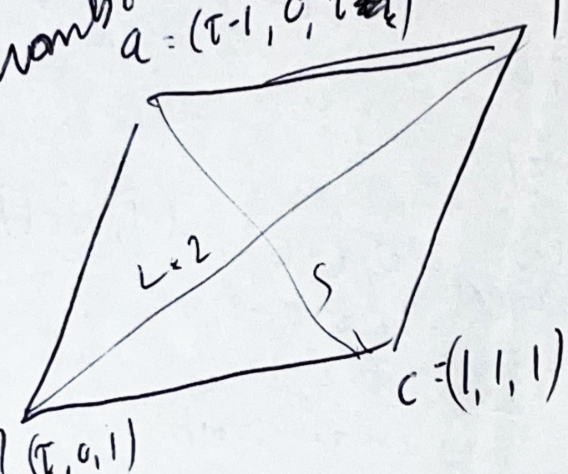
$= (1, \tau-1, -(\tau+1-2\tau+1))$

$= (1, \tau-1, -(2-\tau)) = \boxed{(1, \tau-1, \tau-2)}$

$\boxed{vol_{longR} = 2(\tau-1)}$

$\vec{C} \cdot A \times B = (-1, 0, \tau-1) \cdot (1, \tau-1, \tau-2) = -1 + 0 + (\tau-1)(\tau-2) = -1 + \tau^2 - 3\tau + 2 = -\cancel{1} + \tau + \cancel{1} - 3\tau + 2$
$= -2\tau + 2$

ACUTE rhombohedron



$L = \vec{71} = |(\tau, -1, 1-\tau)| = \sqrt{\tau^2+1+\tau^2-2\tau+1}$

$= \sqrt{\tau+1+1+\cancel{\tau}+1-\cancel{2\tau}+1}$

$= 2$

~~$S = \vec{ca} = |(\tau-2, -1, \tau)| = \sqrt{\tau^2-4\tau+4+1+\tau^2}$~~

$= \sqrt{\cancel{\tau}+1-4\tau+4+1+\cancel{\tau}+1}$

$= \sqrt{-2\tau+7}$

$= 1.94...$

$S = \vec{ca} = |(\tau-2, -1, \tau-1)|$

$= \sqrt{\tau^2-4\tau+4+1+\tau^2-2\tau+1}$

$= \sqrt{\cancel{\tau}+1-4\tau+4+1+\cancel{\tau}+1-\cancel{2\tau}+1}$

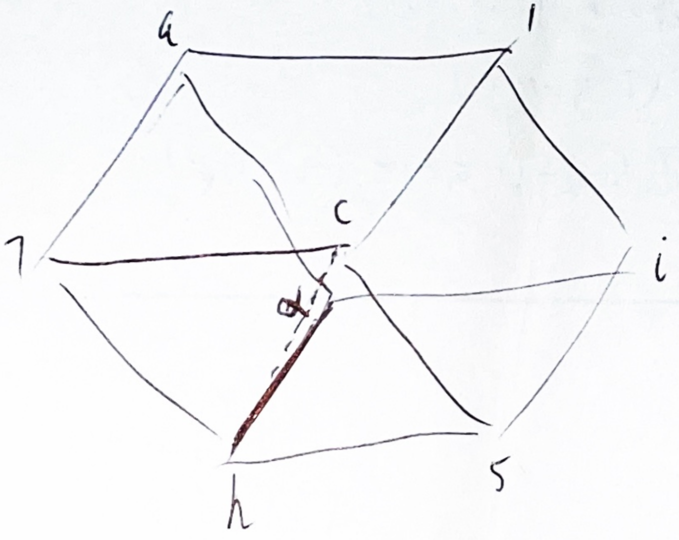
$= \sqrt{8-4\tau}$

$= 1.23606797$

~~$\sqrt{(1+\tau)^2+1}$~~ $\sqrt{(\tau-1)^2+1+0} = \sqrt{\tau^2-2\tau+1+1}$

$= \sqrt{\tau+1-2\tau+2}$

$= \sqrt{3-\tau}$



$S = \sqrt{8-4\tau}$

$L = 2$

$\frac{L}{S} = \tau = \frac{2}{\sqrt{8-4\tau}}$

$\therefore \sqrt{8-4\tau} = \frac{2}{\tau}$

$8-4\tau = \frac{4}{\tau^2} = \frac{4}{\tau+1}$

$8\tau+8-4\tau^2-4 = 4$

$8\tau+8-4\tau-4-4 = 4$

$\frac{1}{\tau} = .618$

$\varphi = 4($