$\rho_1 = r_1 \cos\alpha + R_1 \sin\alpha$

$\rho_2 = r_2 \cos\alpha + R_2 \sin\alpha$

$\rho_{12} \equiv \rho_2 - \rho_1 = r_{12}\cos\alpha + R_{12}\sin\alpha$

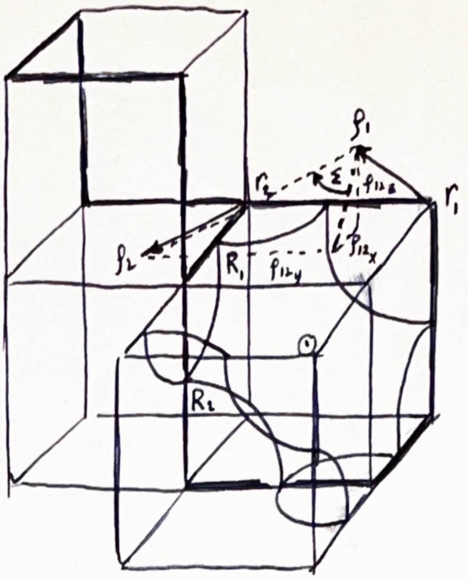
$r_1 = (-1\,1\,1) \quad R_1 = (0\,{-1}\,1)\eta = (0\,{-\eta}\,\eta)$

$r_2 = (-1\,{-1}\,1) \quad R_2 = (1\,{-1}\,0)\eta = (\eta\,{-\eta}\,0)$

$r_{12} = (0\,{-2}\,0)\,; \quad R_{12} = (\eta\,0\,{-\eta})$

$\rho_{12} = (0\,{-2}\,0)\cos\alpha + (\eta\,0\,{-\eta})\sin\alpha$

$= (\eta\sin\alpha,\ -2\cos\alpha,\ -\eta\sin\alpha)$




For a locally finite associate surface, the x-, y-, and z- components of $\vec{\rho}_{12}$ must satisfy the relations

$$\underbrace{(\rho_{12})_{x \text{ or } z}}_{\substack{\frac{1}{2}\times\text{ diameter of circumhelix} \\ =\ \underline{\text{radius of circumhelix}}}} \div \underbrace{(\rho_{12})_y}_{\substack{\frac{1}{4}\times\text{ pitch} \\ \text{of circumhelix}}} = \text{rational \#}.$$

Hence $\dfrac{\eta\sin\alpha}{2\cos\alpha}$ = rational number, or $\dfrac{\eta}{2}\tan\alpha$ = rational number. Hence $\boxed{\eta\tan\alpha = \text{rational number}}$

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$\omega$: $\tan\Sigma$ = steepness of regular helical polygon hole

$$= \frac{|(\rho_{12})_y|}{\sqrt{(\rho_{12})_x^2 + (\rho_{12})_y^2}} = \frac{2\cos\alpha}{\sqrt{2\eta^2\sin^2\alpha}} = \frac{\sqrt{2}}{\eta}\,\text{ctn}\,\alpha$$

$\tan\theta_c = \dfrac{1}{1.279\,261\,57} = .781\,700\,9621$

Hence $\theta_c \approx 38.014\,774\,01^\circ$

$= .663\,482\,970\,1$ rad.

Hence $\tan\Sigma = \dfrac{1}{\left(\frac{\eta\tan\alpha}{\sqrt{2}}\right)}$

But $\dfrac{\eta\tan\alpha}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\,n$ ($n$ = rat. #)

Therefore $\boxed{\sqrt{2}\tan\Sigma = \text{rational number}}$

Since $\dfrac{\eta\tan\alpha}{\sqrt{2}} = \dfrac{1}{\tan\Sigma} = \sqrt{2}\times$ rational #,

$\boxed{\eta\tan\alpha = \text{rational \#}}$

$\eta \approx 1.279\,261\,57\ldots$ (Schwarz, p. 88)

[$\eta$ is about 1.27 or so.
It is the dilation of the $\{6\}$ face of P, inscribed in the same cube as the $\{\tilde{6}\}$ face of D, required to make arc lengths invariant under the adjoint transformation.]