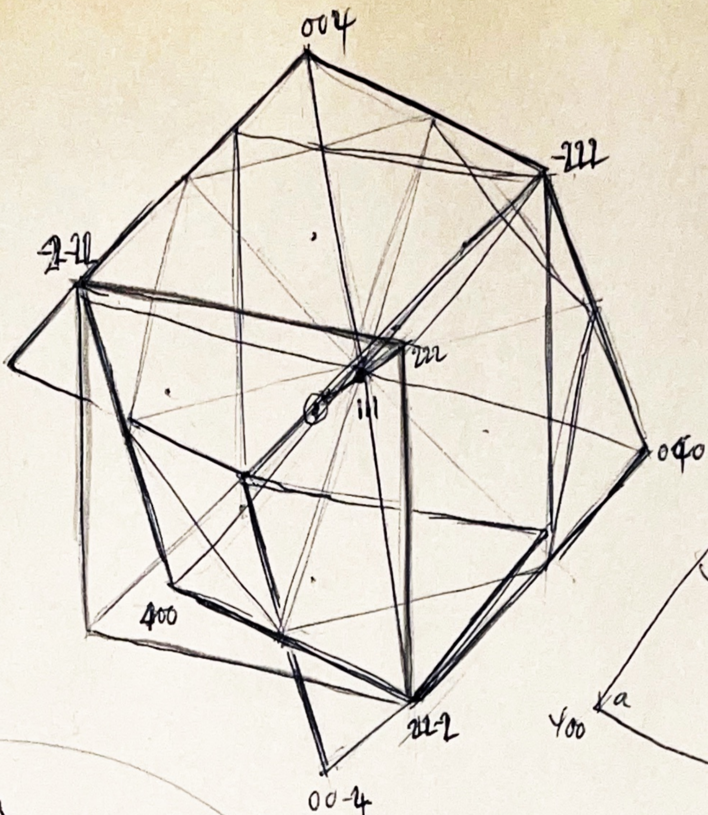
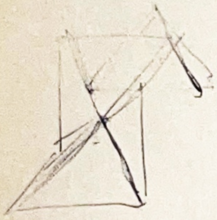
$$\cos\theta = \frac{\vec{ba}\cdot\vec{bc}}{|ba|\cdot|bc|}$$

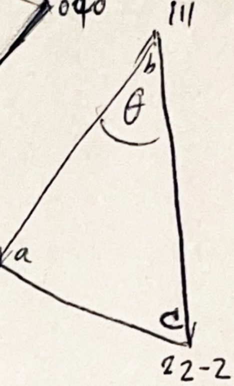
$$= \frac{(3\ {-1}\ {-1})\,(1\ 1\ {-3})}{11}$$

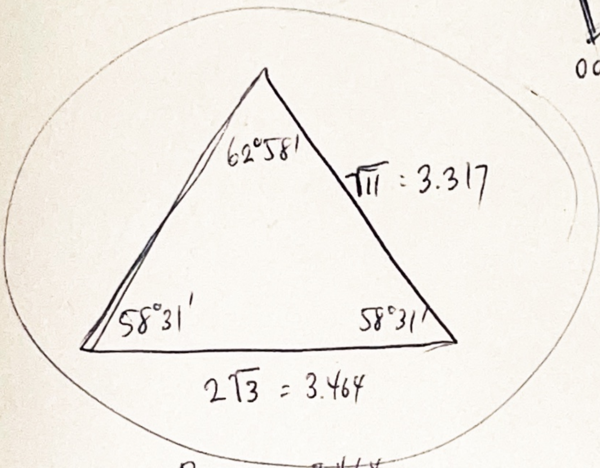
$$= \frac{3-1+3}{11} = +\frac{5}{11}$$

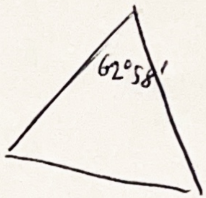




11







$|\vec{ac}| = |(-2\ 2\ {-2})|$

$= \sqrt{4\cdot 3} = 2\sqrt{3}$

$$\begin{array}{r} 1.732 \\ \underline{2} \\ 3.464 \end{array}$$

$$\begin{array}{r} 180 \\ \underline{62°58'} \\ 2\,)\,117°02' \\ 58°31' \end{array}$$

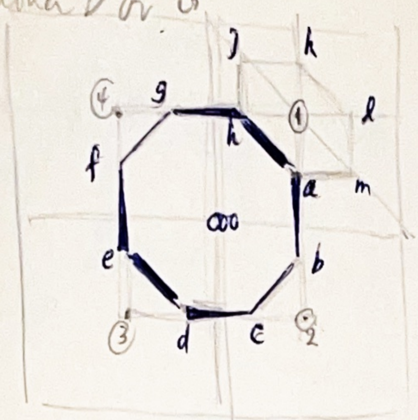
$$\frac{\text{Base}}{\text{Side}} = \frac{3.464}{3.317} \simeq 1.0443$$

☆ Isosceles triangles which fit together to define flat-faceted regular skew hexagon with 109.5° vertex angles.

$t\{4,6\}$ - 8 8

applied to D

(probably doesn't exist on either D or G)



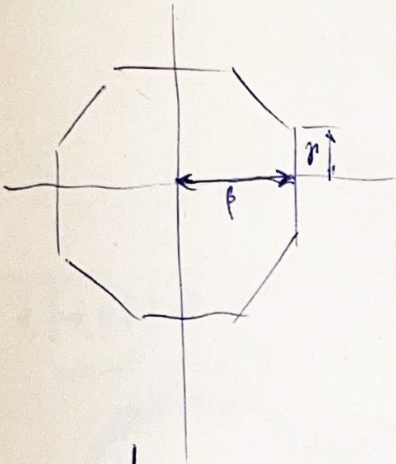
Original octagon is quasi-regular

① (111) ② (-11-1) ③ (1-1-1) ④ (-1-11)

$a \to (\alpha, \beta, \gamma)$

$b \to (-\alpha,$

$c \to (\alpha, \gamma, -\beta)$



Determine position of b:

$\boxed{u^2 + v^2 = \beta^2 + \gamma^2}$ (b lies in same cylinder as a & c)

ab = bc: $|(-2\alpha, u-\beta, v-\gamma)| = |(2\alpha, \gamma-u, -\beta-v)|$

$\cancel{4\alpha^2} + (u-\beta)^2 + (v-\gamma)^2 = \cancel{4\alpha^2} + (\gamma-u)^2 + (-\beta-v)^2$

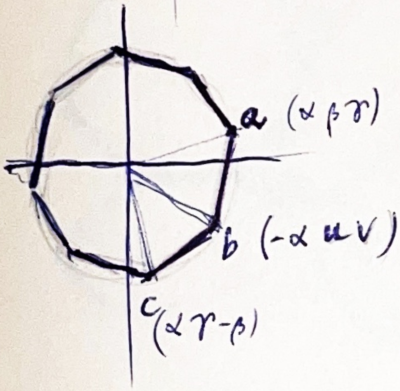
$\cancel{u^2} - 2\beta u + \cancel{\beta^2} + \cancel{v^2} - 2v\gamma + \cancel{\gamma^2}$

$= + \cancel{u^2} + \cancel{\beta^2} + \cancel{v^2} + \cancel{\gamma^2} - 2u\gamma + 2\beta v$

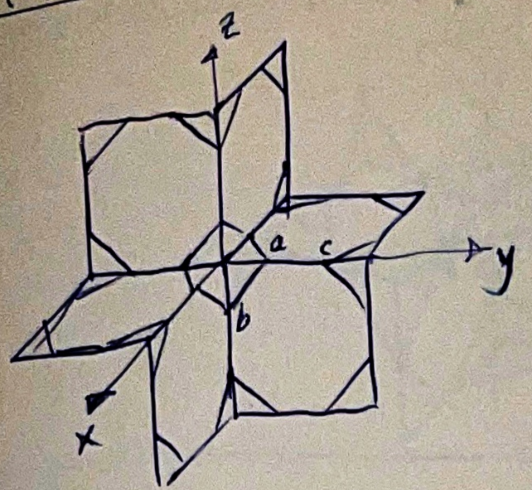
or $-2\beta u - 2v\gamma = -2u\gamma + 2\beta v$

$\boxed{u\beta + v\gamma = u\gamma - v\beta}$

Not finished



$t\{4,6\} = 6\cdot 8^2$ applied to P (not very hard!)



a: $(0\ \beta\ 0)$

b: $(0\ 0\ -\beta)$

c: $(0, 1-\beta, 0)$

$ac = ab$:

$1-2\beta = \sqrt{2\beta^2}$

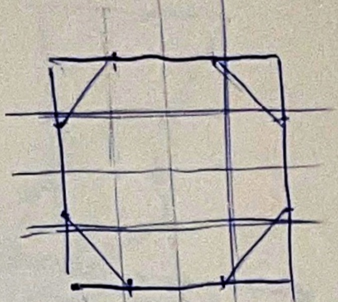
$1-4\beta+4\beta^2 = 2\beta^2$

$1-4\beta+2\beta^2 = 0$

$2\beta^2-4\beta+1=0$

$$\beta = \frac{4 \pm \sqrt{16-8}}{4} = \frac{4\pm\sqrt{8}}{4} = 1 \pm \frac{2\sqrt{2}}{4} = 1 \pm \frac{\sqrt{2}}{2}$$

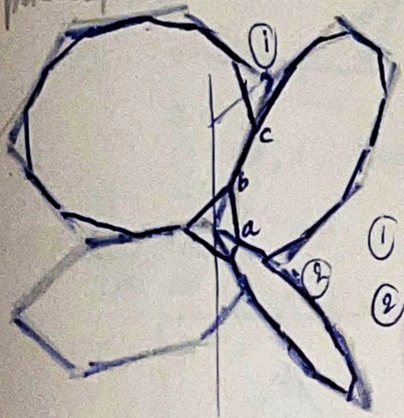
$\boxed{\beta = 1-\frac{\sqrt{2}}{2}} \cong$ 1 − .707 = .293



$t\{6,4\} = 4\cdot 12^2$

probably no G or P

applied to P



① : −101

② = 110

a $(\alpha\ \alpha\ 0)$

b $(-\alpha\ 0\ \alpha)$

c $(-1+\alpha, 0, 1-\alpha)$

$ab = bc$ $|(-2\alpha, -\alpha, \alpha)|$

$= |(-1+2\alpha, 0, 1-2\alpha)|$

$4\alpha^2+\alpha^2+\alpha^2$

$= 2(1-4\alpha+4\alpha^2) = 2-8\alpha+8\alpha^2$

$\therefore 6\alpha^2 = 8\alpha^2 - 8\alpha + 2$

$2\alpha^2 - 8\alpha + 2 = 0$

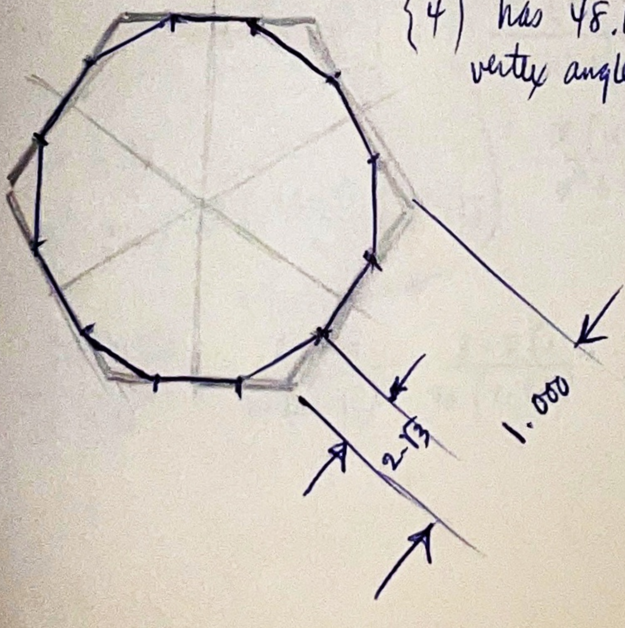
$\boxed{\alpha^2 - 4\alpha + 1} = 0$

$$\alpha = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \frac{\sqrt{12}}{2} = 2 \pm \frac{2\sqrt{3}}{2} = 2-\sqrt{3}$$

$\{\tilde{4}\}$ has 48.19° vertex angles.

$\boxed{\alpha = 2-\sqrt{3}}$

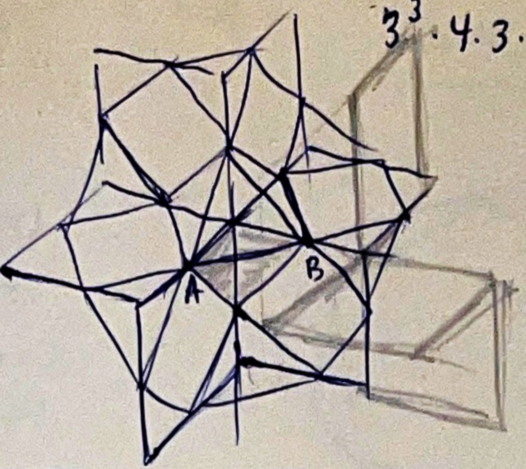
2 − 1.732 = .268



$t\{6,6\} = 6 \cdot 12^2$ on D Flat $\{12\}$ & skew $\{\tilde{6}\}$

s{4,6} on P

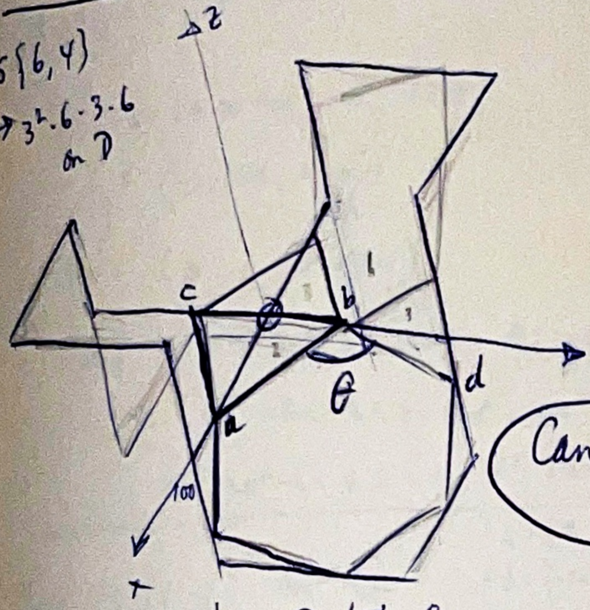
$3^3.4.3.4$



Inscribing 4 equil. Δ's in {6} requires alternating placement of lines equiv to AB — e.g., in one labyrinth, AB lines are all across concave dihedral angles, as shown here.

---

s{6,4}
→ $3^2.6.3.6$ on D



a $(\alpha\ 0\ 0)$

b $(0,\ 1-\alpha,\ 0)$

c $(0,\ -1+\alpha,\ 0)$

$ab = bc$

$|(-\alpha,\ 1-\alpha,\ 0)| = |(0,\ -2+2\alpha,\ 0)|$

$\alpha^2+(1-\alpha)^2 = (-2+2\alpha)^2$

$\cancel{\alpha^2}+\cancel{1}-\cancel{2\alpha}+\cancel{\alpha^2} = \cancel{4}-\cancel{8\alpha}+\cancel{4\alpha^2}$

$+3 \quad -6\alpha \quad 2\alpha^2$

$\alpha^2-3\alpha+1=0$

$\alpha = \frac{3\pm\sqrt{9-4}}{2} = \frac{3\pm\sqrt{5}}{2}$

$\alpha \cong .382 \quad = 1.5 \pm \frac{2.236}{2}$

$= 1.5 \pm 1.118$

$1.5 - 1.118 = .382$

Can this be skewed?

$\alpha = .634$

$2\alpha^2-6\alpha+3=0$

$\alpha = \frac{6\pm\sqrt{36-24}}{4} = \frac{3}{2}\pm\frac{2\sqrt{3}}{4}$

$= 1.5 \pm \frac{\sqrt{3}}{2}$

$\rightarrow\ = 1.5 - \frac{\sqrt{3}}{2}$

$1.500 - .866 = .634$

$1.732 \times 2 = 3.464$

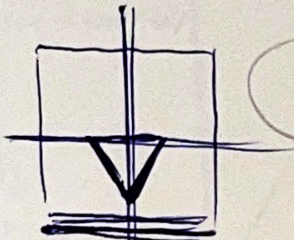
$2 - 1.732 = .268$

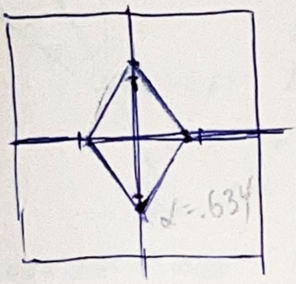
3

b = 0 $1-\alpha$ 0

$d = (0,\ 1,\ -1+\alpha)$

$\cos\theta = \frac{\vec{ba}\cdot\vec{bd}}{ba\cdot bd}$





$\cos\theta = \frac{(\alpha,\ -1+\alpha,\ 0)\cdot(0,\ \alpha,\ -1+\alpha)}{\alpha^2+(1-\alpha)^2}$

$= \frac{\alpha(\alpha-1)}{\alpha^2+1-2\alpha+\alpha^2} = \frac{\alpha^2-\alpha}{2\alpha^2-2\alpha+1} = \frac{\alpha(\alpha-1)}{2\alpha^2-2\alpha+1}$

$= \frac{\left(\frac{3-\sqrt{3}}{2}\right)\left(\frac{1-\sqrt{3}}{2}\right)}{12-6\sqrt{3}-(3-\sqrt{3})+1}$

$\alpha = \frac{3-\sqrt{3}}{2}$

$\alpha^2 = \frac{9-6\sqrt{3}+3}{2} = \frac{12-6\sqrt{3}}{2} = 6-3\sqrt{3}$

$= \frac{\frac{3+3-4\sqrt{3}}{4}}{10-5\sqrt{3}} = \frac{6-4\sqrt{3}}{20(2-\sqrt{3})} = \frac{3-2\sqrt{3}}{40(2-\sqrt{3})} = \frac{-.464}{40(.268)} = -.0433$

$\boxed{\theta \cong 92.4°}$

~~s{4,6}~~

$s\{6,4\}$ on P $3^2\cdot 6\cdot 3\cdot 6$

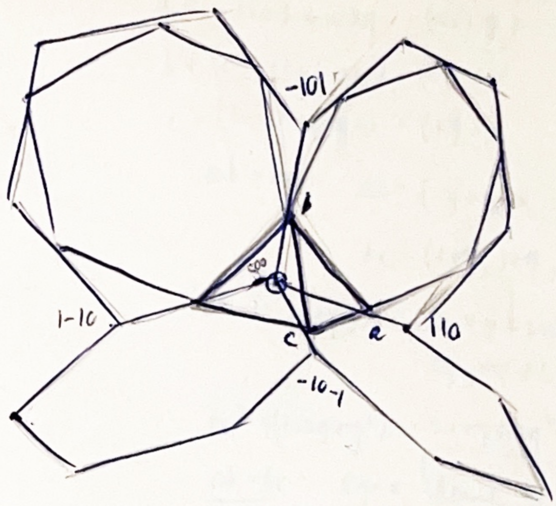
(11-16-98 This looks like a Feb 1971 calculation.)

$\boxed{\text{All flat faces}}$

$a = (\alpha\ \alpha\ 0)$

$b = (-1\,0\,1) + (\alpha\ 0\ -\alpha)$
$= (-1+\alpha,\ 0,\ 1-\alpha)$

$c = (-1\,0\,-1) + (\alpha\ 0\ \alpha)$
$= (-1+\alpha,\ 0,\ -1+\alpha)$



ab = bc :

$ab = (-1+\alpha-\alpha,\ -\alpha,\ 1-\alpha)$
$= (-1, -\alpha, 1-\alpha)$

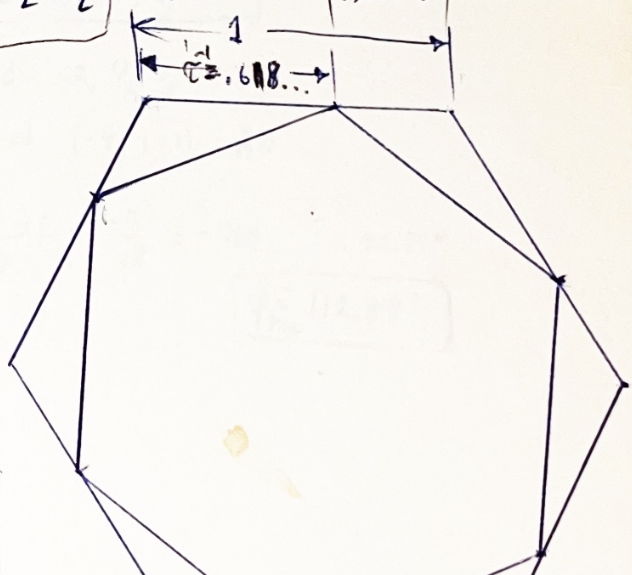
$bc = (0,\ 0,\ -2+2\alpha)$

Hence $\cancel{1}+\alpha^2+\cancel{1}-2\alpha+\alpha^2 = 4(1-2\alpha+\alpha^2)$

or $2\alpha^2-2\alpha+2 = 4-8\alpha+4\alpha^2$

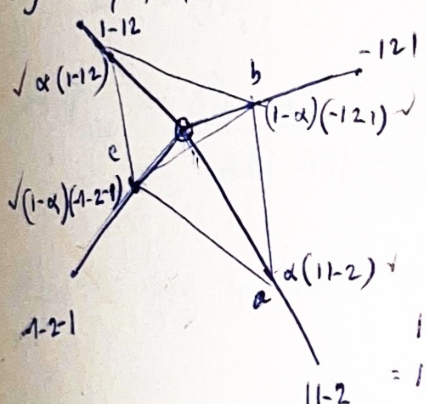
a $2\alpha^2-6\alpha+2=0$ $\alpha^2-3\alpha+1=0$ $\boxed{\alpha = \frac{3}{2}-\frac{\sqrt{5}}{2}}$ $\simeq$ .381966 $= (1-\tau^{-1})$!! 11-16-98

$\tau = \frac{\sqrt{5}-1}{2} = 1.618033989$ $\tau^{-1} = \tau - 1 = .6180\ldots$



Try $s\{6,4\}$ on G.

$a = (\alpha, \alpha, -2\alpha)$
$b = (-1+\alpha,\ 2-2\alpha,\ 1-\alpha)$
$c = (-1+\alpha,\ -2+2\alpha,\ -1+\alpha)$



ab = bc

$ab = (-1,\ 2-3\alpha,\ 1+\alpha)$
$bc = (0,\ -4+4\alpha,\ -2+2\alpha)$

$1+4-12\alpha+9\alpha^2+1+2\alpha+\alpha^2$
$= 16-32\alpha+16\alpha^2+4-8\alpha+4\alpha^2$

$10\alpha^2-10\alpha+6 = 20\alpha^2-40\alpha+20$: $10\alpha^2-30\alpha+14=0$
$5\alpha^2-15\alpha+7=0$

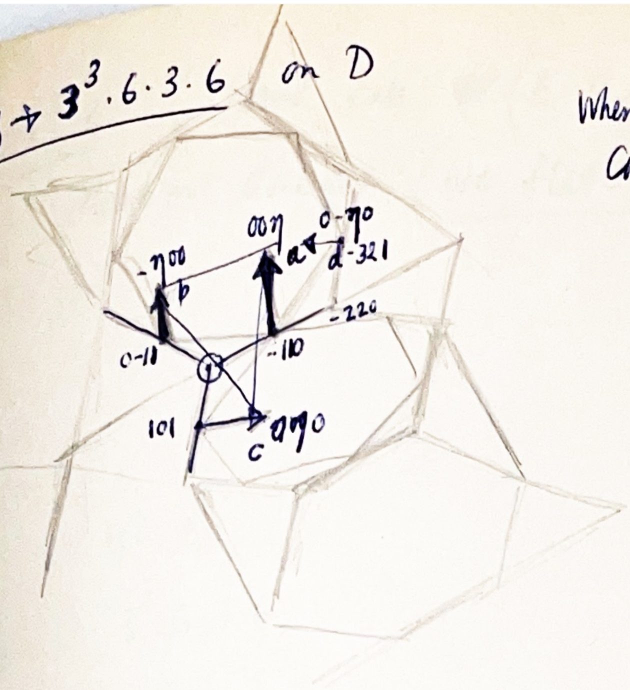
$\alpha = \frac{15\pm\sqrt{225-140}}{10} = \frac{3}{2}\pm\frac{\sqrt{85}}{10}$ 9.220 $= 1.5000 - .9220$
.578045554...

$\boxed{\alpha = .578}$

$(3^2\cdot 6\cdot 3\cdot 6)$ G $\boxed{\alpha = \frac{3}{2}-\frac{\sqrt{85}}{10} \simeq .578\,045\,554\ldots}$ All flat faces

(This calculation was checked and confirmed on 11/16/98.)

s{6,6} → $3^3.6.3.6$ on D



When $\eta = 0$, abc is not a {3}

Compute value of η for abc → {3}

$a = (-110) + 00\eta = (-1\,1\,\eta)$

$b = (0{-}11) + (-\eta\,00) = (-\eta\,{-1}\,1)$

$c = (101) + (0\eta 0) = (1\,\eta\,1)$

$ab = bc$ $\quad ab = (-\eta+1, -2, 1-\eta)$

$bc = (1+\eta, 1+\eta, 0)$

$ab^2 = 2(1-2\eta+\eta^2) + 4 = 2 - 4\eta + 2\eta^2 + 4$

$= 2\eta^2 - 4\eta + 6$

$bc^2 = 2(1+2\eta+\eta^2) = 2 + 4\eta + 2\eta^2$

$\underline{ab = bc}$: $8\eta = 4$ $\quad \boxed{\text{Hence } \eta = 1/2}$

Hence $2a = (-2\,2\,1) = A$ $\quad AB = (1\,{-4}\,1)$

$2b = (-1\,{-2}\,2) = B$ $\quad BC = (3\,3\,0)$ $\quad$ lengths are equal

$2c = (2\,1\,2) = C$ $\quad CA = (-4\,1\,{-1})$

$2d = \boxed{(-6, 3, 2) = D}$

d

$d = -321 + 0{-}\eta 0 = -3, 2-\eta, 1 = (-3, \frac{3}{2}, 1)$

$ad : a = (-1\,1\,\frac{1}{2})$ $\quad (-1\,1\,\frac{1}{2})^a$

$ad = -2, 1/2\ 1/2$

$2 \cdot ad = (-4, 1, 1) = AD$

$$\cos\varphi_{hex} = \frac{\vec{AB}\cdot\vec{AD}}{|AB|\,|AD|} = \frac{(1{-}41)\cdot(-411)}{18} = \frac{-4-4+1}{18} = \frac{-7}{18} = -.389$$

22.89°

$\boxed{\varphi_{hex} \cong 112.89°}$

Have not yet applied to P & G

$\left\{\begin{matrix}4\\6\end{matrix}\right\} \to \underline{4^3\cdot 6}$ (Should call $(6\cdot4\cdot4\cdot4)$)

Have not done yet. (<u>No flat-faced forms, on D, P, or G.</u>)

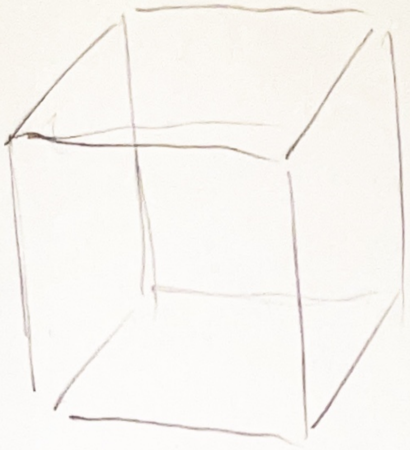
$4^3\cdot 6$

$3^3\cdot6\cdot3\cdot6$

$3^2\,4\cdot3\cdot6$

$3^3\cdot4\cdot3\cdot4$

.6·3·6 on G (4-15-07)



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adv in Sci. Am, not yet available Jan '68.   ≤ 40" width

"Sprocketed film" necessary? (Noll)
registration pins are standard with   Arriflex 16 mm | Mark McLean
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Andy Marchetti, Kingston

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Stereo outfit projection & camera lens
" Keystone adapter rings

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2) hand viewer — for looking at still pictures — & step up ring

3) projection components
   a) polarizer
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