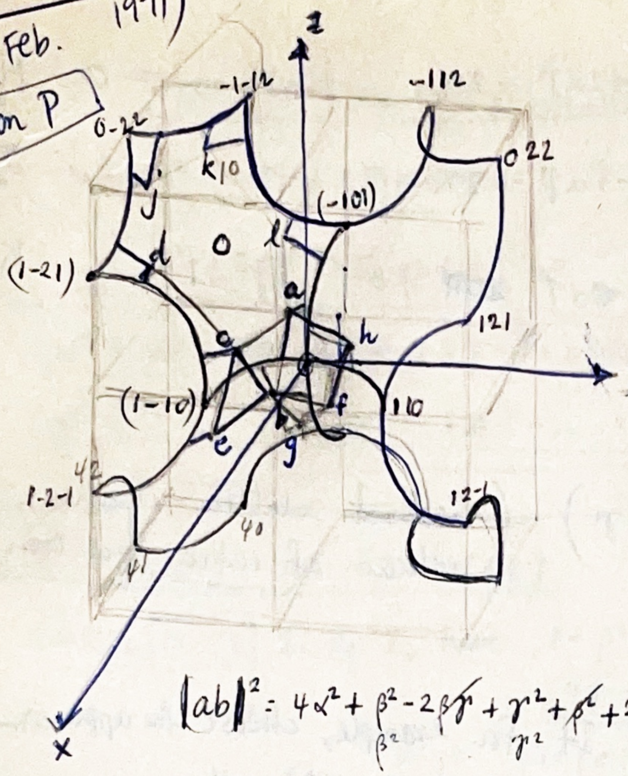
Apollo 14 day
(Feb. 1971)

$\boxed{(3^2\cdot 4\cdot 3\cdot 6) \text{ on } P}$

α, β, & γ are here treated as <u>inherently positive numbers</u>.



$\vec{a} = (+\alpha, -\beta, \gamma)$

$\vec{b} = (-\alpha, -\gamma, -\beta)$

$\vec{c} = (1-\gamma, -1-\alpha, \beta)$

$\vec{d} = (1-\beta, -2+\gamma, 1+\alpha)$

$\vec{e} = (1-\beta, -1+\alpha, -\gamma)$

$\vec{f} = (\alpha, \beta, -\gamma)$

$\vec{g} = (-1+\gamma-\beta, -1-\alpha)$

$\boxed{ab = bc = ca}$

$\vec{ab} = (-2\alpha, -\gamma+\beta, -\beta-\gamma)$

$\vec{bc} = (1-\gamma+\alpha, -1-\alpha+\gamma, 2\beta)$

$\vec{ca} = (\alpha-1+\gamma, -\beta+1+\alpha, \gamma-\beta)$

$|ab|^2 = 4\alpha^2 + \underset{\beta^2}{\beta^2} - 2\cancel{\beta\gamma} + \underset{\gamma^2}{\gamma^2} + \cancel{\beta^2} + 2\cancel{\beta\gamma} + \cancel{\gamma^2} = 4\alpha^2 + 2\beta^2 + 2\gamma^2$ ✓

$|bc|^2 = 1 + 2(\alpha-\gamma) + (\alpha-\gamma)^2 + 1 + 2(\alpha-\gamma) + (\alpha-\gamma)^2 + 4\beta^2$

$= 2[1 + 2\alpha - 2\gamma + \alpha^2 - 2\alpha\gamma + \gamma^2] + 4\beta^2 = 2 + 4\alpha - 4\gamma + \underline{2\alpha^2} - \underline{4\alpha\gamma} + \underline{2\gamma^2} + \underline{4\beta^2}$

$= 2\alpha^2 + 4\beta^2 + 2\gamma^2 - 4\alpha\gamma + 4\alpha - 4\gamma + 2$ ✓

$|ca|^2 = 1 - 2(\alpha+\gamma) + \alpha^2 + 2\alpha\gamma + \gamma^2 + 1 + 2(\alpha-\beta) + \alpha^2 - 2\alpha\beta + \beta^2 + \gamma^2 - 2\beta\gamma + \beta^2$

$= 1 - 2\alpha - 2\gamma + \alpha^2 + 2\alpha\gamma + \gamma^2$

$\underline{+1 + 2\alpha \quad + \alpha^2 \quad + \gamma^2 - 2\beta - 2\alpha\beta + \beta^2 - 2\beta\gamma + \beta^2}$

$= \underline{2} - \underline{2\gamma} + \underline{2\alpha^2} + \underline{2\alpha\gamma} + \underline{2\gamma^2} - \underline{2\beta} - \underline{2\alpha\beta} + \underline{2\beta^2} - \underline{2\beta\gamma}$

$= 2\alpha^2 + 2\beta^2 + 2\gamma^2 - 2\alpha\beta - 2\beta\gamma + 2\alpha\gamma - 2\beta - 2\gamma + 2$

I. $\underline{|ab|^2 = |bc|^2}$: $\overset{2\alpha^2}{4\cancel{\alpha^2}} + \overset{-2\beta^2}{\cancel{2\beta^2}} + \cancel{2\gamma^2} = \cancel{2\alpha^2} + \cancel{4\beta^2} + \cancel{2\gamma^2} - 4\alpha\gamma + 4\alpha - 4\gamma + 2$

$2\alpha^2 - 2\beta^2 + 4\alpha\gamma - 4\alpha + 4\gamma - 2 = 0$, or $\boxed{\alpha^2 - \beta^2 + 2\alpha\gamma - 2\alpha + 2\gamma - 1 = 0}$ ✓

II. $|bc|^2 = |ca|^2$ $\quad \cancel{2\alpha^2} + \cancel{4\beta^2} + \cancel{2\gamma^2} - 4\alpha\gamma + \cancel{4\alpha} - \cancel{4\gamma} + \cancel{2} =$

$\cancel{2\alpha^2} \ \cancel{+2\beta^2} \ \cancel{+2\gamma^2} + 2\alpha\gamma \qquad \underline{-2\alpha\beta} \ \underline{-2\beta\gamma} \ \underline{-2\beta} \ \cancel{-2\gamma} + \cancel{2}$

$\underline{-2\beta^2} \qquad \underline{+6\alpha\gamma} \qquad \underline{-4\alpha} \ \cancel{+4\gamma} - 2 \qquad \underline{+2\gamma}$

$2\beta^2 - 6\alpha\gamma + 2\alpha\beta + 2\beta\gamma + 4\alpha + 2\beta - 2\gamma - \cancel{2} = 0$

$\boxed{\beta^2 - 3\alpha\gamma + \alpha\beta + \beta\gamma + 2\alpha + \beta - \gamma = 0}$ ✓

III. $|ca|^2 = |ab|^2$ $\quad \boxed{\alpha^2 + \alpha\beta + \beta\gamma - \alpha\gamma + \beta + \gamma - 1 = 0}$ ✓

$\boxed{\alpha^2 + 3\beta^2 + 2\gamma^2 + 2a - 2b - 4c - 2ab - 2bc + 3 = 0}$ $\boxed{11/13/98}$

nonsense

☆ I just now discovered this old 1971 error.

$$\text{Ⅰ}\quad \alpha^2 - \beta^2 + 2\alpha\gamma - 2\alpha + 2\gamma - 1 = 0 \qquad H_\alpha(\beta,\gamma)=$$

$$\text{Ⅱ}\quad \beta^2 + \alpha\beta + \beta\gamma - 3\alpha\gamma + 2\alpha + \beta - \gamma = 0 \qquad J_\alpha(\beta,\gamma)=0$$

$$\text{Ⅲ}\quad \alpha^2 + \alpha\beta + \beta\gamma - [illegible]\alpha\gamma\ [illegible] + \beta + \gamma - 1 = 0 \qquad K_\alpha(\beta,\gamma)=$$

Solutions for $\vec{R} = (\alpha\beta\gamma)$ (General solution requires solution of *cubic* equation)

	α	β	γ	
		etc.		
(Approximate)	-2	$\frac{11}{4}$	$-\frac{1}{23}$	
(Exact)	-1	$\sqrt{2}$	0	
	$-.1$	~~[illegible]~~	~~[illegible]~~	
(Approximate)	0	$\frac{3}{10}$	$\frac{109}{200}$	$\{\tilde{4}\} \equiv \{4\}$
~Exact	0	.29558	.543683 7682	← Better! (graphical solution)
(Exact)	1	0	$\frac{1}{2}$	(5/7/78) Face 7 of $\{\tilde{6}\}$ = 60°
(Approximate)	2	$-\frac{17}{12}$	$\frac{15}{29}$	

etc.

If, for example, choose the approximate solution $(\alpha\beta\gamma) \cong (0, \frac{3}{10}, \frac{109}{200})$, then

~~E~~ $H_0(\beta,\gamma) = 0$

$J_0(\beta,\gamma) = .0085$ (*not* zero)

$K_0(\beta,\gamma) = .0085$ (*not* zero)

(not *too* far off)

III $\alpha^2 + \cancel{\alpha\beta} + \cancel{\beta\gamma} - \alpha\gamma + \cancel{\beta} + \gamma - 1 = 0$ ✓

II $\beta^2 + \cancel{\alpha\beta} + \cancel{\beta\gamma} - 3\alpha\gamma + \cancel{\beta} - \gamma + 2\alpha = 0$ ✓

III − II: $\alpha^2 - \beta^2 + 2\alpha\gamma + 2\gamma - 2\alpha$ [illegible] $1 = 0$ ✓

I $\alpha^2 - \beta^2 + 2\alpha\gamma + 2\gamma - 2\alpha - 1 = 0$ ✓

III $\alpha^2 + \alpha\beta + \beta\gamma - \alpha\gamma + \beta + \gamma - 1 = 0$

I $\alpha^2 - \beta^2 + 2\alpha\gamma + 2\gamma - 1 - 2\alpha = 0$

There are only 2 independent equations.

Hence, we have a certain amount of freedom of choice for one of the three variables.

Choose, e.g., $\alpha = -1$

Then $\boxed{(\alpha\ \beta\ \gamma) = (-1, \sqrt{2}, 0)}$

Derive allowed range for $\vec{R} = (\alpha\beta\gamma)$

Ex: Choose $\boxed{\alpha = 1}$ In eq I, have $\cancel{1} - \beta^2 + 2\gamma - 2 + 2\gamma - \cancel{1} = 0$

$$\boxed{4\gamma = \beta^2 + 2}$$

II → $\beta^2 + \beta + \beta\gamma - 3\gamma + 2 + \beta - \gamma = 0$

$$\boxed{\beta^2 + 2\beta + \beta\gamma - 4\gamma + 2 = 0}$$

$$\boxed{\beta^2 + (\gamma + 2)\beta - 4\gamma + 2 = 0}$$

III → $\cancel{1} + \beta + \beta\gamma - \cancel{\gamma} + \beta + \gamma - \cancel{1} = 0$

$$2\beta + \beta\gamma = 0$$

$$\beta(\gamma + 2) = 0$$

Let $\beta = 0$

Then from I, $\gamma = 1/2$

Solution ≡ $(\alpha\beta\gamma) = (1\ 0\ \tfrac{1}{2})$

Ex: Choose $\boxed{\alpha = 0}$

I → $-\beta^2 + 2\gamma - 1 = 0$
II → $\beta^2 + \beta\gamma + \beta - \gamma = 0$
III → $\beta\gamma + \beta + \gamma - 1 = 0$

Hence $\gamma = \frac{\beta^2 + 1}{2}$ & $\boxed{\beta^3 + \beta^2 + 3\beta - 1 = 0}$

Let $F(\beta) = \beta^3 + \beta^2 + 3\beta - 1 \ (\equiv 0)$.

	β	$F(\beta)$
0	0	−1
.25	$\frac{1}{4}$	$-\frac{11}{64}$
.3	$\frac{3}{10}$	$\frac{17}{1000}$
.3333	$\frac{1}{3}$	$\frac{4}{27}$

Hence $\beta \cong .3$

$\therefore \gamma \cong \frac{109}{200}$

Check $I(\beta, \gamma) \equiv 0$ (!)

$II(\beta, \gamma) = \frac{17}{2000} = .0085$

$III(\beta, \gamma) = \frac{17}{2000} = .0085$

Hence solution is

APPROXIMATELY

$$(\alpha\beta\gamma) \cong \left(0, \frac{3}{10}, \frac{109}{200}\right)$$

This corresponds to flat square faces for the {4}'s.

★

$\vec{j} = [0+(-\alpha), -2+\beta, 2+(-\gamma)] = (-\alpha, -2+\beta, 2-\gamma)$

$\vec{k} = [-1+\gamma, -1+\alpha, 2-\beta] = (-1+\gamma, -1+\alpha, 2-\beta)$

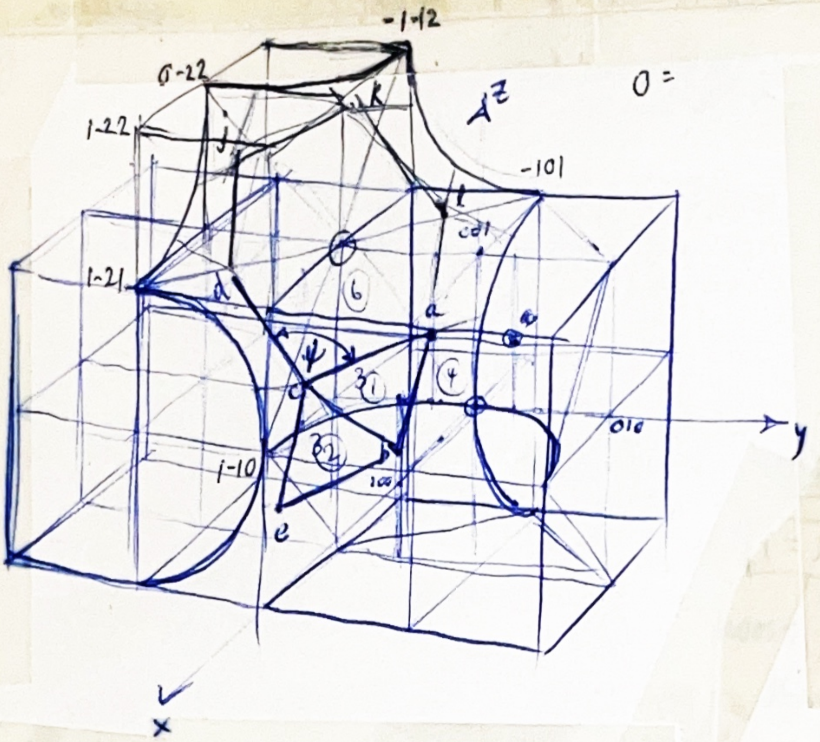
$\vec{l} = [-1+\beta, 0-\gamma, 1-\alpha] = (-1+\beta, -\gamma, 1-\alpha)$

$0 =$

$6|3 \cong 160°$

$3|3 \cong 129°$

$4|3 \cong 110°$



$n_{3_2} = (.382, .529, .113)$

$n_{3_1} = (.529, .156, -.113)$

$n_6 = (.608, .382, .024)$

$n_4 = (100)$

$a = (0, -\frac{3}{10}, \frac{1}{2})$

$b = (0 -\frac{1}{2} -\frac{3}{10})$

$c = (\frac{1}{2}, -1, \frac{3}{10})$

$d = (1-\beta, -2+\gamma, 1+\alpha)$ $\quad 0\ \frac{3}{10}\ \frac{1}{2}$

$= (\frac{7}{10}, -\frac{3}{2}, 1)$

Let $\psi = \measuredangle[\vec{cd}, \vec{ca}]$

$a \propto (0\,-3\,5)$

$b \propto (0\,-5\,-3)$

$c \propto (5\,-10\,3)$

$d \propto (7\,-15\,10)$

$\vec{cd} : (2\,-5\,7)$

$\vec{ca} : (-5\,7\,2)$

$\underset{\text{approx}}{\psi} = \frac{\vec{cd}\cdot\vec{ca}}{(cd)(ca)} = \frac{-10-35+14}{4+25+49} = -\frac{31}{78} \cong .3974$

Hence $\psi \cong 113°25'$ (Shucks!)

Improved values. $\vec{e} = (1-\beta, -1+\alpha, -\gamma) = (1-.29558, -1, -.54368) = (.70442, -1, -.54368)$

$\vec{ce} = \left(\begin{array}{c}.70442\\-.45632\end{array}, \begin{array}{c}-1\\+1\end{array}, \begin{array}{c}-.54368\\-.29558\\ \hline -.83926\end{array}\right) = (.24810, 0, \cancel{.24810}\ -.83926)$

$\vec{ce}\times\vec{cb} \equiv \vec{n}_{3_2} = \begin{vmatrix} i & j & k \\ .248 & 0 & -.839 \\ -.456 & .456 & -.591 \end{vmatrix} = \left((+.839)(.456), +\{\overset{-.1466}{(+.248)(-.591)} + \overset{.3825}{(.456)(+.839)}\}, \overset{.113}{\{(.248)(.456)\}}\right)$ $\quad .839^2 = .704$

$(+.3825, \cancel{.3564}\ .5291, .113)$

$b = (-\alpha -\gamma -\beta)$
$= (0, -.54368, -.29558)$

With <u>improved</u> solution $(\alpha\ \beta\ \gamma) = \boxed{(0, .29558, .54368)}$ ☆

have

$a = (0, -.29558, .54368)$ ✓

$c = (\overset{.45632}{\cancel{.54368}}, -1, .29558)$ ✓

$d = (.70442, \overset{-1.45632}{\cancel{-1.54368}}, 1)$ ✓

$\vec{cd} = (\overset{.24810}{\cancel{.16074}}, \overset{-.45632}{\cancel{-.54368}}, .70442)$

$\vec{ca} = (\overset{-.45632}{\cancel{-.54368}}, +.70442, .24810)$

$\vec{cb} = (-.45632, +.45632, -.59116)$

Hence

$$\cos\psi = \frac{\vec{cd}\cdot\vec{ca}}{|cd||ca|} = \frac{\overset{-.11321}{(.24810)(-.45632)} + \overset{-.32144}{(-.45632)(.70442)} + \overset{.17477}{(.70442)(.24810)}}{(.24810)^2 + (.45632)^2 + (.70442)^2}$$

$|cd| \cong .876$

$$= \frac{\cancel{-.38300}\ -.25988}{061554 + .20823 + .49621 = [.76599]} \cong -.3397 \cong \cancel{-\tfrac{1}{2}}$$

Have $\psi =$

$.32144$
$.17477$
$.49621$
$-.11321$
$+.38300$

$\sin 120° \cong .866$

$-.32144$
$-.11321$
$.43465$

$.43465$
$.17477$
$.25988$

~~Hallelujah!~~

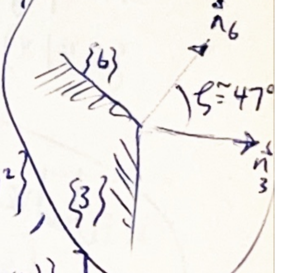
$[\cos(120°) = -1/2]$

Hence $\psi = 109°52'$

Hence ~~both quadrilaterals and hexagons~~ are ~~flat!!!~~ (WRONG)

A very interesting snub polyhedron (uniform).

What is dihedral angle between $\{6\}$ (a c d etc.) and 3 (a b c)?



$$\cancel{\vec{ac}\times}\ \vec{ca}\times\vec{cd} \underset{(def.)}{\equiv} \vec{n}_6 \cong \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -.456 & .704 & .248 \\ .248 & -.456 & .704 \end{vmatrix} = \Big[\overset{.495}{(.704)^2} + \overset{.1131}{(.248)(.456)}, + \{\overset{.321}{(+.456)(.704)} + \overset{.0615}{(.248)^2}\}, \{\overset{.208}{(-.456)^2} - (.704)(.248)\}\Big]$$

$$= \Big[\overset{.608}{(.495 + .1131)}, + \overset{.382}{(.321 + .0615)}, \overset{.0335}{(.208 - .1745)}\Big] = (.608, .382, .0335) \cong \vec{n}_6$$

$.495$
$.6081$

$.321$
$.3825$

$.1745$
$.0335$

$$\vec{cb}\times\vec{ca} \underset{(def.)}{\equiv} \vec{n}_{3_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -.456 & .456 & -.591 \\ -.456 & .704 & .248 \end{vmatrix} = \Big[\overset{.113}{(.456)(.248)} + \overset{.416}{(.704)(.591)} - \{\overset{.113}{(-.456)(.248)} - \overset{.2695}{(.456)(.591)}\}, \{\overset{.321}{(-.456)(.704)} + \overset{.208}{(.456)^2}\}\Big] = (.529, .1565, -.113) = \vec{n}_{3_1}$$

$$\cos\zeta = \measuredangle(\vec{n}_6, \vec{n}_3) = \frac{(.608)(.529) + (.382)(.1565) - (.0235)(.113)}{(.608)^2 + (.382)^2 + (.0235)^2} = \frac{.3212 + .0598 - .00266}{.515} = \frac{.3783}{.515} = .735$$

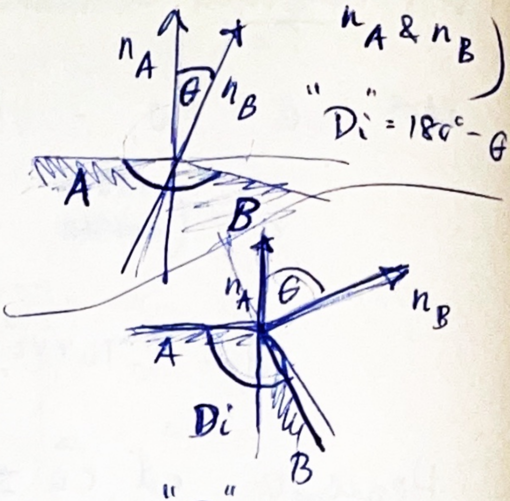
$\Sigma\ .369 \quad .1459 + \quad .000555 = .515$

$\zeta = 47°20'$ <u>Good value.</u>

dihedral ∡ between faces A & B = ~~$\frac{\pi}{2} + \theta$~~ (= angle between normals n_A & n_B)

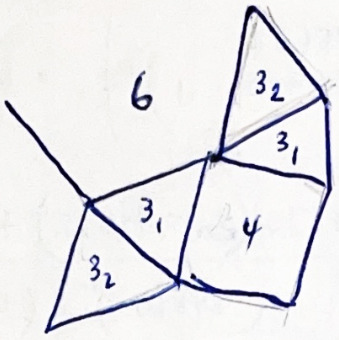
"Di" = 180° − θ

Summary of dihedral angles:



$$\text{"Di"} = \frac{\pi}{2} + \frac{\pi}{2} - \theta$$

$$= 180° - \theta$$

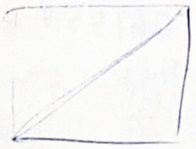


Dihedral angles:

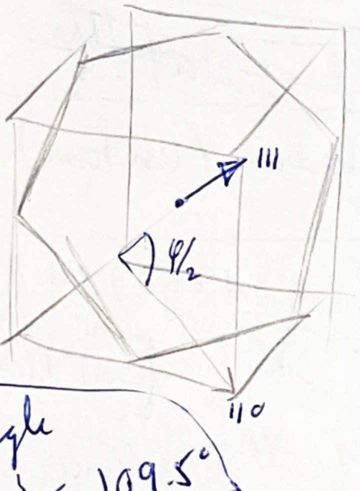
6|3 ≅ 160°

3|3 ≅ 129° ✓

4|3 ≅ 110°



Cf. regular skew polyhedron {6,4|4}



dihedral angle of $\{6,4|4\}_p = 109.5°$

$$\cos\frac{\varphi}{2} = \frac{(111)\cdot(110)}{\sqrt{3}\cdot\sqrt{2}}$$

$$= \frac{1+1}{\sqrt{6}} = \frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3} = \frac{2.449}{3}$$

$$\cong .8163$$

Hence $\frac{\varphi}{2} = 35° 15'$

$\varphi = 70.5°$

$180 - \varphi = 109.5°$

$$\cos\xi = \measuredangle(\vec{n}_{3_2}, \vec{n}_{3_1}) = \frac{\overset{.206}{(.3825)(.529)} + \overset{.0828}{(\overset{.529}{\cancel{.559}})(.1565)} + \overset{.0128}{(.113)(-.113)}}{(3825)^2 + \underset{(.529)^2}{\cancel{(.2334)^2}} + (.113)^2}$$

$$= \frac{.206 + \overset{.0828}{\cancel{.0369}} - .0128}{.1462 + \underset{.280}{\cancel{.0556}} + .0128} = \cancel{.2301} \; \frac{.276}{.439} = .630$$

Hence $\boxed{\xi \cong 51^\circ}$ (!)

.206
0369
.2429
-.0128
.2301

.1462
0556
0128
.2146

.206
0828
.2888
0128
.2760

1462
280
0128
4390

<u>To summarize:</u> Two adjoining triangles have dihedral angle of ~ 51°

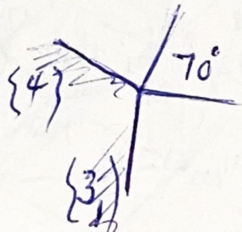
A hexagon and triangle: ~ 47°

Now compute dihedral angle between quadrilateral and single adjoining $\Delta\ (3_3)$
and "double" " " (3_1)

$\boxed{\vec{n}_4 = (100)}$ $\vec{n}_{3_1} = (.529, .1565, -.113)$ Hence $\cos\measuredangle = \dfrac{.529}{(1)\left[.529^2 + .1565^2 + .113^2\right]^{1/2}} = .563$

$$= \frac{.529}{\sqrt{.279 + +.0245 + .0128}} = \frac{.529}{.563} = .940 \quad \boxed{\measuredangle_{4,3_1} \cong 70^\circ}$$

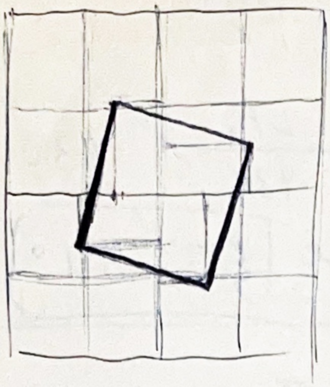
.0245
0128
.3163

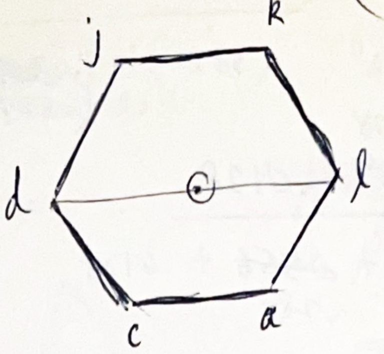


<u>Correct earlier calcn:</u> $\cos\measuredangle(n_6, n_3) = \dfrac{.3783}{\sqrt{.515}\,\underbrace{\sqrt{.529^2 + .156^2 + .113^2}}_{.3163}} = \dfrac{.3783}{\underbrace{[.718][.563]}_{.404}} = .938$

$\boxed{\measuredangle\{n_6, n_3\} \cong 20^\circ}$ (not 47°)

Note signs for α, β, γ
in circuit of vertices of hexag
(Alternate + and −).





$\vec{ca} = (-1+\alpha+\gamma, 1+\alpha-\beta, -\beta+\gamma)$

$\vec{cd} = (1-\beta+\gamma, -1+\alpha+\gamma, 1+\alpha-\beta)$

$\vec{ca}\Big|_{\alpha=0} = (-1+\gamma, 1-\beta, -\beta+\gamma)$

$\vec{cd}\Big|_{\alpha=0} = (1-\beta+\gamma, -1+\gamma, 1-\beta)$

$$\left\{\begin{array}{lllll} a & (\alpha & -\beta & \gamma) & 000 \\ c & (1-\gamma & -1-\alpha & \beta) & 1-10 \\ d & (1-\beta & -2+\gamma & 1+\alpha) & 1-21 \\ j & (-\alpha & -2+\beta & 2-\gamma) & 0-22 \\ k & (-1+\gamma & -1+\alpha & 2-\beta) & -1-12 \\ l & (-1+\beta & -\gamma & 1-\alpha) & -1\ 01 \end{array}\right.$$

$$o \quad (0 \quad -1 \quad 1)$$

$\vec{ca} = (-.45632, .70442, .24810)$

$\vec{cd} = (.24810, -.45632\ .70442)$

$$\vec{ca} \times \vec{cd} = \begin{vmatrix} i & j & k \\ -.45632 & .70442 & .24810 \\ .24810 & -.45632 & .70442 \end{vmatrix} = \Big[(\overset{.495}{.70442})^2 + \overset{(.1132)}{(.24810)(.45632)},$$

$$-\{\overset{-.3215}{(-.45632)(.70442)} - \overset{.0616}{(.24810)^2}\},$$

$$\overset{.208}{(-.45632)^2} - \overset{.1747}{(.24810)(.70442)}\Big]$$

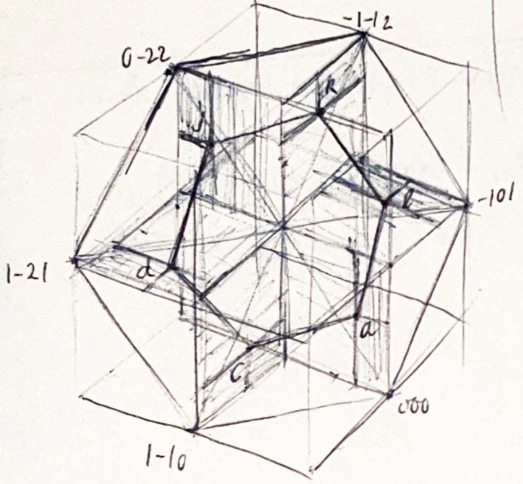
$= (.608, .3831, .0333)$?

$$\begin{array}{r} .495 \\ \underline{1132} \\ .6082 \end{array} \qquad \begin{array}{r} .3215 \\ \underline{0616} \\ .3831 \end{array} \qquad \begin{array}{r} .2080 \\ \underline{-1747} \\ 0333 \end{array}$$

$\alpha \beta \gamma = (0, .29558, .54368)$

$\alpha = 0$
$\beta = .29558$
$\gamma = .54368$

a	(0	-.29558	.54368)
b	(0	-.54368	-.29558)
c	(.45632	-1	.29558)
d	(.70442	-1.45632	1)
e	(.70442	~~.7044~~ -1	-.54368)
f	(0	.29558	-.54368)
g	(-.45632	-.29558	-1)
h	(0	.54368	.29558)
o	(		



Hexagon is not plane.
It appears to have face angles of about 109°52′ (but I haven't calculated this very accurately).

Recalculate $\cos\psi$ more accurately.

$$\boxed{\cos\psi = \frac{\vec{cd}\cdot\vec{ca}}{|cd|\,|ca|}}$$

$\vec{cd} = (\cancel{1}-\beta+\gamma, -1+\alpha+\gamma, 1+\alpha-\beta)$

$\vec{ca} = (-1+\gamma, 1-\beta, -\beta+\gamma)$

β = root of $\underbrace{\beta^3+\beta^2+3\beta-1}_{A(\beta)}=0$; $\gamma = \frac{\beta^2+1}{2}$

$A(\beta)\Big|_{\beta=.295600} = +.000\,008$

$A(\beta)\Big|_{\beta=.295598} = +.000\,001$

$\vec{cd} = (.248\,091\,30, -.456\,311, .704\,4023)$

$\vec{ca} = (-.456311, .704\,4023, .248\,09130)$

Try $\boxed{\beta = .295\,597\,7}$

Prods: $-.113206$ $\quad -.321426$ $\quad .194755$

$\beta^2 \cong .087\,377\,999\,2$

or $\boxed{\beta^2 \cong .087\,378\,00}$

$\Sigma = -.259877$

$\gamma = \frac{1.087\,378\,00}{2} = .543\,689\,00$

$(.061549 + .208219 + .496182) = .765950$

$= -.339287$ Hence $\boxed{\psi \cong 109.834^\circ}$

Nov. 5, 1972 I thought I had already considered $(3^2.4.3.6)$ on P with flat hexagonal faces, but I can't find such calculations.

If $\{6\}$ is flat, then $\vec{oa}\cdot(111) = \vec{oc}\cdot(111)$, i.e., $[(\alpha,-\beta,\gamma)-(0,-1,1)]\cdot(111)$

$= [(1-\gamma,-1-\alpha,\beta)-(0,-1,1)]\cdot(111)$

or $(\alpha,-\beta+1,\gamma-1)\cdot(111) = (1-\gamma,-\alpha,\beta-1)\cdot(111)$

$\alpha-\beta+\cancel{1}+\gamma-\cancel{1} = \cancel{1}-\gamma-\alpha+\beta-\cancel{1}$

i.e., $\alpha-\beta+\gamma = -\alpha+\beta-\gamma$

or $2\alpha-2\beta+2\gamma=0$

or $\alpha-\beta+\gamma=0$

~~Substitute $\beta=\alpha+\gamma$ in ① with $\alpha=-1$ }: $\gamma^2-2\gamma+.8=0 \Rightarrow \gamma = 1 \pm .4472$ If $\gamma=.5528$, $\alpha=-1$, $\beta=.4528$~~