For $\alpha \cong 40°$ or $50°$ (i.e., on either side of cubic graph $[\alpha = 45°]$ configuration of collapsed Laves graph), the 14 extra faces appear very small.

Feb 11 1972

for $\alpha' \cong \tan^{-1}[.848528]$ $\left(= \frac{3\sqrt{2}}{5}\right)$, $\tau'_\alpha \vec{r}_{AB=(-110)} \propto (-1, 4, 0)$

$$\boxed{\alpha' = 40.3157°}$$

$$\lambda = \Lambda \tan\alpha = \left(\frac{\sqrt{6}}{4}\right)\left(\frac{3\sqrt{2}}{5}\right) = \frac{3}{10}\sqrt{3} = \boxed{\frac{3}{20}(2\sqrt{3})}$$

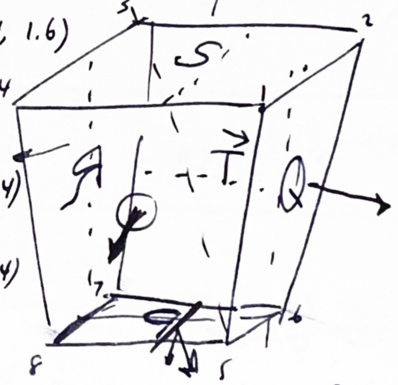
Thus $\lambda = \frac{3}{20}$ of the separation between similarly oriented Laves graph vertices at (000) & (1-1-1).

If $\lambda = \left(\frac{3}{18} = \frac{1}{6}\right)(2\sqrt{3})$, then $\alpha' = 43.3140°$

→ Add .3 (P[111]) to each RST etc

R (1-10) →	(.7 / .3	-1.3 / .3	-.3 / -.3)	→ (.4, -1.6, 0)
S (0, 1, 1)	(.3 / .3	.7 / .3	1.3 / -.3)	→ (0, .4, 1.6)
T (-1 0 -1)	(-1.3 / .3	.3 / .3	-.7 / -.3)	→ (-1.6, 0, -.4)
O (3 0 -1)	(2.7 / .3	.3 / .3	-.7 / -.3)	→ (2.4, 0, -.4)
P (0 1 -3)	(.3 / .3	.7 / .3	-2.7 / -.3)	→ (0, .4, -2.4)
Q (1 3 0)	(.7 / .3	2.7 / .3	-.3 / -.3)	
	.4	2.4	0	

Origin is displaced by .3(11-1) = (.3, .3, -.3)



R = (.4, -1.6, 0)
S = (0, .4, 1.6)
T = (-1.6, 0, -.4)
O = (2.4, 0, -.4)
P = (0, .4, -2.4)
Q = (.4, 2.4, 0)

1 = QRS
2 = QST
3 = RST
4 = ORS
5 = OPQ
6 = PQT
7 = PRT
8 = OPR

☆ Mult. R, S etc by 2.5 →

R	=	1	-4	0
S		0	1	4
T		-4	0	-1
O		6	0	-1
P		0	1	-6
Q		1	6	0

✓

① = ~~(.25, 2, 3.25)~~ ~~(25, 2, 13/4)~~

QRS (6.66434, 5.05594, 2.98601) $\frac{1}{143}(953, 723, 427)$

② = (−4.87629, 6.97938, 2.50515) $\frac{1}{97}(-473, 677, 243)$

QST

③ = (−5.66667, −5.66667, 5.66667) $\frac{17}{3}(-1, -1, 1)$

RST

④ (6.97938, −2.50515, 4.87629) $\frac{1}{97}(677, -243, 473)$

ORS

⑤ (5.28571, 5.28571, −5.28571) $\frac{1147}{217}(1 1 -1)$

OPQ

⑥ ~~(−3.25, 25, −2)~~ ~~(−13/14, 25, 2)~~

PQT (−2.98601, 6.66434, −5.05594)

⑦ (−2.50515, −4.87629, −6.97938) $\frac{1}{97}(-243, -473, -677)$

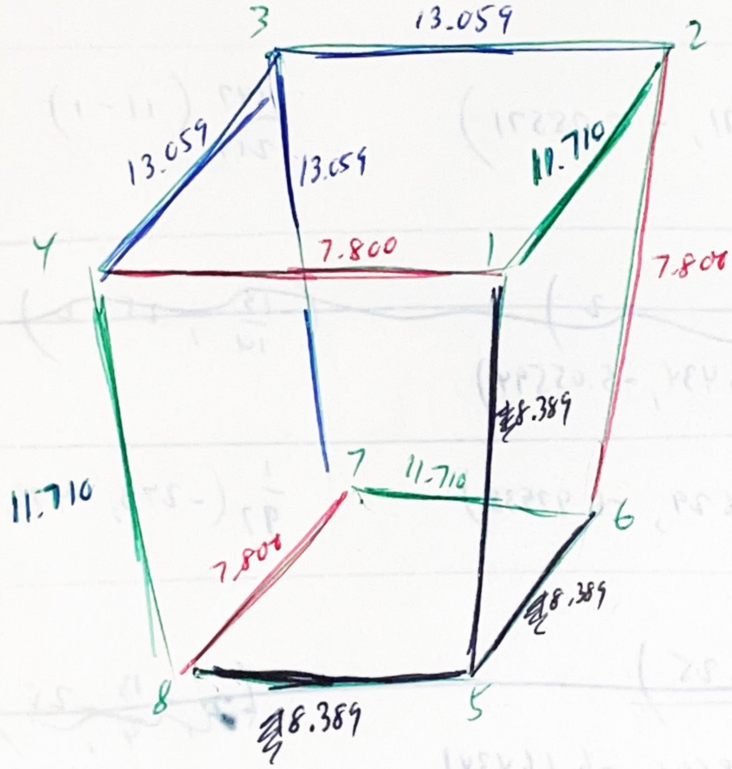
~~PRT~~

⑧ ~~(2, −3.25, −13/4, −25)~~ ~~(2, −13/4, 25)~~

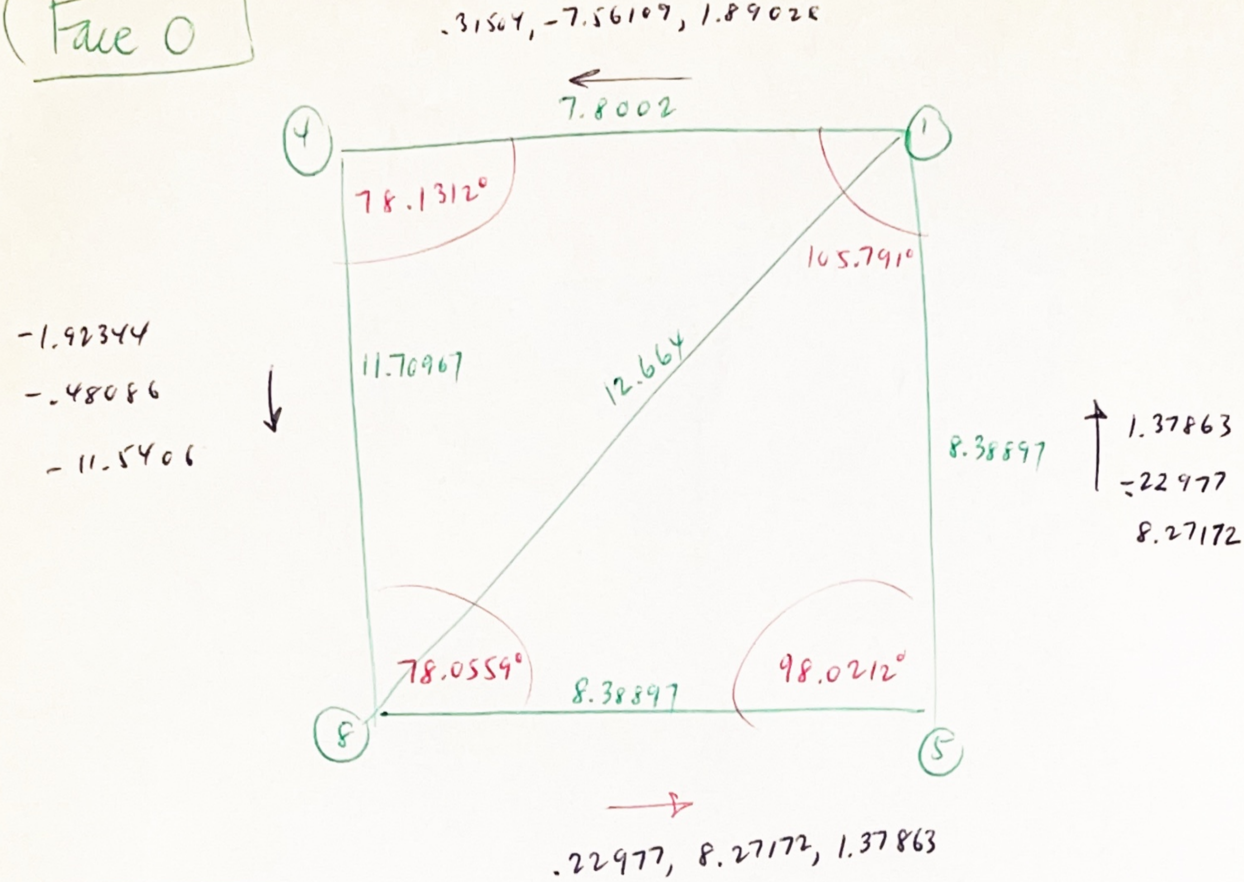
OPR (5.05594, −2.98601, −6.66434)

1 → 8 → 6 (CCW) by $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}$

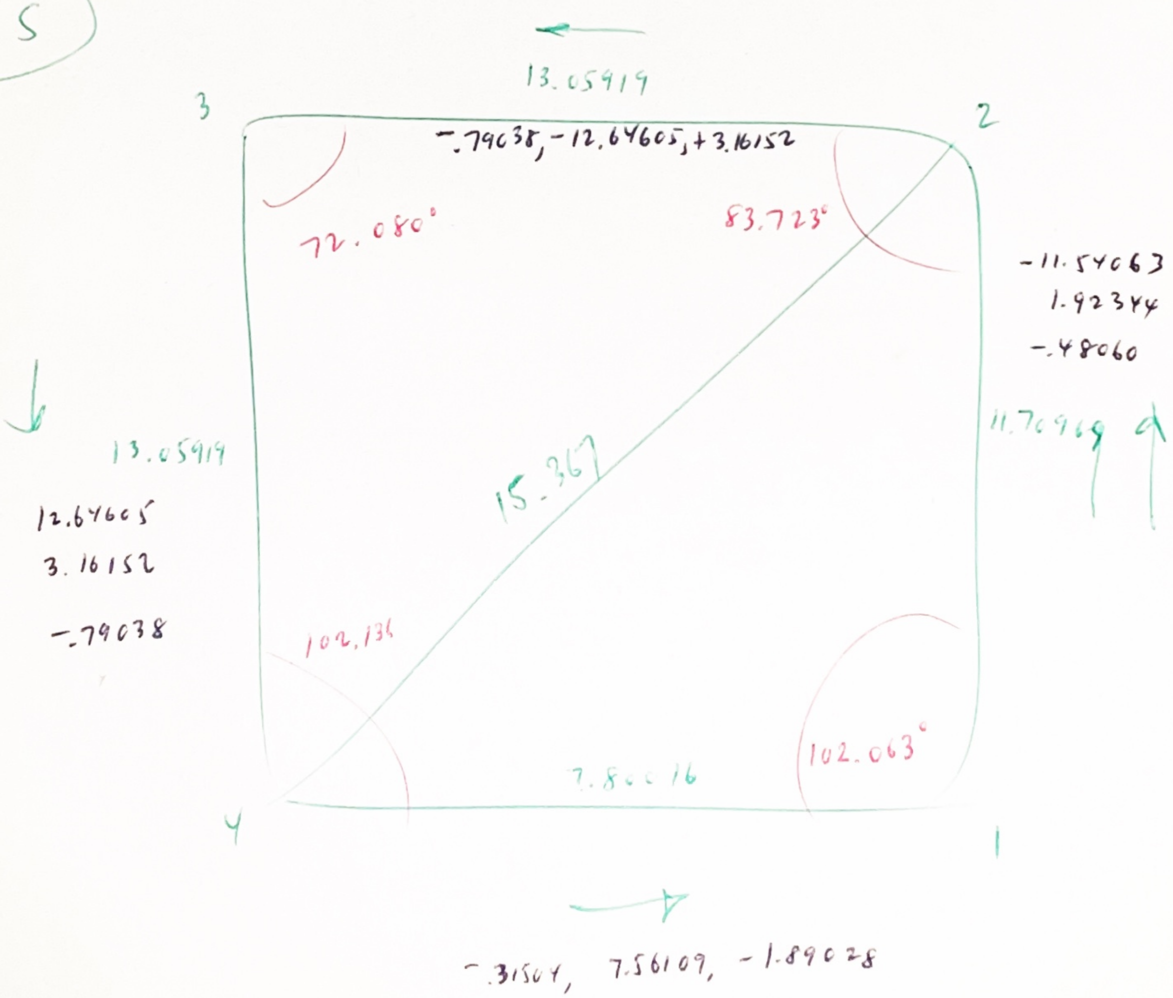
2 → 4 → 7 "

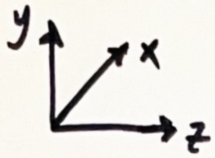

3
13.059
2
13.059
13.059
11.710
4
7.800
1
7.808
8.389
7
11.710
6
11.710
7.808
8.389
8
5
8.389

Face 0

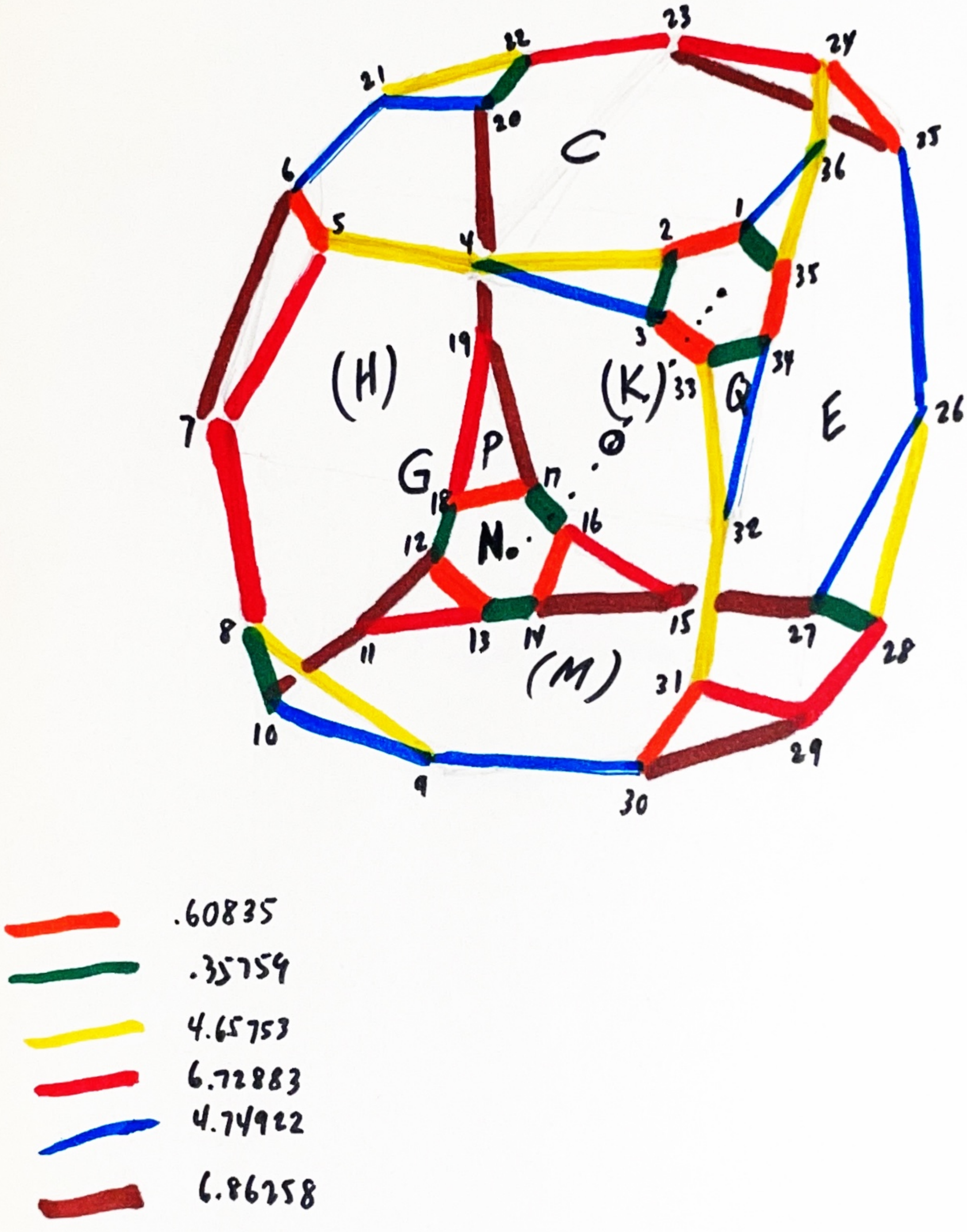

.31504, -7.56109, 1.89028
7.8002
4
1
78.1312°
105.791°
-1.92344
-.48086
-11.5406
11.70967
12.664
8.38897
1.37863
-.22977
8.27172
78.0559°
98.0212°
8.38897
8
5
.22977, 8.27172, 1.37863

Face 5

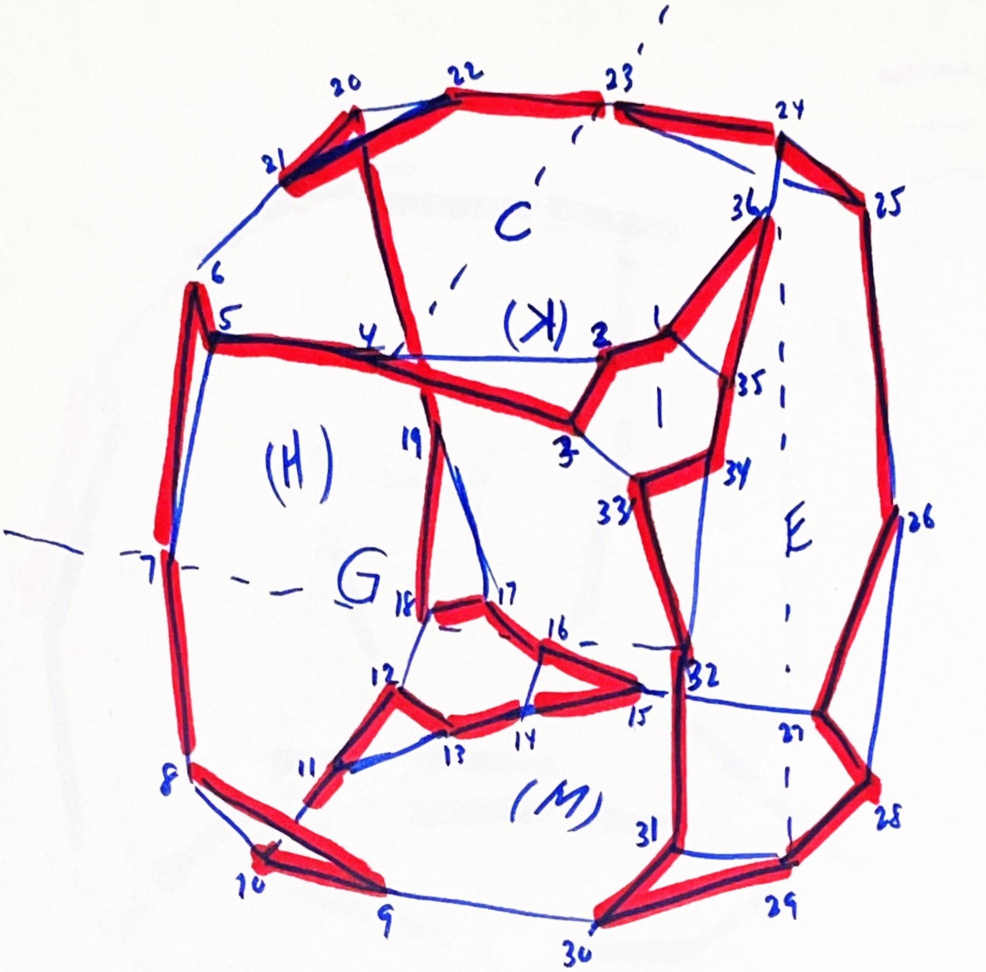

13.05919
3
2
-.79038, -12.64605, +3.16152
72.080°
83.723°
-11.54063
1.92344
-.48060
11.70969
13.05919
15.367
12.64605
3.16152
-.79038
102.136
102.063°
7.80016
4
1
-.31504, 7.56109, -1.89028



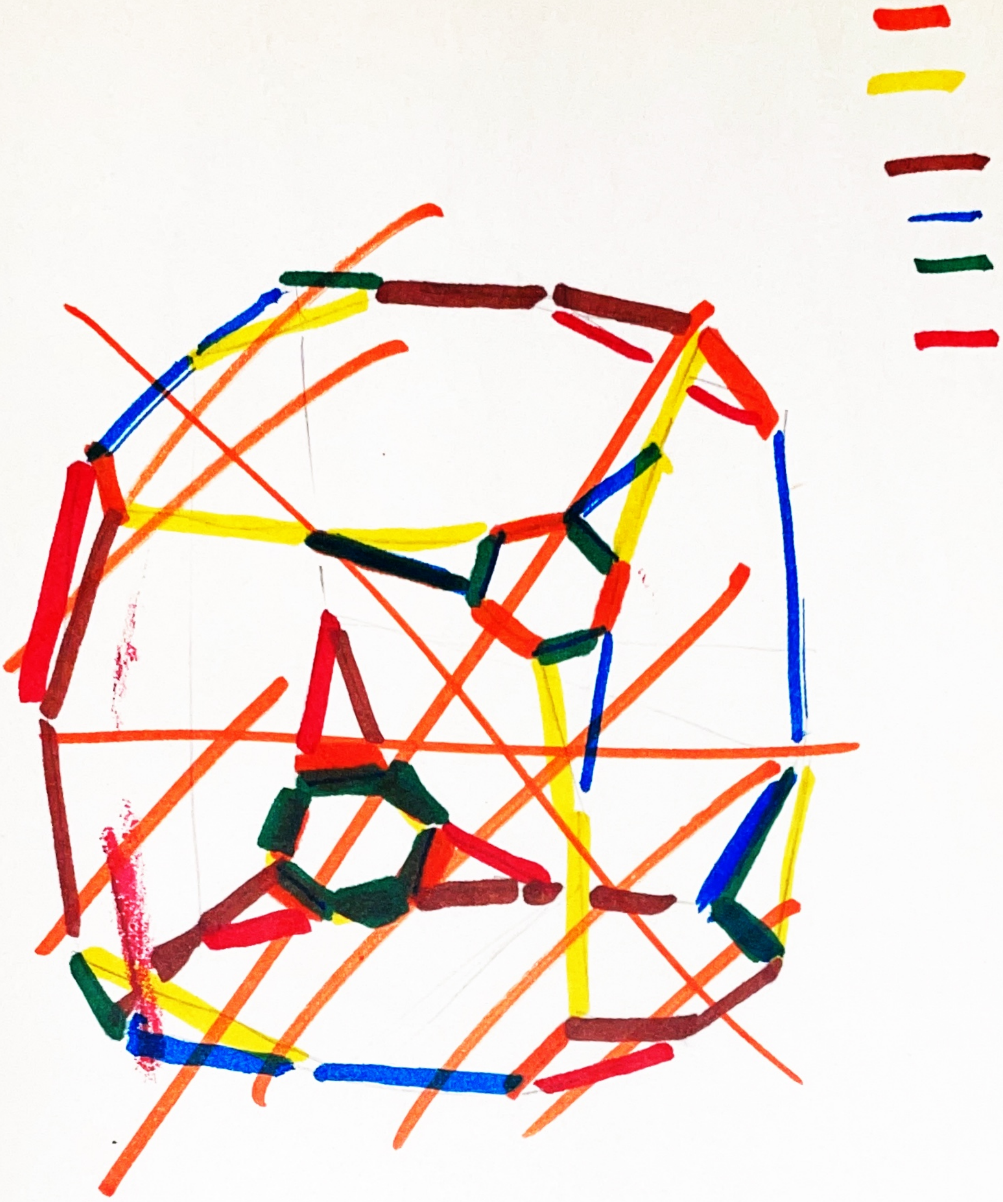
$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}$$ rotates $1 \to 3 \to 34 \to 1$ etc.



(Pare 11)



20
22
23
24
21
25
36
C
6
5
4
(K)
2
1
35
3
I
19
34
(H)
33
26
E
7
G
18
17
16
32
12
15
27
11
13
14
8
(M)
28
31
10
29
9
30



$\vec{C} \cdot \vec{A} \times \vec{B}$

$$A \times B = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

$flat_R$

A $(1, 0, 1-\tau)$

B $(1-\tau, 1, 0)$

C $(0, \tau-1, -1)$

$(\tau-1)^2 = \tau^2 - 2\tau + 1$

$= \tau + 1 - 2\tau + 1$

$= -\tau + 2$

$-(\tau-1)^2 = 2 - \tau$

$$B \times C = \begin{vmatrix} i & j & k \\ 1-\tau & 1 & 0 \\ 0 & \tau-1 & -1 \end{vmatrix} = \begin{vmatrix} i & j & h \\ -.61803 & 1 & 0 \\ 0 & .61803 & -1 \end{vmatrix}$$

$= (-1, 1-\tau, -(\tau-1)^2) = (-1, 1-\tau, 2-\tau)$

$A \cdot B \times C = (1, 0, 1-\tau) \cdot (-1, 1-\tau, 2-\tau)$

$= -1 + (2-\tau)(1-\tau) = -1 + 2 - 3\tau + \tau^2 = 1 - 3\tau + \tau + 1$

$= \boxed{-2\tau + 2}$

$\frac{r_1 + r_7}{2} = \frac{1}{2}\left[(0, 1, \tau) + (\tau, 0, 1)\right]$

$= \frac{1}{2}\left[(\tau, 1, \tau+1)\right]$ $\vec{radius} = \frac{1}{2}[\tau, 1, \tau+1]$

$|\vec{radius}| = \frac{1}{2}\sqrt{\tau^2 + 1 + \tau^2 + 2\tau + 1}$

$= \frac{1}{2}\sqrt{\tau + 1 + 1 + \tau + 1 + 2\tau + 1}$

$= \frac{1}{2}\sqrt{4\tau + 4} = \sqrt{\tau + 1}$

inradius $= \sqrt{\tau + 1} = \tau$

(Tr... ")

Rhombic triacontahedron "Kepler ball" (Michael Longuet-Higgins)

March 6, 2001 (Mathematica notebooks)

1	0	1	τ	a	τ-1	0	τ		l	1	1	-1		
2	0	-1	τ	b	1	-1	1		m	0	τ	$-(\tau-1)$		
3	0	1	$-\tau$	c	1	1	1		n	$-\tau$	$\tau-1$	0		
4	0	-1	$-\tau$	d	$-(\tau-1)$	0	τ		o	$-\tau$	$-(\tau-1)$	0		
5	1	τ	0	e	-1	-1	1		p	-1	-1	-1		
6	1	$-\tau$	0	f	0	$-\tau$	$\tau-1$		q	$\tau-1$	0	$-\tau$		
7	τ	0	1	g	τ	$-(\tau-1)$	0		r	-1	1	-1		
8	τ	0	-1	h	τ	$\tau-1$	0		s	$-(\tau-1)$	0	$-\tau$		
9	-1	τ	0	i	0	τ	$\tau-1$		t	0	$-\tau$	$-(\tau-1)$		
10	-1	$-\tau$	0	j	-1	1	1							
11	$-\tau$	0	1	k	1	-1	-1							
12	$-\tau$	0	-1											

The ratio of volumes of the two rhombohedra is τ:

Obtuse rh. vol. $V_o = 2(\tau-1)$ ratio: $\dfrac{2(\tau-1)}{2(2-\tau)} = \tau$

Acute " " $V_a = 2(2-\tau)$

$2(2-\tau) = .763932$

$2(\tau-1) \cong 1.23606$

$(\Sigma = 2.00000)$

"Altitude angle" φ for acute rh. $= \cos^{-1}\left(\dfrac{1}{\sqrt{3-\tau}}\right) \cong 31.71747...°$

" ξ " obtuse " $=$ $\sin^{-1}(\ " \ ) \cong 58.28252558...°$

---

insphere radius $= \tau$

$V_{insphere} = \dfrac{4\pi}{3}\tau^3 = 17.7440000 5 1C...$

$V_{triacontahedron} = 10\left[\underbrace{2(2-\tau)+2(\tau-1)}_{2}\right] = 20$

$Area_{triacontahedron} = 30\left[2(\tau-1)\right] = 60(\tau-1) \cong 37.082039325$

$edgelength_{triacontahedron} = \sqrt{3-\tau}$

Area of one rhombic face $= 2(\tau-1)$

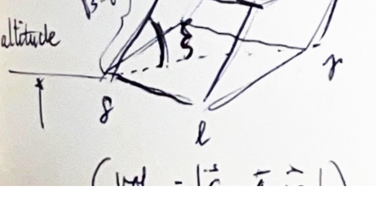
$Altitude_{acute\ rh.} = \tau-1$

$Altitude_{obtuse\ rh.} = 1$

But normalized to unit sphere, instead of sphere of rad. $= \tau$, area $= \dfrac{60(\tau-1)}{\tau^2}$

$\cong 14.16407805$

($\sim n = 27$ "best" polyhedron)



$\vec{h} = (\tau, \tau-1, 0)$

$\vec{s} = (1, \tau, 0)$

$\vec{g} = (\tau, 0, -1)$

$\vec{l} = (1, 1, -1)$

$|\vec{g r}| = 2$

Altitude = edge length $\times \sin\xi$ $= \sqrt{3-\tau} \cdot \dfrac{1}{\sqrt{3-\tau}} = 1$ ☆

$\cos\xi = \dfrac{\tau-1}{\ }$ $\Rightarrow \sin\xi = \dfrac{1}{\sqrt{3-\tau}}$

Rhombic triacontahedron "Kepler ball" (Michael Longuet-Higgins)

March 6, 2001 (Mathematica notebooks)

1	0	1	τ
2	0	-1	τ
3	0	1	$-\tau$
4	0	-1	$-\tau$
5	1	τ	0
6	1	$-\tau$	0
7	τ	0	1
8	τ	0	-1
9	-1	τ	0
10	-1	$-\tau$	0
11	$-\tau$	0	1
12	$-\tau$	0	-1

a	$\tau-1$	0	τ
b	1	-1	1
c	1	1	1
d	$-(\tau-1)$	0	τ
e	-1	-1	1
f	0	$-\tau$	$\tau-1$
g	τ	$-(\tau-1)$	0
h	τ	$\tau-1$	0
i	0	τ	$\tau-1$
j	-1	1	1
k	1	-1	-1

l	1	1	-1
m	0	τ	$-(\tau-1)$
n	$-\tau$	$\tau-1$	0
o	$-\tau$	$-(\tau-1)$	0
p	-1	-1	-1
q	$\tau-1$	0	$-\tau$
r	-1	1	-1
s	$-(\tau-1)$	0	$-\tau$
t	0	$-\tau$	$-(\tau-1)$

The ratio of volumes of the two rhombohedra is τ:

Obtuse rh. vol. $V_o = 2(\tau-1)$ ratio $= \dfrac{2(\tau-1)}{2(2-\tau)} = \tau$

Acute " " $V_a = 2(2-\tau)$

$2(2-\tau) \cong .763932$

$2(\tau-1) \cong 1.23606$

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" ξ " obtuse " $= \sin^{-1}(\;"\;) \cong 58.28252558\ldots^\circ$

---

insphere radius $= \tau$

$V_{\text{insphere}} = \dfrac{4\pi}{3}\tau^3 = 17.7440000510\ldots$

$V_{\text{triacontahedron}} = 10\left[\underbrace{2(2-\tau)+2(\tau-1)}_{2}\right] = 20$

$\text{Area}_{\text{triacontahedron}} = 30\,[2(\tau-1)] = 60(\tau-1) \cong 37.082039325$

$\text{edgelength}_{\text{triacontahedron}} = \sqrt{3-\tau}$

Area of one rhombic face $= 2(\tau-1)$

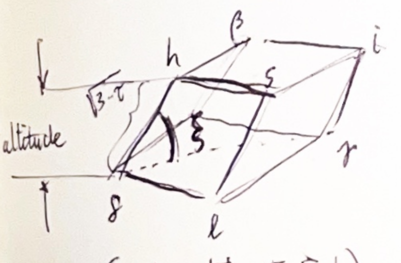
$\text{Altitude}_{\text{acute rh.}} = \tau-1$

$\text{Altitude}_{\text{obtuse rh.}} = 1$

But normalized to unit sphere, instead of sphere of rad. $= \tau$, area $= \dfrac{60(\tau-1)}{\tau^2}$

$\cong 14.16407805$

($\sim n = 27$ "best" polyhedron)



(vol. $= |\vec{C}\cdot\vec{A}\times\vec{B}|$)

$\vec{h} = (\tau, \tau-1, 0)$

$\vec{5} = (1, \tau, 0)$

$\vec{8} = (\tau, 0, -1)$

$\vec{l} = (1, 1, -1)$

$|\vec{8r}| = 2$

$(\vec{8h}\cdot\vec{8r}) = 2(\tau-1) = 2\sqrt{3-\tau}\cos\xi \therefore \cos\xi = \dfrac{\tau-1}{\sqrt{3-\tau}} \Rightarrow \sin\xi = \dfrac{1}{\sqrt{3-\tau}}$

Altitude = edge length $\times \sin\xi$

$= \sqrt{3-\tau}\cdot\dfrac{1}{\sqrt{3-\tau}} = 1$ ☆