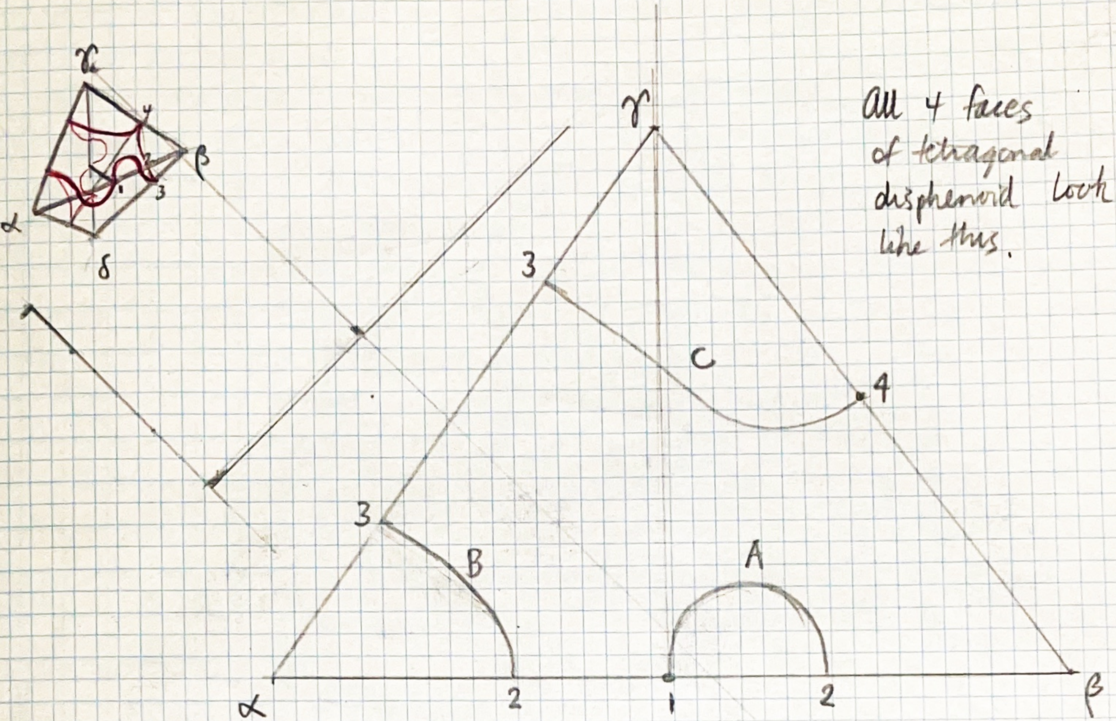
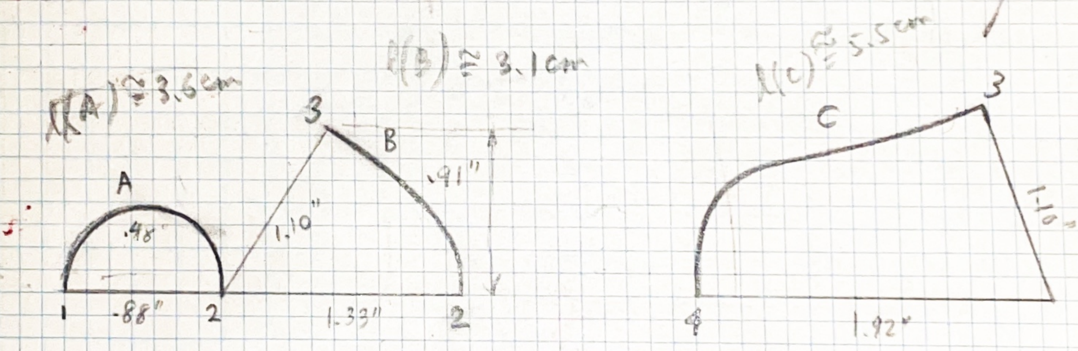
3-6-72
AHS

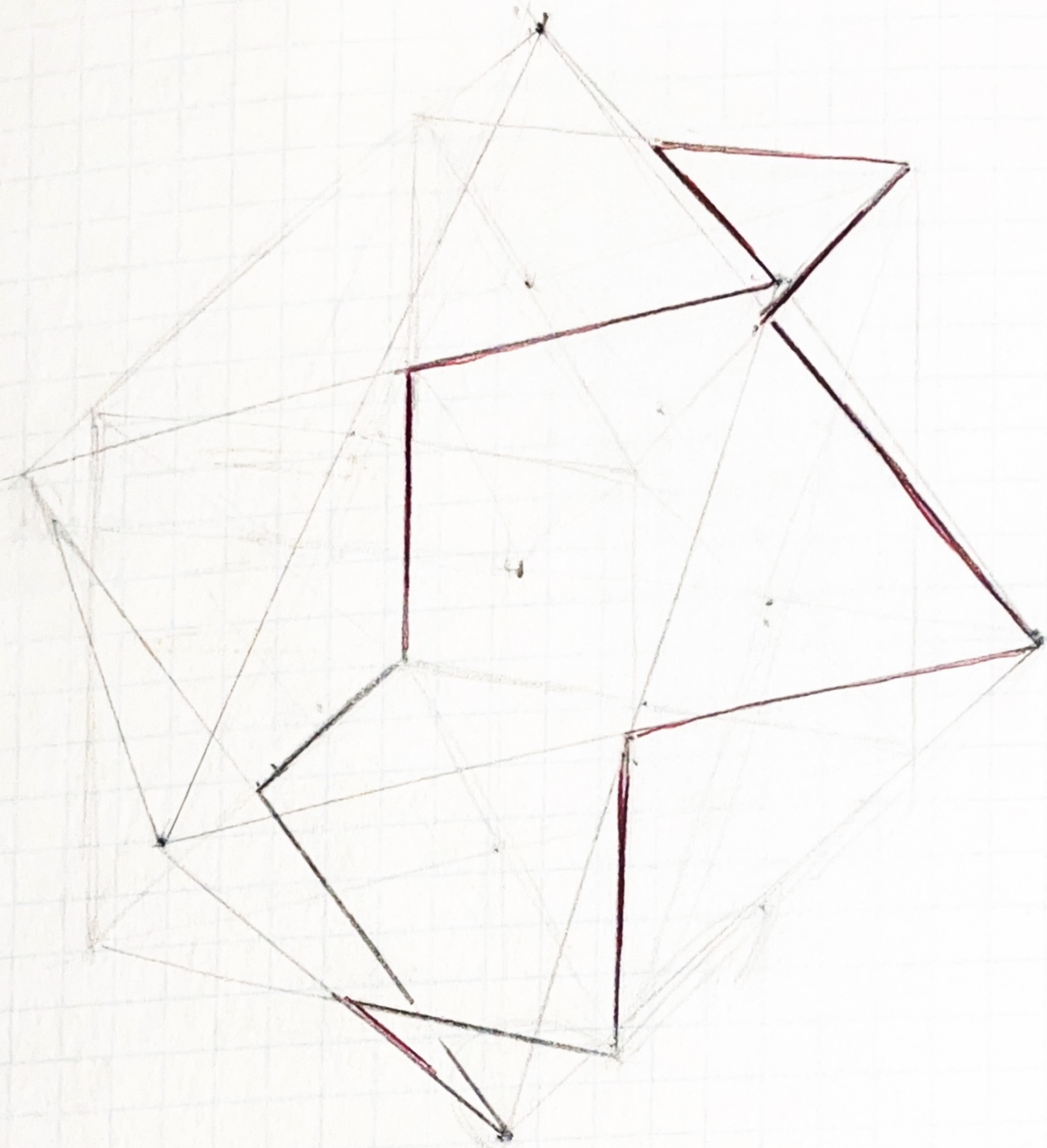


All 4 faces of tetragonal disphenoid look like this.



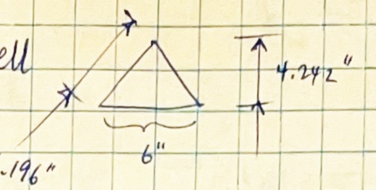
These curves were carefully derived from soap film measurements

Alan H. Schoen, 5126 Elkmont Drive, Palos Verdes Peninsula, California 90274 213-378-2149

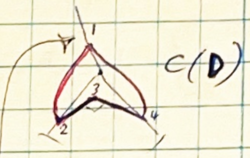


Tuesday, March 7, 1972

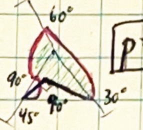
This morning, as soon as I had built a somewhat larger tetragonal disphenoid film cell



and installed the appropriate (100) & (110) threads, I found all 3 of the modified minimal surfaces I had imagined might exist:



A) the one on p. 1:

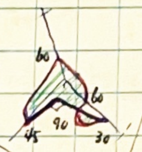


$\boxed{p=31}$

$(2-n)\pi+\sum \iota_k = -3\pi + \frac{7\pi}{4} = -\frac{5\pi}{4}$

$\iint KdA = (4)(24)\left(-\frac{5\pi}{4}\right) = -120\pi \quad d_g = -30 \quad \boxed{p=31}$

B) its topological "mirror twin":

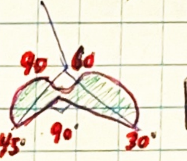


$\boxed{p=35}$

$\sum \iota_k = 285°$ Hence $\iint_P KdA = \frac{-17\pi}{12} \quad 2\pi\chi = 96\left(\frac{-17\pi}{12}\right) = -136\pi$

$d_g = -34 \quad p = 35$

C) the one obtained by "pulling the corner [(1) of C(D)] over the vertex".



$\boxed{p=31}$

$\iint_P KdA = 2\pi - \sum \varepsilon_k$

$\sum \varepsilon_k = 135° + 90° + 120° + 150° + 90° = 585°$ (585)

$2\pi - \sum \varepsilon_k = -225° = -\frac{5\pi}{4}$

$\iint_M KdA = (4)(24)\left(-\frac{5\pi}{4}\right) = -120\pi = 2\pi\chi$

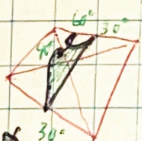
$\chi = -60 \quad d_g = -30$

$\boxed{p=31}$

These are all derived from C(D)

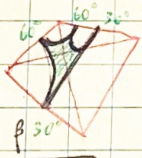
I have also produced 2 derived from P

(green shaded portion of front face is missing from cell.)



α $\boxed{p=11}$

and



β $\boxed{p=13}$

This one is somewhat less easy than α, until the inside faces drain quite dry.

α) $\sum \iota_k = 210° = \frac{7\pi}{6} \quad \iint_P KdA = (2-4)\pi + \frac{7\pi}{6} = \frac{(-12+7)\pi}{6} = -\frac{5\pi}{6} \quad \iint_M KdA = 48\left(-\frac{5\pi}{6}\right) = -40\pi$

$d_g = -10 \quad \boxed{p=11}$

β) $\sum \iota_k = 180° = \pi \quad \iint_P KdA = -\pi \quad \iint_M KdA = -48\pi \quad d_g = -12 \quad \boxed{p=13}$

(Review!)

Note: (12/12/2002)

For C(D) [genus p=19],

∃ 8 vertices for surface patch in tetragonal disphenoid.

For this skew 8-gon, $\sum \iota_k = 90° + 60° + 60° + 60° + 90° + 60° + 60° + 60° = 3\pi$

$\therefore \iint_P KdA = (2-8)\pi + 3\pi = -3\pi. \quad \iint_M KdA = (24)(-3\pi) = -72\pi = 2\pi\chi$

$\therefore \chi = -36 = 2-2p \Rightarrow \boxed{p=19}$

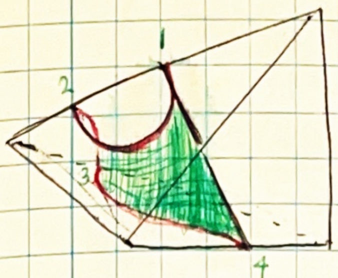
Cf. regular plane polygons

$\iint_P KdA = (2-n)\pi + \sum \iota_k$

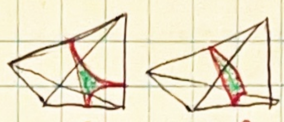
$= 0$ for all n

Sunday, Mar 12, 1972

quite "stable" complement of P



$C_2(P)$



C(P)   P

$\iota_1 = 30°$	$\Sigma_1 = 150°$
$\iota_2 = 60°$	$\Sigma_2 = 120°$
$\iota_3 = 90°$	$\Sigma_3 = 90°$
$\iota_4 = 45°$	$\Sigma_4 = 135°$

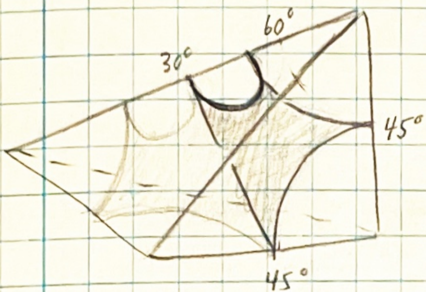
$$\sum_{K=1}^{4} \Sigma_K = 495°$$

$$\iint_P K\,dA = 2\pi - \sum \Sigma_K = -135° = -\frac{3\pi}{4}$$

$$\iint_M K\,dA = (2)(48)\left(-\frac{3\pi}{4}\right) = -72\pi = 2\pi\chi$$

$$\therefore \chi = -36;\ d_g = -18;\ \boxed{p = 19}$$

Also (Wed., Mar 22, 1972)



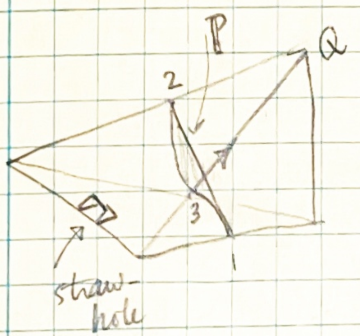
$\sum \iota_K = \pi$

$$\iint_P K\,dA = (2-n)\pi + \sum \iota_K = -2\pi + \pi = -\pi$$

$$\iint_M K\,dA = (2)(48)(-\pi) = -96\pi = 2\pi\chi$$

$$\therefore \chi = -48;\ p = 1 - \chi/2$$

or $\boxed{p = 25}$



(Blow corner 3 toward Q to produce $C_{25}(P)$.)

(If you blow corner 1 toward Q, you get the surface

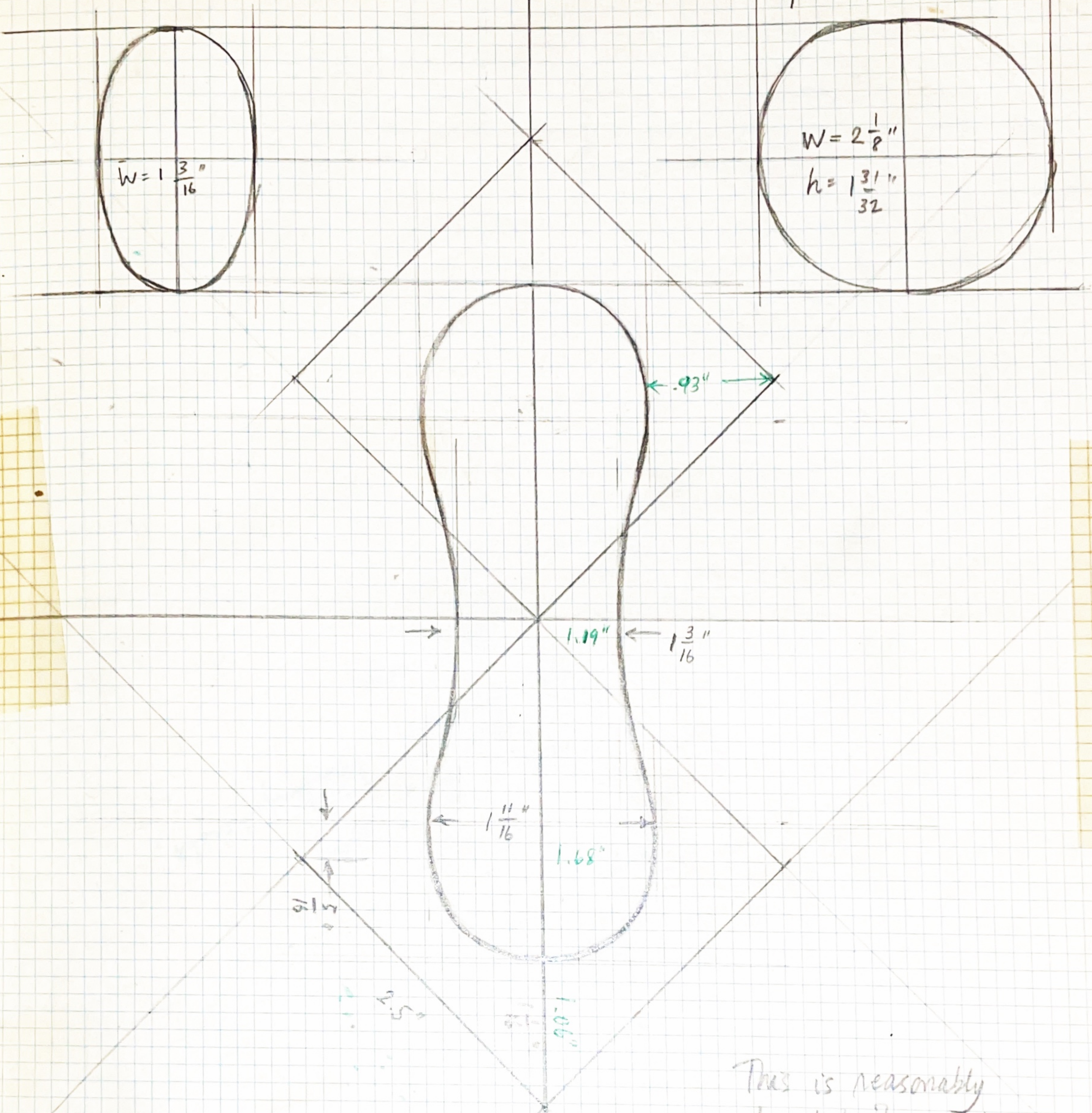
of genus 33: $C_{33}(P)$.

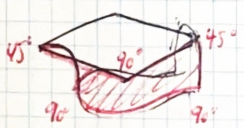
self-intersecting)

X see p.18 (12-11-02)

Sunday, Mar 12, 1972

vertical separation $h = 1\frac{31}{32}$"





$2\pi\chi = (32)\left[(2-5)\pi + 2\pi\right] = -32\pi$

$d_g = -8$; $\boxed{p = 9}$

This is reasonably accurate, but it would be better to photograph the film.

Anyway, I'd better solder a more accurate pair of frames before making tools for this surface.

a new one — very beautiful!

Wed., Mar 23, 1972

$$\iint_P K\,dS = (2-n)\pi + \sum \iota_k$$

$$= -3\pi + \frac{5\pi}{3} = \frac{-9\pi+5\pi}{3} = \frac{-4\pi}{3}$$

$$\iint_M K\,dS = (96)\left(\frac{-4\pi}{3}\right) = -128\pi = 2\pi\chi$$

$\chi = -64$ $\quad \frac{\chi}{2} = -32$

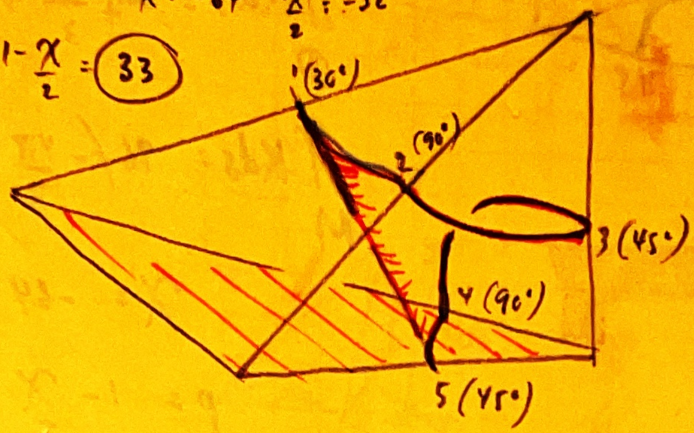
Hence $p = 1 - \frac{\chi}{2} = 33$

$\sum \iota_k =$ 30
90
45
45
90

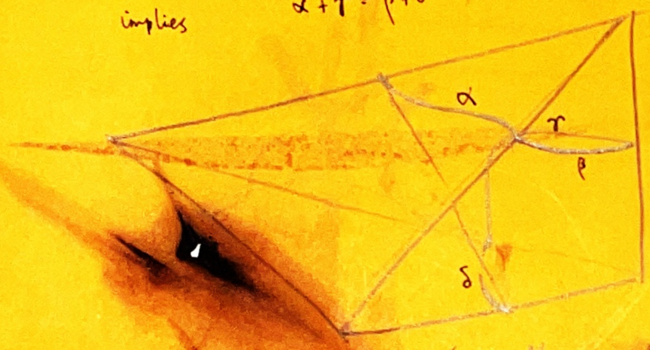
$= 300° = \frac{3\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{3}$

2 tiles per cell

48 cells per f.n.



Closure on full octagon implies $\alpha + \gamma = \beta + \delta$



(slightly more accurate)

Thus, the complements of P so far are:

$C_9(P)$ [Neovius]

$C_{19}(P)$

$C_{21}(P)$

$C_{25}(P)$

$C_{33}(P)$

$C_{33'}(P)$

$C_{39}(P)$

The complements of D are $C_{19}(D)$

$C_{31}(D)$

$C_{31'}(D)$

$C_{35}(D)$

a new one — very beautiful!

Wed., Mar 23, 1972

$$\iint_P K\,dS = (2-n)\pi + \Sigma \iota_K = -3\pi + \frac{5\pi}{3} = \frac{-9\pi+5\pi}{3} = -\frac{4\pi}{3}$$

$$\Sigma \iota_K = \begin{array}{r} 30 \\ 90 \\ 45 \\ 45 \\ 90 \\ \hline \end{array} = 300^\circ = \frac{3\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{3}$$

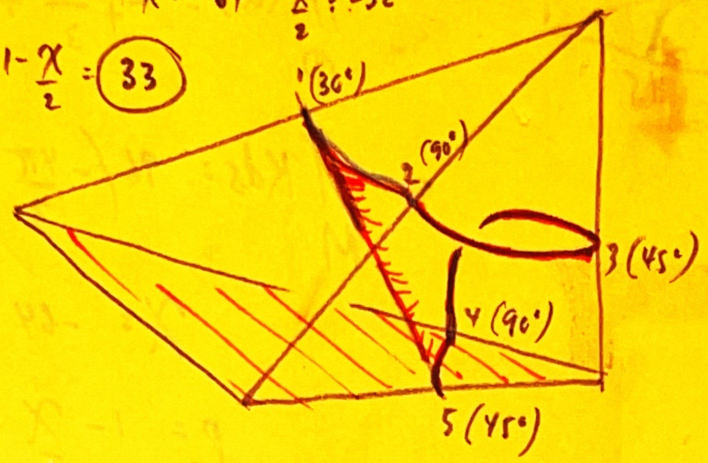
$$\iint_M K\,dS = (96)\left(-\frac{4\pi}{3}\right) = -128\pi = 2\pi\chi$$

$$\chi = -64 \qquad \frac{\chi}{2} = -32$$

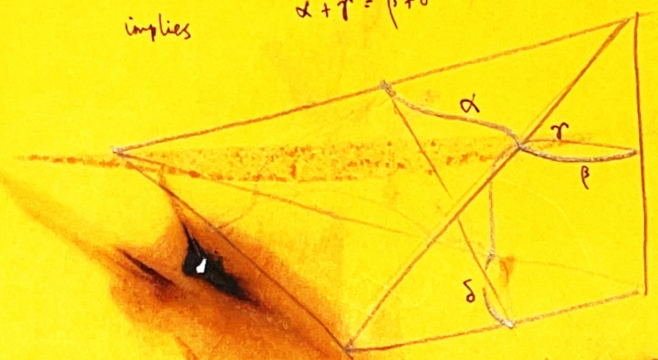
Hence $p = 1 - \frac{\chi}{2} = 33$

2 films per cell

48 cells per f.r.



Closure on full octagon implies $\alpha + \gamma = \beta + \delta$



(slightly more accurate)

Thus, the complements of P so far are:

$C_9(P)$ [Neovius]

$C_{19}(P)$

$C_{21}(P)$

$C_{25}(P)$

$C_{33}(P)$

$C_{33'}(P)$

$C_{39}(P)$

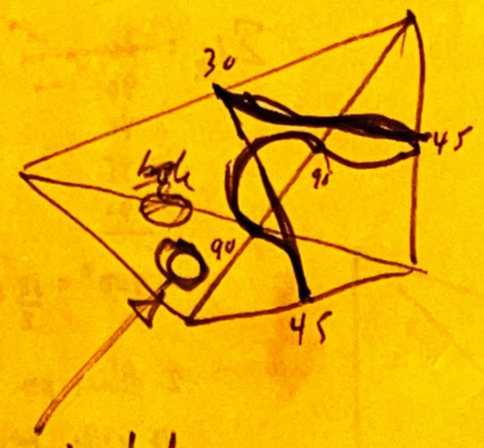
The complements of D are $C_{19}(D)$

$C_{31}(D)$

$C_{33'}(D)$

$C_{35}(D)$

J Brennan Thurs Mar 23 1972



☆ hole

$$\sum i_K = 300^\circ = \frac{3\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{3}$$

~~2π~~

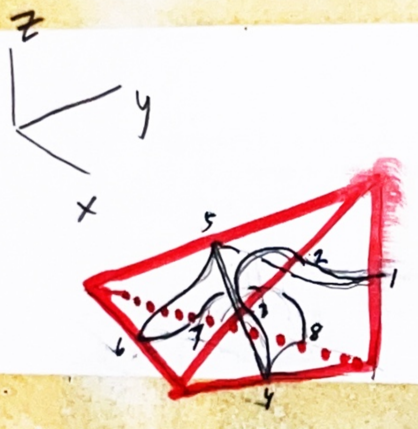
$$\iint_{\Phi} K\,ds = \frac{5\pi}{3} + (-3\pi) = -\frac{4\pi}{3}$$

$$\iint_{M} K\,ds = 96\left(-\frac{4\pi}{3}\right) = -128\pi = 2\pi\chi$$

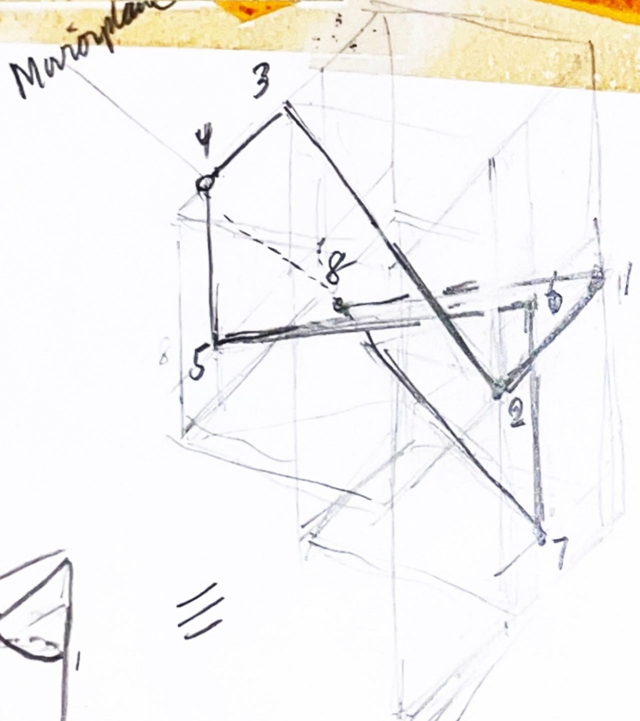
$$\chi = -64$$

$$p = 1 - \frac{\chi}{2} = 1 - (-32)$$

$= 33$



$p = 33$



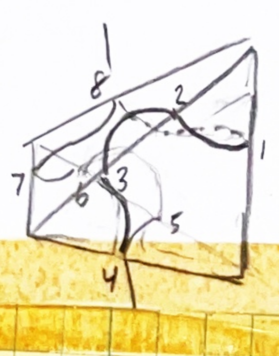
1 2 3 4 5 6 7 8

7 6 5 4 3 2 1 8

necessarily self-intersecting

☆ boundary polygon!

12-11-02

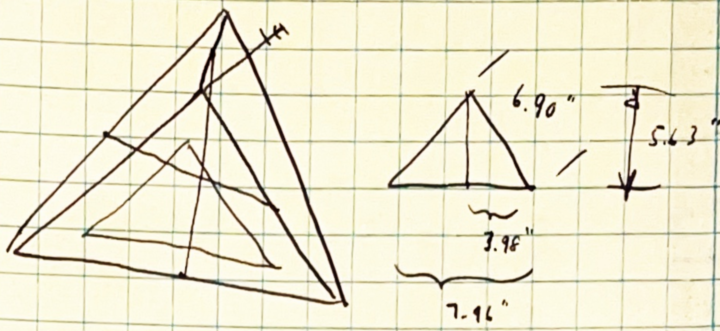


≡

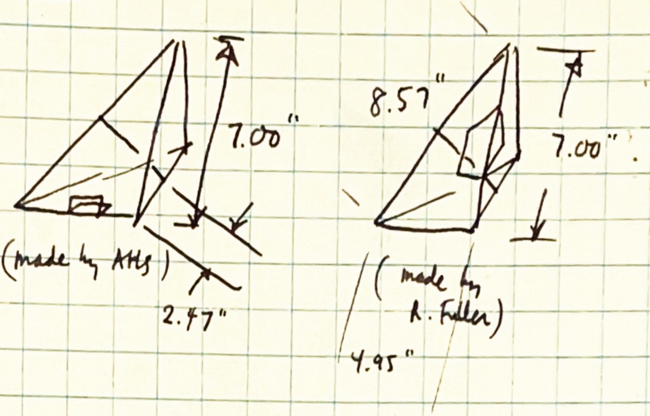
3-26-72

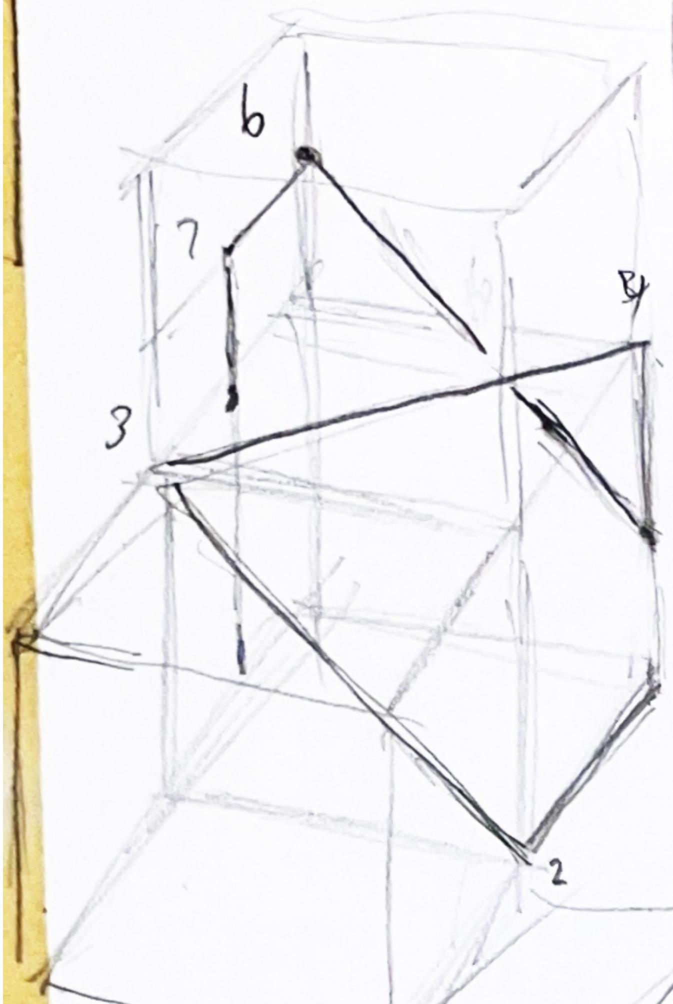
The dimensions of the 2 kaleidoscopic cells used to contain modules of new IPMS in the marathon photographing session yesterday, Sunday, with Bob Fuller & John Brennan are:

Tetragonal disphenoid:



Quadrirectangular tetrahedron:





8	1	2	3	4	5	6	7
6	5	4	3	2	1	8	7

$12 \equiv 54$

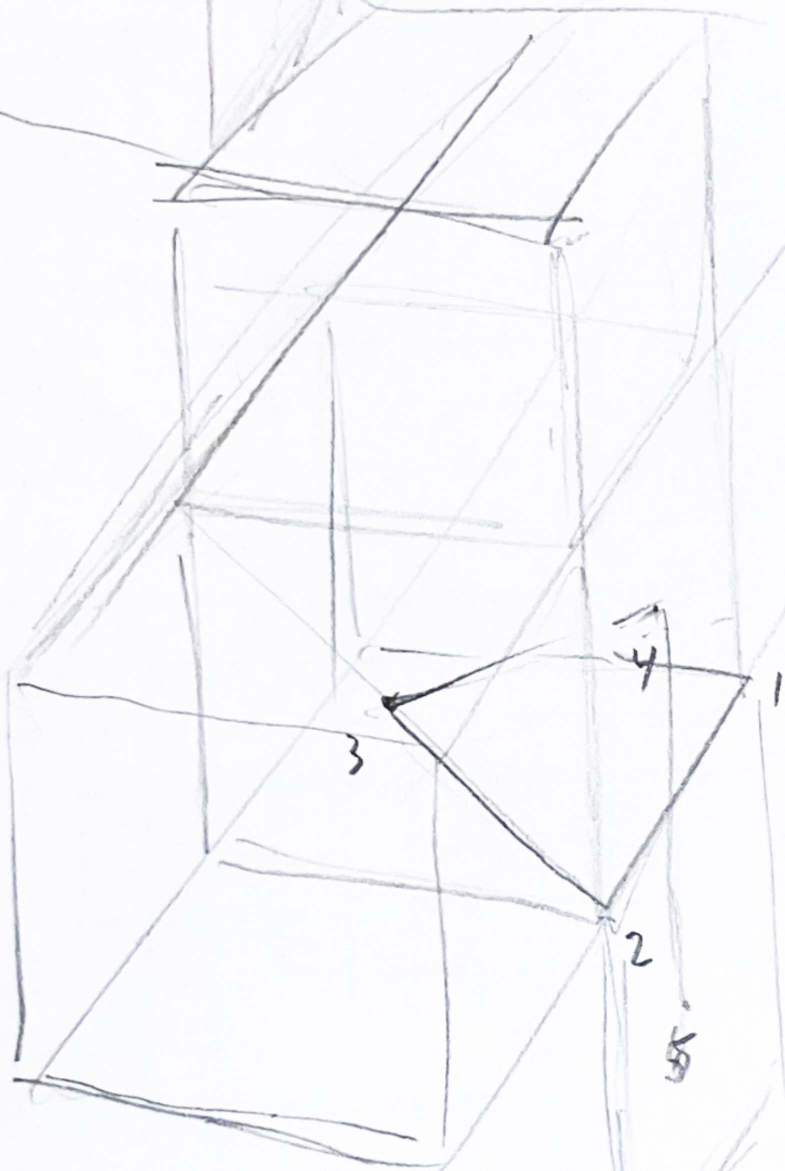
$34 \equiv 32$

$56 \equiv 18$

$78 \equiv 76$

X

12-11-02

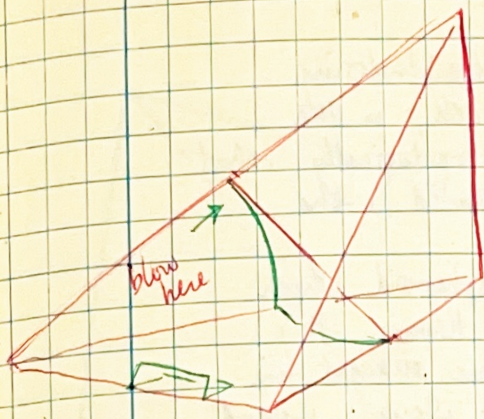


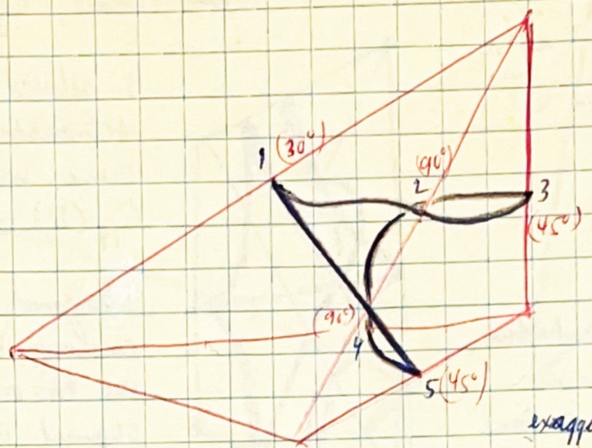
X

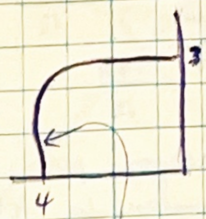
Necessarily self-intersecting boundary polygon

$C_{33}(P)$ — Sunday, April 2, 197[illegible]

Another new one* in the quadrirectangular tetrahedron. $\boxed{\text{Quite "stable"}}$







This, shown much exaggerated here, is undoubtedly curved this way.

1	30°
2	90°
3	45°
4	90°
5	45°

$\Sigma = 300° = \frac{10\pi}{6} = \frac{5\pi}{3}$ $\quad \iint_P K\,dS = -3\pi + \frac{10\pi}{6} = -\frac{8\pi}{6} = -\frac{4\pi}{3}$

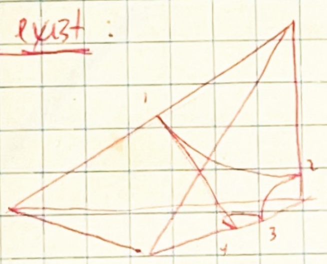
$\iint_M K\,dS = (96)\left(-\frac{4\pi}{3}\right) = -128\pi = 2\pi\chi. \quad \chi = -64 \quad \frac{\chi}{2} = -32 = d_g$

$\boxed{p = 1 - d_g = 33}$

* NOT NEW: see bottom of p. 13

---

1 have found today that this simple complement of P obtained from $C_7(P)$ does exist:



1	30°
2	45°
3	90°
4	45°

$\Sigma \iota_k = 210° = \frac{7\pi}{6}$

$\iint_P K\,dA = -2\pi + \frac{7\pi}{6} = -\frac{5\pi}{6} \quad \iint_M K\,dA = 96\left(-\frac{5\pi}{6}\right) = -80\pi \quad \chi = -40$

Hence $\boxed{p = 21}$

$$\iint_P K\,dS = (2-n)\pi + \sum l_K$$
$$= -3\pi + \frac{17\pi}{12}$$
$$= \frac{-19\pi}{12}$$

$\exists$ 2 pentagons per quadrirectangular tetrahedron.

The symmetry of the surface is the same as that of P.

Hence:
$\exists$ 96 pentagons per fundamental region.

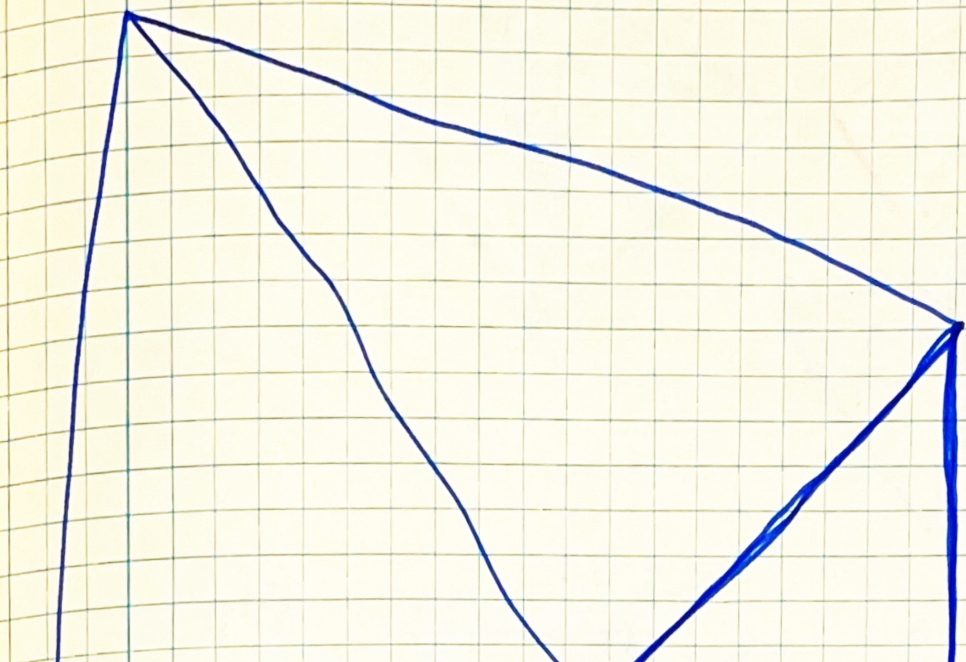
Hence $\iint_M K\,dS = (96)\left(\frac{-19\pi}{12}\right) = -152\pi$
$= 2\pi\chi$

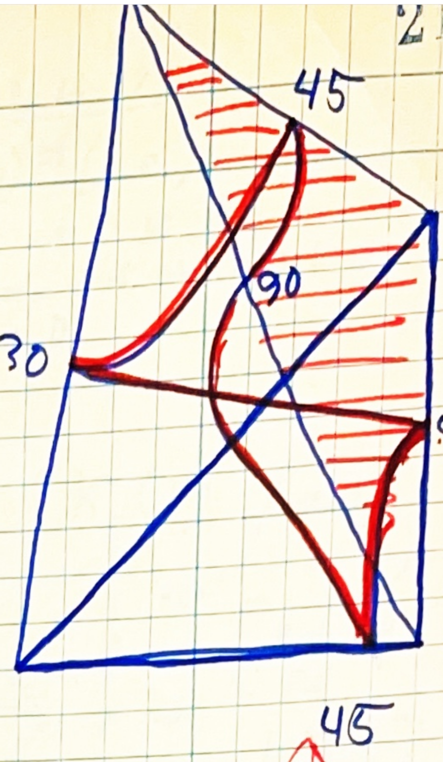
This surface, $C_{39}(P)$, is related to the enclosing tetrahedral cell, in the same way, topologically, that $C_{19}(D)$ is related to the tetragonal disphenoid which encloses it. However, this one has only one straight line segment in its interior instead of two. It is surprisingly easy to produce and hold in a quasi-stable way, but it looks like a real bear to vacuum-form, because of the unpleasantly negative draft angles.

$\therefore \chi = -76$
$p = 1 - \frac{\chi}{2} = 39$! (The record so far!)

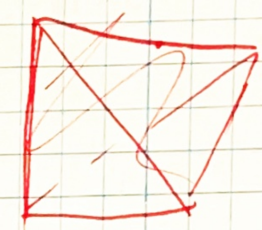
March 21, 1972 Brennan - Fuller - Schoen discovery

a new complement of P produced in quadri-rectangular tetrahedron having a thread 2-fold axis.

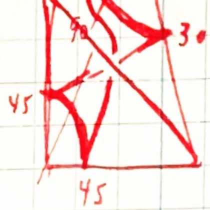




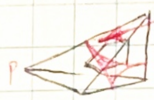
$$\begin{array}{r} 135 \\ 120 \\ \hline \end{array}$$
$$\Sigma \iota_K = 255° = \frac{17}{12}\pi$$



Mirror Image of above



Another view

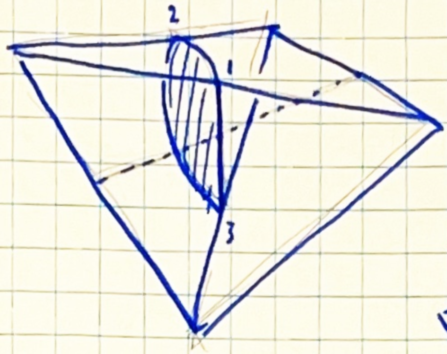


Blow Neovius toward P like this to produce $C_{39}(P)$

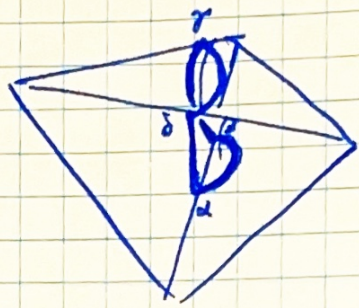
$C_{39}(P)$

<u>Sunday 3-26-72</u>

2 new K(D) * (i.e., complements of D in the restricted sense that only (100) lines remain in D



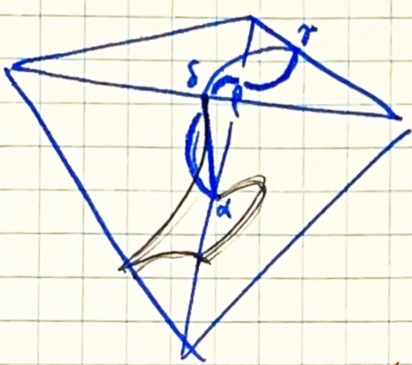
" $K_x(D)$ " (actually $x = \cancel{17}\,9$)



α 45°
β 90°
γ 60°
δ 45°

[Better call this $K'_{\cancel{17}\,9}(D)$]

$K_{\cancel{17}\,9}(D)$



α 45°
β 90°
γ = 60°
δ = 45°

* These are the <u>same</u> surface!!

The genus is not 17; it is 9

$$\iint_{\rho} K\,dA = -2\pi + \frac{4\pi}{3} = -\frac{2\pi}{3}$$

$$\iint_{M} K\,dA = 2\pi\chi = 48\left(-\frac{2\pi}{3}\right) = -32 \quad \text{Hence } \chi = -16$$

and $\boxed{p = 9}$